Competitive Cheap Talk

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April 21, 2014

Abstract

This paper studies a competitive cheap talk model with two senders. Each sender, who is responsible for a single project, only observes the return of his own project. Exactly one project will be implemented. Both senders share some common interests with the receiver, but at the same time have own project biases. Under simultaneous communication, all equilibria are shown to be partition equilibria, and the partitions of the two agents are intimately related: the interior partition points of the two agents have an alternating structure. In the most informative equilibrium, the agent with a smaller bias always has the sure option/veto power to determine which alternative is implemented and weakly more messages. Simultaneous communication, sequential communication and simple delegation are essentially all outcome equivalent. As the number of agents increases, each agent transmits more information in symmetric equilibrium.

JEL Classification Numbers: D23, D74, D82

Keywords: Cheap talk; Multiple senders; Competition

1 Introduction

Decision makers often seek advice from multiple experts. For instance, consider an economics department trying to hire a junior faculty member. The two targeted fields are, say micro theory and macro. Due to budget constraints exactly one position will be filled. In each field a single candidate is identified. The theory group of the department observes the quality of the theory candidate but not that of the macro candidate. Similarly, the macro group observes the quality of the macro candidate but not that of the theory candidate. The department chair, say a labor economist, does not observe the quality of either candidate. The chair prefers to hiring the candidate of higher quality. For each group, though they also prefer the higher quality candidate being hired, they have own-field biases: if the candidate of a group is hired that group derives an additional positive private benefit.

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†This paper combines two independently developed papers: Rantakari (2014), and Li and Yang (2013). We would like to thank Yaron Azrieli, Johannes Horner, Vijay Krishna, Jim Peck, Nicola Persico, and the audience of the Midwest Economic Theory Meetings (Michigan, Fall 2013), and the 12th Columbia/Duke/MIT/Northwestern IO Theory Conference (Duke, Fall 2013), for helpful comments and suggestions.

The above example has several distinguishing features. (i) A decision maker (DM) consults two experts regarding two alternative options (projects). (ii) The experts' interests are largely aligned with the DM's, but each expert has his own-project bias. (iii) The two experts only observe the return of his own project. (iv) The DM's action is binary (which project to adopt) and exactly one project will be adopted. The two agents are thus essentially competing with each other in having their own projects adopted. The purpose of this paper is to study communication or information transmission in the above setting, with communication being modeled as cheap talk (Crawford and Sobel, 1982, CS hereafter). The novelty of the paper is that we introduce an aspect of competition explicitly into cheap talk models with multiple senders: each sender has an extra incentive to have his own project implemented at the expense of the other sender.

Real world situations of competitive cheap talk, which share the above features, abound. For instance, consider a CEO of a firm deciding on launching one of two alternative new products (projects). The CEO consults two managers, who each are responsible for one of the two products, regarding the profitability of each product. Each manager only knows the profitability of his own product and has an extra incentive to have his own product launched. Alternatively, consider the President weighing between two alternative policies to address a particular environmental issue. The President consults two experts who each are responsible for investigating the effectiveness of one policy. Each expert only observes the effectiveness of his own policy but has an extra incentive to have his own policy adopted.

Specifically, there are two symmetric projects and the return of each project is uniformly distributed. The DM's payoff is just the return of the adopted project. Each agent's payoff has two components. The first component is the return of the adopted project. This component implies that the two experts' and the DM's interests are largely aligned: all prefer to implement a project with higher return. The second component is a private benefit: an agent receives this additional payoff if and only if his own project is adopted. We call this component the agent's own-project bias, and allow it to vary across the agents. This own-project bias creates a conflict of interests: for two projects of equal value to the DM, each agent prefers having his own project implemented. Given that exactly one project will be implemented, the own-project biases of the two agents create competition between them.

We first study a situation in which the two agents send messages simultaneously. As in standard cheap talk models, all equilibria are shown to be partition equilibria in which each agent only indicates to which interval the return of his own project belongs. Within the set of equilibria, we focus on asymmetric equilibria where the messages of the two agents can be strictly ranked according to the posterior induced and the DM will thus have a strict preference for one project over the other for all combinations of messages. The reason for this focus is two-fold. First, these equilibria are ex post equilibria, whereby the agents don't want to change their messages even after learning the message of the other agent and thus the decision induced. Second, while there also exists a sequence of symmetric equilibria where the agents send messages that have the same information content and thus ties are possible, these equilibria (i) require exact randomization by the DM to sustain them and thus are not robust to even small perturbations in beliefs and (ii) are shown to be dominated by the asymmetric equilibria at least in a subset of cases.

The first main result of the paper is that the equilibrium information transmissions of the two agents are intimately related. In particular, in equilibrium the messages of the two agents must exhibit an alternating ranking structure: for any message belonging to one agent, the two messages of adjacent rankings must belong to the other agent. Correspondingly, the two agents' interior partition points also have an alternating or staggering feature: one agent's partition point must be neighbored by two partition points of the other agent. As a result, in equilibrium the two agents have either the same number of distinct messages, or the number of messages differs by one. This implies that the amount of (meaningful) information transmitted by the two agents cannot be too far apart. Moreover, if one agent's bias decreases, then both agents will transmit more information in the most informative equilibrium. Thus in some sense the two agents' information transmissions are strategic complements. The underlying reason for these features is as follows. The DM's problem is to select the better project to implement. Thus it is the comparison of the two projects' returns that matters. If one agent transmits much more information than the other agent does, then some information transmitted by the first agent will be wasted as it cannot improve the DM's decision making. When one agent's bias decreases, this agent will naturally transmit more information, and this also allows the other agent to transmit more (meaningful) information.

Within the full set of messages used by the two agents, the lowest message has the feature that it guarantees a rejection against all recommendations by the other agent. Similarly, the highest message guarantees acceptance against all recommendations by the other agent. We will call these lowest and highest overall messages as the give-up option and the sure option, respectively. Given that the rest of the communication equilibrium responds to the allocation of these options, there are four qualitatively different equilibria, as determined by the allocation of these two messages among the two agents. We will call an equilibrium with agent i having the give-up option and agent j having the sure option an $i\mathrm{G}j\mathrm{S}$ equilibrium.

Our second set of results examines how the give-up and sure options should be allocated among the two agents to maximize the informativeness of communication and thus the DM's expected payoff. We begin by considering the case where the agents' private benefit consists of a multiplicative component only. In this case, the agents' interests become perfectly aligned as their alternatives become worthless. Therefore, the allocation of the give-up option does not matter in equilibrium. The sure option, on the other hand, should always be allocated to the less biased agent. The reason is that the less biased agent will be more conservative in exercising the sure option, which directly benefits the DM and further helps the more biased agent to also be more conservative in his recommendations due to the complementarity identified above.

We then consider the case where the agents' private benefit consists of an additive component only. The allocation of the give-up option will matter as well now, because there is no point where the agents would agree on the value of a given project. The first result is that allocating the give-up option to the less biased agent will lead to weakly more equilibrium messages, because it maximizes the use of the give-up option and thus benefits the rest of the communication equilibrium due to its recursive structure. The second result is that, other things equal, the sure option should be allocated to the less biased agent. This is because making the sure option more precise is more important since it is more likely to be exercised in equilibrium than the give-up option.[1] Based on these two results, the DM would like to allocate both the sure and the give-up options to the less biased agent ($1G1S$ equilibrium). However, because the equilibrium can sustain only a finite number of messages, allocating the give-up option to one agent may necessitate allocating the sure option to the other agent in the equilibrium that maximizes the number of distinct messages. In other words, allocating both options to the less biased agent may reduce the number of messages by one. The resolution of this tradeoff is as follows. The less biased agent should always have the sure option. This is because the sure option is more important in determining the overall informativeness of communication. Given that the less biased agent has the sure option, the give-up option will then be allocated to either agent to maximize the number of equilibrium messages.

Thus, only $1G1S$ equilibria and $2G1S$ equilibria can arise as the most informative equilibrium. This implies that in the most informative equilibrium the less biased agent has weakly more unique messages. Further, noting that having the sure option is equivalent to having veto power to prevent the implementation of a project, each agent would naturally prefer having the sure option for themselves as it increases the likelihood of having their alternative implemented. The DM, however, always prefers allocating this option to the less biased agent. And thus the less biased agent is always better off relative to the more biased agent. Finally, an interesting feature worth emphasizing is that the most informative equilibrium might not be the equilibrium with the maximum number of messages. In particular, the most informative equilibrium could be an $1G1S$ equilibrium while an $1G2S$ equilibrium with one more message exists.

We also study quasi-symmetric mixed strategy equilibrium (QSMSE), in which the two agents have the same set of messages and the DM implements both projects with strictly positive probabilities whenever there is a tie. In QSMSE, the give-up option and sure option are allocated randomly. When the two agents' biases are sufficiently similar, then QSMSE can never improve on the best asymmetric equilibrium. When the two agents' biases are enough apart, however, QSMSE can generate a higher expected payoff to the DM than the most informative pure strategy equilibrium.[2] This implies that sometimes it is beneficial to give the more biased agent some authority/veto power through randomization.

We then ask the following comparative statics question: fixing the combined bias of the two agents, will the DM be better or worse off when the two agents' biases become relatively more unequal? Intuitively, the bias(es) of the agent(s) who hold(s) the sure and give-up options are most important for determining the equilibrium quality of communication. Thus, in the case of multiplicative bias, where only the sure option matters, reducing the bias of the agent with the sure option is relatively more important. And since the sure option is always allocated to the less biased agent, asymmetry in biases thus improves expected performance. In the case of additive bias, asymmetry unambiguously improves the performance under the $1G1S$ equilibrium, where

[1] More precisely, the efficiency loss is increasing and convex in partition sizes. Thus, reducing the size of the largest partition (that of the sure option) is the dominant concern.

[2] In particular, randomization cannot improve the outcome when $1G1S$ is optimal, but for some cases can improve upon $2G1S$ equilibrium.

both options are allocated to the less biased agent. The result is ambiguous, however, in the $2G1S$ equilibrium: on one hand, the use of the sure option is improved, but at the same time the use of the give-up option is worsened, and the comparison can go either way. Finally, because the DM's payoff depends on both the combined bias and the distribution of the biases between the two agents, the DM's payoff in the most informative equilibrium may increase even when the combined bias increases.

Having analyzed the case of simultaneous talk, we then consider both sequential communication, where the two agents send public messages in a sequence, and simple delegation, under which the DM delegates the decision right to one of the agents. Here, we first establish an outcome-equivalence between simultaneous and sequential communication. The rough intuition behind this result is that, even under simultaneous talk, when the marginal type of one agent decides which message to send, he conditions his choice on the other agent's message having adjacent rankings and thus his choice of message actually being consequential for the final outcome. This implies that, under sequential talk, the second agent's ability to directly condition his message on the first agent's message does not matter. Second, delegation is essentially equivalent to sequential talk. The reason is that there exists an equilibrium in the sequential talk setting where the DM always follows the recommendation of the second agent, which in turn is equivalent to the agent having the sure option in the case of simultaneous communication. Therefore, simultaneous talk, sequential talk, and simple delegation are all outcome equivalent in terms of the most informative equilibrium. This is quite surprising, as in other cheap talk models delegation, sequential talk, and simultaneous talk usually lead to different equilibrium outcomes. Following the results under simultaneous talk, the DM always prefer delegating the decision rights to the less biased agent.

We conclude by considering the case with more than two agents. We simplify the setting to consider symmetric agents with the same bias, and the resulting symmetric communication equilibrium. We show that as the number of agents increases, each agent transmits more information, suggesting that more intense competition among agents leads to more information transmission. Intuitively, with more agents it is more likely that there is at least one agent whose project has a higher return. This means that the cost of sending a higher message increases for each agent, which reduces each agent's incentive to exaggerate the return of his own project.

This paper is related to the growing literature on cheap talk with multiple senders. For some models (Gilligan and Krehbiel, 1989; Epstein 1998; Krishna and Morgan, 2001a, 2001b; Li, 2010), the state space is one dimensional and both senders perfectly observe the same realized state. In Austen-Smith (1993), senders receive correlated (conditionally independent) signals regarding the state.[3] The main differences to this literature are two-fold. First, in our model the two senders observe non-overlapping private information (each only observes the return of his own project), which makes the cross-checking of recommendations in hopes of inducing more precise information transmission impossible, a topic which has been the main focus of the above literature. Second,

[3] This line of inquiry is extended in Battaglini (2002) and Ambrus and Takahashi (2008), who study multidimensional cheap talk models with multiple senders. In both models, each sender observes the realized states in all dimensions and the decision is a two-dimensional vector. In this setting, full information revelation can be typically achieved in equilibrium.

the binary nature of the final decision introduces an explicit element of competition that is absent in the other models and leads to different interactions between the sources of information.[4]

Hori (2006) and Yang and McGee (2013) study cheap talk models in which two senders have partial and non-overlapping private information, and where the receiver's action space is one-dimensional but continuous. Alonso et al. (2008), Rantakari (2008), and Yang (2013) study models of coordinated adaptation where the need for communication arises from the need to coordinate decisions across different senders. Hagenbach and Koessler (2010) and Galeotti et al. (2013) study strategic communication in network, where the need for communication again arises from the value of coordination and each agent is a sender and a receiver at the same time. Highlighting the differential reasons for communication, the coordination models exhibit either independence or substitutability across the sources of information.

In a two-stage auction setting, Quint and Hendricks (2013) model the first stage indicative bidding as a cheap talk game. The two bidders who send the highest messages will be selected by the seller (receiver) to advance to the second stage of auction. In some sense, bidders in the first stage are competing with each other for the two spots in the second stage through cheap talk, an aspect closely related to our paper. The most important difference is that in their model there is only pure conflict of interests among the bidders (senders), while in our setting senders have some common interests as they care about the quality of the adopted project.

Our paper is the first paper that studies a general model of competitive cheap talk. In an extension, Rantakari (2014) considers how uncertainty over the agents' biases affects the allocation of the sure option when the bias is multiplicative. Rantakari (2013a) considers the effects of allowing the receiver/principal to investigate the proposals after the cheap talk stage and Rantakari (2013b) considers how the level of conflict arises endogenously through incentive contracts if the agents need to be motivated to generate the alternatives in the first place.

This paper is also related to "comparative" cheap talk (Chakraboty and Harbaugh, 2007, 2010; Che et al., 2013). In those models, a single expert observes the realized returns of multiple projects, and makes recommendation to the receiver, who then makes decision about which project to implement. Under certain conditions, Chakraboty and Harbaugh (2007, 2010) show that some information can be credibly transmitted by the expert by making comparative statements. Focusing on asymmetric projects, Che et al. (2013) find that pandering is possible: the expert sometimes might recommend a "conditionally better-looking" project whose realized return is lower than that of the other project. Our paper is related to these papers in that the receiver's action is binary (which project to implement),[5] but we consider the complementary problem, where instead of a singe agent ranking multiple projects, multiple agents advocate for their own alternatives.

The rest of the paper is organized as follows. Section 2 sets up the model and offers some

[4] For example, in Krishna and Morgan (2001a) the competition between two senders is implicit in that the receiver can combine the information transmitted by both senders and fine tune his action continuously, as his action space is continuous.

[5] Jindapon and Oyarzun (2013) also study a one-sender cheap talk model in which the receiver takes a binary action as to whether to accept a good recommended by the sender. The sender has two possible types, honest or biased, and his type is unobservable to the receiver.

preliminary analysis. In Section 3 we study simultaneous communication with asymmetric agents, both for the case of multiplicative bias and the case of additive bias. Section 4 studies sequential communication and simple delegation, and the case of more than two agents is investigated in Section 5. Section 6 offers conclusions and discussions. All the proofs can be found in the Appendix.

2 Model and Preliminary Analysis

Consider a principal or a decision maker (DM) who is facing a choice between two alternative projects. The return of project i, $i = 1, 2$, is θ_i, which is uniformly distributed on $[0, 1]$. We assume that θ_1 and θ_2 are independent from each other. There are two agents, with each agent i being responsible for investigating project i. The realization of θ_i is only observed by agent i.[6] The DM has to adopt exactly one project. Adopting both projects is not feasible, which could be due to budget or technological constraints.[7] In short, the two projects will be competing for implementation. Adopting neither project is not an option either.[8]

In the basic model we consider one round of simultaneous and non-mediated communication, where the two agents send messages to the DM who then makes the final decision. Denote agent i's message as m_i. After hearing messages m_1 and m_2, the DM decides which project to adopt. Let $d \in \{1, 2\}$ be the DM's decision, with $d = i$ indicating that project i is adopted.

Given the project choice d, the DM's payoff is $U_P(d) = \theta_d$. The DM thus cares only about the return on the implemented project. Agent i's payoff, on the other hand, is given by

$$U_i(d) = \begin{cases} \theta_d & \text{if } d \neq i \\ c_i\theta_d + b_i & \text{if } d = i \end{cases},$$

where $c_i \geq 1$ and $b_i \in [0, 1)$ capture agent i's own project bias, which we allow to arise both from a fixed benefit b_i and a multiplicative benefit c_i. There is thus some alignment in the interests among the DM and the two agents: all of them care about the return to the adopted project and want to choose the project with a higher return, other things equal. Each agent, however, has a bias to have his own project adopted, with this bias increasing in both c_i and b_i. In particular, given θ_i, agent i's private benefit of implementing project i is $(c_i - 1)\theta_i + b_i$. If $b_i = 0$ and $c_i = 1$ for both agents, then the agents' interests would be perfectly aligned with both the DM and each other. We allow the biases to differ between the agents and to ensure that some information can be transmitted in equilibrium, we further assume that $b_1 \leq b_2 < 1/2$.[9] Both b_i and c_i, $i = 1, 2$, are common knowledge. All players are expected utility maximizers.

[6] This feature that different agents observe different information is understudied in the cheap talk literature. It is reasonable due to specialization in the modern world: in organizations such as firms and governments, different divisions (groups) specialize in different functional areas.

[7] For example, integrating two product improvements in the same design may be technologically infeasible.

[8] In Section 6 we will discuss what will happen if there is a third option of implementing neither project.

[9] In particular, this condition implies that if one agent babbles then it is possible for the other agent to transmit some information.

There are multiple potential sources for an agent to have either a fixed bias and/or a value-dependent bias in favor of his alternative. For instance, consider the hiring example. An individual proposing a particular hire may receive both a fixed benefit $b_i > 0$ from the particular individual, independent of his quality, and also benefit disproportionately more from the overall talent level of the individual (e.g. a microeconomics group may derive some fixed benefit from just having an additional microeconomist but then also benefit disproportionately more from the quality of the microeconomist relative to the macroeconomics group). Alternatively, the multiplicative element can arise if the two agents are responsible for two separate divisions or units of a firm, and their compensation contracts have a division-level component in addition to a firm-level component (equivalently, if each agent is only compensated based on firm-level performance, $c_i = 1$). Similarly, the additive component can arise through career concerns, where the acceptance of a proposal may be positive news regarding an individual's ability, whether in firms or in public office, or the manager whose project is chosen is likely to be the one who will carry out the project, which can bring private benefits.

Under simultaneous communication, a strategy for agent i then specifies a message m_i for each θ_i, which is denoted as the communication rule $\mu_i(m_i|\theta_i)$. A strategy for the DM specifies an action d for each message pair (m_1, m_2), which is denoted as decision rule $d(m_1, m_2)$. Let the belief function $g(\theta_1, \theta_2|m_1, m_2)$ be the DM's posterior beliefs on θ_1 and θ_2 after hearing messages m_1 and m_2. Since θ_1 and θ_2 are independent and agent i observes only θ_i, the belief function can be decomposed into distinct belief functions $g_1(\theta_1|m_1)$ and $g_2(\theta_2|m_2)$.

Our solution concept is Perfect Bayesian Equilibrium (PBE), which requires:

(i) Given the DM's decision rule $d(m_1, m_2)$ and agent j's communication rule $\mu_j(m_j|\theta_j)$, for each i, agent i's communication rule $\mu_i(m_i|\theta_i)$ is optimal.

(ii) The DM's decision rule $d(m_1, m_2)$ is optimal given beliefs $g_1(\theta_1|m_1)$ and $g_2(\theta_2|m_2)$.

(iii) The belief functions $g_i(\theta_i|m_i)$ are derived from the agents' communication rules $\mu_i(m_i|\theta_i)$ according to Bayes rule whenever possible.

Given the two agents' strategies, the DM's optimal decision is just to implement the project that has a higher expected return. That is, the optimal decision can be written as

$$d(m_1, m_2) = \begin{cases} i & \text{if } E[\theta_i|m_i] > E[\theta_j|m_j] \\ j & \text{if } E[\theta_i|m_i] < E[\theta_j|m_j] \\ i \text{ or } j & \text{if } E[\theta_i|m_i] = E[\theta_j|m_j] \end{cases} . \tag{1}$$

And the DM's expected (interim) payoff given m_1 and m_2 is given by $E[U_p(m_1, m_2)] = \max\{E[\theta_1|m_1], E[\theta_2|m_2]\}$.

Let $\{m_{i,n}\}$ be a set of messages for agent i. Given a message pair $(m_{i,n}, m_{j,n'})$, denote $\Pr(d = i|m_{i,n}, m_{j,n'})$ as the probability that project i is implemented. Note that the DM's decision rule is embodied in $\Pr(d = i|m_{i,n}, m_{j,n'})$. Denote $\Pr(d = i|m_{i,n})$ as the probability that project i is implemented if agent i sends message $m_{i,n}$, and $E(\theta_j|d = j, m_{i,n})$ as the expected return of project

j given that agent i sends message $m_{i,n}$ but project j is implemented. By these definitions, we have

$$\begin{aligned} \Pr(d &= i|m_{i,n}) = \sum_{n'} \Pr(m_{j,n'}) \Pr(d = i|m_{i,n}, m_{j,n'}), \\ E\left(\theta_j|d = j, m_{i,n}\right) &= \sum_{n'} \Pr(m_{j,n'}) \frac{\Pr(d = j|m_{i,n}, m_{j,n'})}{\Pr(d = j|m_{i,n})} E\left(\theta_j|m_{j,n'}\right). \end{aligned}$$

Finally, before considering the exact equilibrium communication outcome, let us establish the structure of potential communication equilibria in this game. It turns out that, as in CS, all PBE are interval equilibria. Specifically, the state space $[0, 1]$ is partitioned into intervals and agent i only reveals to which interval θ_i belongs.

Proposition 1 *All PBE in the simultaneous communication game must be interval equilibria.*

Intuitively, the single-crossing condition is satisfied in the present setting because the value of inducing acceptance is increasing in the value of the agent's alternative. In short, because of the own project bias, each agent tries to overstate the return of his own project to some extent. The benefit of overstating, say by agent 1, is that agent 1's project will more likely be implemented and thus agent 1 is more likely to reap the private benefit. On the other hand, there is a cost of overstating: overstating by agent 1 reduces the probability that agent 2's project will be implemented, which might have a higher return. Consider two different types of agent 1 reporting as the same (higher) type. Compared to the lower type, the overstating of the higher type involves a smaller cost. This is simply because a higher type project 1 is more likely to be the better project than a given project 2. Therefore, a higher type of agent 1 will try to induce a higher posterior, which implies that all PBE must be interval equilibrium. Therefore, while the language itself is indeterminate (as in any cheap talk game), we can order the messages and interpret them in terms of the strength of the claim in favor of a given alternative, and thus read the messages as claims to the alternative being "poor," "mediocre," "good," "fantastic," and so on.

3 Equilibrium communication

Having established that all communication equilibria of the game must be interval equilibria, let N_i be the number of intervals in the partition, and $a_i = (a_{i,0}, a_{i,1}, ..., a_{i,N_i})$ be the partition points, for agent i. Given the state space, note that $a_{i,0} = 0$ and $a_{i,N_i} = 1$. Agent i sends message $m_{i,n}$ if $\theta_i \in [a_{i,n-1}, a_{i,n}]$. For most of this section, we rule out the possibility that $E\left(\theta_i|m_{i,n}\right) = E(\theta_j|m_{j,n'})$ for any (n, n'). That is, no pair of messages by the two agents induces exactly the same posterior to the DM. This is generic when $c_i \neq c_j$ and/or $b_i \neq b_j$, so that the biases and thus the credibility of the two agents differ and so similar claims by the two agents will generally have different informational content. For example, if two agents both claim that their projects are "great," it may be natural for the DM to discount the claim of the more biased agent more heavily and thus choose the alternative of the less biased agent. However, as we will see shortly, because the content of the messages in terms of $E(\theta_i|m_{i,n})$ depends partly on how the message is interpreted, multiple equilibria will be sustainable.[10]

To establish the nature of equilibrium communication, we will first offer two definitions.

Definition 1 *Two messages of agent i are said to be outcome equivalent if, regardless of the message sent by agent j, sending either of the two messages always leads to the same outcome as to which project is implemented. A set of messages of agent i is said to be irreducible if any pair of messages in the set are not outcome equivalent.*

We will mainly focus on the sets of messages that are irreducible, since adding additional outcome-equivalent messages will not affect the outcome (unless introducing reducible messages makes the analysis easier). Second, recall that for all possible messages (for both players) associated with an equilibrium the DM's induced posteriors can be strictly ranked. A particular ranking structure is described in the following definition.

Definition 2 *A set of messages is said to have an alternating ranking structure between two agents if (i) the messages having the highest, the 3rd highest, the 5th highest, and so on, posteriors belong to agent i, and (ii) the messages having the 2nd highest, the 4th highest, the 6th highest, and so on, posteriors belong to agent j.*

The following lemma shows the relationship between irreducible sets of messages and the alternating ranking structure, which is the key behind the structure of equilibrium communication.

Lemma 1 *If a set of messages is irreducible, then (i) it must exhibit an alternating ranking structure, and (ii) the number of messages used by each agent can differ at most by one.*

To establish this Lemma, suppose that agent j has N_j messages, with induced posteriors $E(\theta_j|m_{j,1}) < ... < E(\theta_j|m_{j,N_j})$, while agent i has N_i messages, with induced posteriors $E(\theta_i|m_{i,1}) < ... < E(\theta_i|m_{i,N_i})$. Now, if for any two messages, n and $n+1$, it is the case that $E(\theta_j|m_{j,n'}) < E(\theta_i|m_{i,n}) < E(\theta_i|m_{i,n+1}) < E(\theta_j|m_{j,n'+1})$ (agent i's two messages have consecutive overall rankings), then the messages $m_{i,n}$ and $m_{i,n+1}$ are outcome-equivalent and can be combined into one message: both induce acceptance (of project i) against all messages $m_j \leq m_{j,n'}$ while conceding against all messages $m_j \geq m_{j,n'+1}$. Second, because of this alternating ranking structure, it is immediate that the number of messages used by each agent can differ at most by one. In other words, when considering the irreducible set of messages, either $N_i = N_j$ or $N_i = N_j \pm 1$.

The key implication of Lemma 1 is that the amount of meaningful information transmission by the two agents is intimately related, a result which follows from the observation that the key piece of information for the DM is the comparison of the two projects' returns. In particular, the amount of meaningful information transmission by the two agents cannot be "too far apart," in the sense that the number of meaningful messages used by two agents can at most differ by one. To illustrate this implication, consider an extreme case in which agent 1 has no bias ($c_1 = 1$, $b_1 = 0$) and agent

[10] We will discuss the case where $E(\theta_i|m_{i,n}) = E(\theta_j|m_{j,n'})$ is possible in Subsection 3.1, 3.5, and Section 5.

2 has a very large bias ($b_2 > 1$). In this case, agent 1 will fully reveal his information and agent 2 will reveal no information (babble). Note that although agent 1 fully reveals his information, given that agent 2 reveals no information, his information cannot be fully utilized by the DM in decision making. Actually, the amount of information of agent 1 that can be utilized in decision making is at most a two-element partition: whether θ_1 is below 1/2 (the unconditional mean of θ_2), or above 1/2. If agent 2 reveals more information (say has N messages), then the meaningful amount of information that can be transmitted by agent 1 increases as well (has $N+1$ messages).

To solve for the most informative communication equilibrium, note first that the alternating ranking structure implies that one of the agents will have the lowest overall message. If this message is sent by the agent, then his alternative will never be implemented regardless of the other agent's message. For this reason, we call the lowest overall message the "give-up option." Correspondingly, one of the agents will have the highest overall message, and if sent, this guarantees the agent's project will be implemented for sure. We thus call the highest overall message the "sure option." When considering the optimal communication strategy, the allocation of these options will be crucial, and we will return to them in more detail below. For now, note that their allocation will depend on the number of messages used by each agent. In particular, if $N_i = N_j + 1$, then agent i will have both the sure and the give-up options. Conversely, if $N_i = N_j$, then the two options are split between the agents.

Next, we write a type θ_i of agent i's expected payoff from sending message $m_{i,n}$ as

$$EU_i(\theta_i, m_{i,n}) = \sum_{n' \in N_j} \Pr\left(m_{j,n'}\right)\left[\Pr(d = i | m_{i,n}, m_{j,n'})\left(c_i\theta_i + b_i\right) + \Pr(d = j | m_{i,n}, m_{j,n'}) E(\theta_j | m_{j,n'})\right].$$

Let the give-up option be allocated to agent i. That is, $E(\theta_i | m_{i,1}) < E(\theta_j | m_{j,1})$. Given the alternating ranking structure, $E(\theta_i | m_{i,n}) < E(\theta_j | m_{j,n})$, which, by the DM's optimal decision rule, implies that $\Pr(d = i | m_{i,n}, m_{j,n'}) = 1$ for $n' < n$ and $\Pr(d = i | m_{i,n}, m_{j,n'}) = 0$ for $n' \geq n$. Then, we can write the indifference condition that defines the (interior) partition point $a_{i,n}$, $1 \leq n < N_i$, between messages $m_{i,n}$ and $m_{i,n+1}$ as

$$\Pr(m_{j,n})\left[(c_i a_{i,n} + b_i) - E(\theta_j | m_{j,n})\right] = 0 \Leftrightarrow a_{i,n} = \frac{E(\theta_j | m_{j,n}) - b_i}{c_i}. \tag{2}$$

Intuitively, when choosing between the messages $m_{i,n}$ and $m_{i,n+1}$, the type $a_{i,n}$ of agent i knows that his choice will not matter if $m_j < m_{j,n}$, because then both messages will induce acceptance (of project i), nor when $m_j > m_{j,n}$, because then both messages will lead to rejection. Thus, agent i knows that his choice of message will be pivotal only when agent j sends exactly $m_{j,n}$ (the message whose overall ranking lies between $m_{i,n}$ and $m_{i,n+1}$), and optimizes his response to that. The same logic then applies to the type $a_{j,n}$ of agent j, with the exception that since $E(\theta_i | m_{i,n}) < E(\theta_j | m_{j,n})$, his choice between $m_{j,n}$ and $m_{j,n+1}$ matters only against $m_{i,n+1}$. Therefore, his (interior) partition point $a_{j,n}$, $1 \leq n < N_j$, satisfies

$$a_{j,n} = \frac{E\left(\theta_i | m_{i,n+1}\right) - b_j}{c_j}. \tag{3}$$

Since agent i is allocated the give-up option, we can then apply (2) and (3) recursively to solve for the difference equations that define the communication equilibria. In particular, for interior partition points (the meaning of interior will be made precise later),

$$\text{agent } i \quad : \quad \left(a_{i,n+1} - a_{i,n}\right) = \left(a_{i,n} - a_{i,n-1}\right) + 4\left(c_i c_j - 1\right) a_{i,n} + 4\left(b_j + c_j b_i\right), \tag{4}$$

$$\text{agent } j \quad : \quad \left(a_{j,n+1} - a_{j,n}\right) = \left(a_{j,n} - a_{j,n-1}\right) + 4\left(c_i c_j - 1\right) a_{j,n} + 4\left(b_i + c_i b_j\right). \tag{5}$$

As in CS, the interior elements of the partition thus grow in size to counter the agents' incentives to push for their own alternative. The differences to the standard CS solution are two-fold. First, the rate at which the intervals of each agent grow depend on the bias of both agents. The reason is that if agent j becomes more biased, $E\left(\theta_j | m_{j,k}\right)$ will decrease because he starts to push more aggressively for his alternative. But since $E\left(\theta_j | m_{j,k}\right)$ decreases, that will lower the cost of exaggeration for agent i as well, lowering $E\left(\theta_i | m_{i,k}\right)$. Second, the boundary elements do not follow (4) or (5). For agent i, who has the give-up option, the first partition point $a_{i,1}$ satisfies

$$a_{i,1} = \frac{a_{j,1} - 2b_i}{2c_i}.$$

Similarly, for the agent with the sure option, his largest interior partition point satisfies

$$\text{If agent } i \text{ has the sure option:} \qquad a_{i,N_i-1} = \frac{\left(a_{j,N_j-1} + 1\right) - 2b_i}{2c_i},$$

$$\text{If agent } j \text{ has the sure option} \quad : \quad a_{j,N_j-1} = \frac{\left(a_{i,N_i-1} + 1\right) - 2b_j}{2c_j}.$$

Now we clarify the interior elements of the partition that satisfy (4) or (5). If agent i has the sure option, then (4) holds for $2 \leq n \leq N_i - 2$, and (5) holds for $1 \leq n \leq N_j - 1$. If agent j has the sure option, then (4) holds for $2 \leq n \leq N_i - 1$, and (5) holds for $1 \leq n \leq N_j - 2$.

Example 1 *Suppose* $c_1 = c_2 = 1$*,* $b_1 = 0.02$*, and* $b_2 = 0.05$*. Figure 1 illustrates an equilibrium with agent 1 having the give-up option and the agent 2 having the sure option, with each agent having 3 distinct messages.*

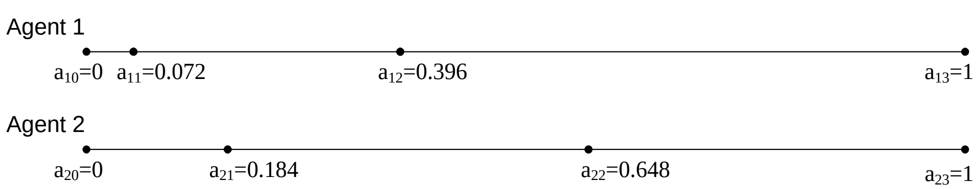

Figure 1: Asymmetric Equilibrium

We conclude this section with three observations. First, for the agent with the sure option, his highest message may be more precise than his second highest message. This is embodied in Example 1: the size of agent 2's 3rd interval is 0.352, which is smaller than the size of his 2nd interval, 0.464.[11] This observation differs from standard cheap talk models (where the precision of messages is always decreasing in the direction of agents' biases) and arises because of the competitive nature of cheap talk: for the highest marginal type of the agent with the sure option, the indifference condition relates only to the size of the highest element of the *other* agent, not his own.

Second, Example 1 illustrates how the alternating ranking structure of the two agents' messages implies that the agents' partition points have the following alternating or staggering feature: for any interior partition points $a_{i,n}$, it must be neighbored by the two partition points of the other agent. That is, we either have $a_{i,n} \in (a_{j,n-1}, a_{j,n})$ for all n, or we have $a_{i,n} \in (a_{j,n}, a_{j,n+1})$. This pattern holds generally. To see this, note that $a_{i,1} < a_{j,1}$ since agent i has the give-up option. Now, type $a_{j,1}$ of agent j's indifference condition (3) implies that $a_{j,1} < a_{i,2}$. Applying the indifference conditions (2) and (3) recursively, we have $... < a_{j,n-1} < a_{i,n} < a_{j,n} < a_{i,n+1} < ....$

Third, there are four different ways of allocating the give-up and sure options among the two agents, which lead then to four different types of equilibria. We call equilibria in which agent i has both the give-up option and the sure option as *iGiS equilibria*, and equilibria in which agent i has the give-up option and agent j has the sure option as iGjS *equilibria*. Given the alternating ranking structure, in iGiS equilibria agent i has one more message than agent j, $N_i = N_j + 1$, and the total number of messages is odd. In iGjS equilibria, the two agents have the same number of messages, $N_i = N_j$, and the total number of messages is even. Sometimes we use the terminology AiG equilibria, which includes both iGiS equilibria and iGjS equilibria, as in both cases agent i has the give-up option. The following proposition summarizes the results we derived so far.

Proposition 2 *There are four types of equilibria. In 1G1S equilibria, $N_1 = N_2 + 1$, and in 1G2S equilibria, $N_1 = N_2$; and in both types of equilibria, two agents' partitions have the following staggering feature: $a_{1,n} \in (a_{2,n-1}, a_{2,n})$ for all interior n, and $a_{2,n} \in (a_{1,n}, a_{1,n+1})$ for all interior n. In 2G1S equilibria, $N_1 = N_2$, and in 2G2S equilibria, $N_2 = N_1 + 1$; and in both types of equilibria, two agents' partitions have the following staggering feature: $a_{1,n} \in (a_{2,n}, a_{2,n+1})$ for all interior n, and $a_{2,n} \in (a_{1,n-1}, a_{1,n})$ for all interior n.*

The key part of the analysis is then to consider which type of equilibrium maximizes the principal's expected payoff. But before considering the expected performance, we will introduce an outcome-equivalent communication equilibrium that is easier to analyze in terms of the relevant difference equations.

3.1 Quasi-symmetric equilibria

First, define *quasi-symmetric (pure strategy) equilibria* (QSE) as follows:

[11] It can also be verified that this exception applies only to the highest message of the agent with the sure option.

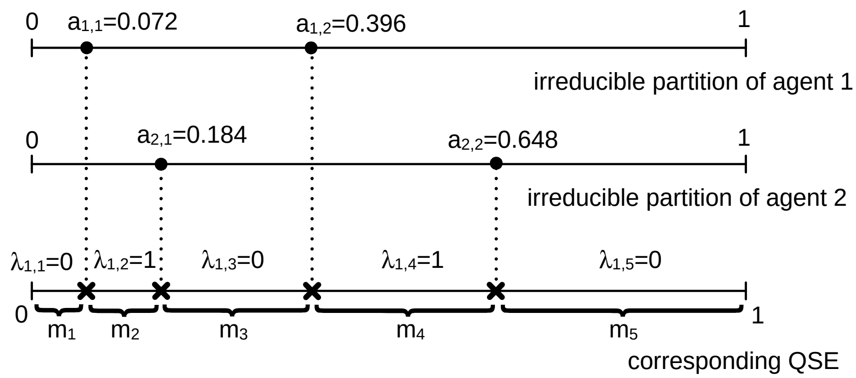

Figure 2: The equivalence between QSE and irreducible partitions.

Definition 3 *QSE are equilibria with the following properties: two agents have the same partition (hence the same set of messages) and the DM implements one of the projects with probability 1 whenever two agents send the same message.*

These equilibria thus involve both agents having the same partition, and we let $N \geq 2$ be the number of elements in the partition and $\{a_n\}$ be the partition points. In the case that both agents send the same message m_n (there is a tie), denote the probability that agent i's project is adopted as $\lambda_{i,n} \in \{0,1\}$.

The relationship between QSE and regular equilibria with irreducible message sets (and how to construct the QSE from irreducible messages) is illustrated in Figure 2 (Example 1). First, we overlay the cutoffs of the irreducible partitions of the two agents. This leads to a new partition with cutoffs $[0, a_{1,1}, a_{2,1}, a_{1,2}, ..., 1]$. Second, we give each agent access to the full set of messages generated by this new partition. For now, assume that the agents use the new message set truthfully. Then, from the perspective of performance it is clear that the choice by the DM can be wrong only when the agents send the same message.[12]

Third, we need to replicate the outcomes under the original irreducible messages. We do this as follows. First, note that the lowest and the highest messages of the QSE are equivalent to the regions over which the agents exercised their give-up and sure options of the irreducible message set, and we thus set $\lambda_{i,n}$ to match those outcomes. Second, for all the interior messages, the QSE partition splits the original messages in two. In particular, agent i used to send message $m_{i,k}$ for $\theta_i \in [a_{i,k-1}, a_{i,k}]$, which then induced acceptance against $\theta_j \leq a_{j,k-1}$ while leading to rejection against $\theta_j > a_{j,k-1}$. Now, the agent will have two messages for the same region, with $m_n \in [a_{i,k-1}, a_{j,k-1}]$ and $m_{n+1} \in [a_{j,k-1}, a_{i,k}]$. To replicate the outcome (and thus the incentive-compatibility) of the original partition, we need to make the use of these two messages outcome-equivalent to the original single message. We achieve this by setting $\lambda_{i,n} = 1$, so that in case of a tie on the lower message, agent i's alternative is chosen, while having $\lambda_{i,n+1} = 0$, so that in case of a tie on the higher message, agent j's alternative is chosen. This procedure then replicates the

[12] In particular, following the receipt of same messages, we have that $E(\theta_i|m_i) = E(\theta_j|m_j)$, and so choosing either alternative incurs an expected loss of $E[\max(\theta_i, \theta_j)|m_i, m_j] - E(\theta_i|m_i)$. Conversely, when $m_i \neq m_j$, it is known with probability one which alternative is better.

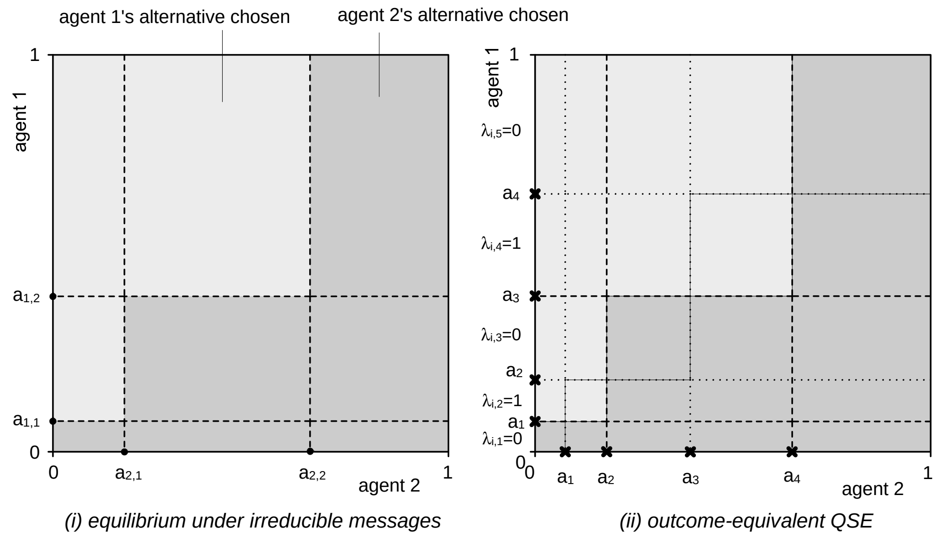

Figure 3: Outcome-equivalence between QSE and irreducible messages

original outcome of acceptance against $\theta_j \leq a_{j,k-1}$ while rejection against $\theta_j > a_{j,k-1}$. Repeating this procedure over all original messages, an outcome-equivalent QSE must exhibit *alternatingly favored tie-breaking rule:* for any n, if $\lambda_{i,n} = 0$ then $\lambda_{i,n+1} = 1$, and if $\lambda_{i,n} = 1$ then $\lambda_{i,n+1} = 0$.[13]

This equivalence is further illustrated in Figure 3, which graphs the equilibrium outcome for all state realizations under the equilibrium under irreducible messages and the corresponding QSE. In panel (ii), the diagonal is highlighted to point out that it is only in these areas that the DM may make the wrong choice, and how the alternating tie-breaking rule restores the original outcome.

Note that the alternatingly favored tie-breaking rule for QSE is parallel to the alternating ranking structure for regular equilibria. For QSE, $\lambda_{i,1} = 0$ means that agent i has the give-up option, and $\lambda_{i,N} = 1$ means that agent i has the sure option. Thus we can classify QSE according to which agent has the give-up option and which agent has the sure option, and we adopt the same terminology, iGiS and iGjS, as those for the regular equilibria.

Suppose agent i has the give-up option. By the alternatingly favored tie-breaking rule, for odd n, $\lambda_{i,n} = 0$, and for even n, $\lambda_{i,n} = 1$. Under these tie-breaking rules, note that for any odd (interior) partition points, agent j's indifference condition is always satisfied, while for any even (interior) partition points agent i's indifference condition is always satisfied. The intuition is quite simple: with the tie-breaking rule, for odd n, agent j's messages m_n and m_{n+1} have the same overall ranking and thus are outcome equivalent. And for even n, agent i's messages m_n and m_{n+1} have the same overall ranking and thus are outcome equivalent.

Now there are two sets of equilibrium conditions left: agent i's indifference conditions at odd (interior) partition points, and agent j's indifference conditions at even (interior) partition points. They can be explicitly written as

[13] Actually, it can be shown that in any QSE the tie-breaking rule must be alternatingly favored, as otherwise agents' incentive compatibility conditions cannot be satisfied.

$$\text{For } n \text{ odd (agent } i \text{ binding):} \qquad a_n = \frac{E(\theta_j | m_n, m_{n+1}) - b_i}{c_i}, \tag{6}$$

$$\text{For } n \text{ even (agent } j \text{ binding):} \qquad a_n = \frac{E(\theta_i | m_n, m_{n+1}) - b_j}{c_j}.$$

The above conditions can be simplified as

$$\text{For } n \text{ odd (agent } i \text{ binding):} \quad (a_{n+1} - a_n) - (a_n - a_{n-1}) = 2(c_i - 1) a_n + 2b_i, \tag{7}$$

$$\text{For } n \text{ even (agent } j \text{ binding):} \quad (a_{n+1} - a_n) - (a_n - a_{n-1}) = 2(c_j - 1) a_n + 2b_j.$$

As an example of the equivalence, consider an iGjS equilibrium with $N_i = N_j = \widetilde{N}$ (as in the figure). This equilibrium is then equivalent to an iGjS QSE with $2N - 1$ messages. To see this, consider the partition points a' of the iGjS QSE. Combine all the outcome equivalent messages of the QSE, and we get partition points (a_i, a_j), which consists of a regular iGjS equilibrium.[14] This is because the indifference conditions (2) and (3) for the asymmetric equilibrium are exactly the same as (6) for QSE, after rearranging the numbering of messages.

To analyze the optimal allocation of the sure and give-up options among the two agents, we need to solve for the DM's expected payoffs under the different arrangements. Unfortunately, the difference equation for the general case is not tractable, and we will thus illustrate the results for two particular cases: (i) $c_i, c_j > 1, b_i = b_j = 0$ and (ii) $c_i = c_j = 1$, $b_i, b_j > 0$. We thus simplify the analysis to consider the performance under pure multiplicative bias and pure additive bias.

3.2 Multiplicative bias

We will first consider the case of pure multiplicative bias. The key element in this case is that as θ_i goes to 0, agent i's own-project bias in absolute terms (or private benefit) $(c_i - 1)\theta_i$ goes to 0 as well. As a result, agent i is willing to tell the truth when θ_i approaches 0, and thus the give-up option has no bite. This also implies that in the most informative equilibrium the number of distinct messages is infinite (will be verified later). Due to this feature, it is more convenient to arrange the partition points in a decreasing order. Specifically, let $a_i = \{a_{i,n}\}$ be a sequence of partition points of agent i, and $a_{i,n}$ is strictly decreasing in n, with $a_{i,0} = 1$ and $a_{i,N_i} = 0$. Suppose agent i has the sure option. By (4) or (5), the indifference conditions for the partition points can

[14] In the other direction, if we combine all interior points of a_i and a_j in the asymmetric equilibrium and rearrange them into an increasing sequence, then we arrive at the partition points a' for the QSE.

be written as

$$
\begin{aligned}
(a_{j,n-1} - a_{j,n}) - (a_{j,n} - a_{j,n+1}) &= 4(c_j c_i - 1)a_{j,n} \text{ for } n \geq 1, \\
(a_{i,n-1} - a_{i,n}) - (a_{i,n} - a_{i,n+1}) &= 4(c_j c_i - 1)a_{i,n} \text{ for } n \geq 2, \\
a_{i,1} &= \frac{1 + a_{j,1}}{2c_i}.
\end{aligned} \tag{8}
$$

The difference equation of (8) has a similar form as in Alonso et al. (2008) and Rantakari (2008). The most informative equilibrium, which will be our focus, has a countably infinite number of partition elements. Let $\varphi \equiv \frac{1}{c_j c_i - 1}$ and $\alpha(\varphi) = \frac{\varphi}{\left(1+\sqrt{1+\varphi}\right)^2}$. The solutions to the above difference equations can be computed as:

$$
\begin{aligned}
a_{j,n} &= [\alpha(\varphi)]^n, \\
a_{i,n} &= \frac{[\alpha(\varphi)]^{n-1}[1 + \alpha(\varphi)]}{2c_i}.
\end{aligned} \tag{9}
$$

The key observation is that the main determinant behind the precision of communication by both agents is the relative conflict between them, $c_j c_i$. Indeed, for the agent without the sure option, it is the lone determinant, while for the agent with the sure option there is an additional direct effect of his bias, c_i. As $c_j c_i \to 1$, communication becomes perfect, while as $c_j c_i \to \infty$, communication becomes fully uninformative.

Let a superscript iS denote agent i having the sure option. The principal's expected payoff can then be derived as follows.

$$
\begin{aligned}
E(U_P^{iS}) &= \sum_{n=1}^{\infty} \Pr(m_j^n)\{\Pr(\theta_i \geq a_{i,n})E(\theta_i | \theta_i \geq a_{i,n}) + \Pr(\theta_i < a_{i,n})E\left(\theta_j | m_j^n\right)\} \\
&= \sum_{n=1}^{\infty} \Pr(m_j^n)\left[\frac{1}{2} + \frac{1}{4c_i}\left[1 - \frac{1}{2c_i}\right][\alpha(\varphi)]^{2(n-1)}(1 + \alpha(\varphi))^2\right] \\
&= 1 + \frac{c_j c_i}{4c_j c_i - 1}\left[\frac{2c_i - 1}{c_i^2}\right] \qquad (10)
\end{aligned}
$$

By (10), it can be readily seen that $E(U_P^{iS}) > E(U_P^{jS})$ if and only if $c_i < c_j$. It follows that the sure option should be given to the less biased agent. The following proposition summarizes the above analysis.

Proposition 3 *With pure multiplicative biases, to maximize the principal's expected equilibrium payoff, the agent with a smaller bias should have the sure option.*

To understand the intuition behind Proposition 3, first observe that, no matter who has the sure option, the equilibrium partition for the agent without the sure option is always the same, because the precision of communication depended only on the relative conflict, $c_i c_j$. On the other hand, when the less biased agent has the sure option, his equilibrium partition points are higher than the more biased agent's equilibrium partition points when the more biased agent has the sure option,[15] since the less biased agent has a smaller incentive to exaggerate for a given precision of communication by the other agent. Thus, the first case leads to a more even partition as the elements of the partition grow with the state, and so the principal gets a higher expected payoff when the less biased agent has the sure option.

3.3 Additive bias

We assume that $b_1 < b_2$, so that agent 1 is less biased. In contrast to the multiplicative bias, the case of additive bias has no point of congruence and thus no fully revealing messages. In particular, the allocation of the give-up option will now be meaningful as it will influence the recursion that results from the first partition element. To solve for the DM's expected payoff, we invoke the equivalence between regular equilibria and QSE: an iGjS equilibrium with $N+1$ messages is equivalent to an iGjS QSE with N messages, and an iGiS equilibrium with $N+1$ messages is equivalent to an iGiS QSE with N messages.

Suppose agent i is allocated the give-up option. With pure additive biases, the difference equations (7) that characterize QSE can be simplified as

$$\begin{aligned} &\text{For } n \text{ odd (agent } i \text{ binding):} \quad (a_{n+1} - a_n) - (a_n - a_{n-1}) = 2b_i, \\ &\text{For } n \text{ even (agent } j \text{ binding):} \quad (a_{n+1} - a_n) - (a_n - a_{n-1}) = 2b_j. \end{aligned} \tag{11}$$

Denote $\overline{N}^{iGiS}$ and $\overline{N}^{iGjS}$ as the maximum numbers of messages for iGiS QSE and iGjS QSE, respectively. And let $\overline{N}^{AiG}$ be the maximum number of messages for AiG QSE. That is, $\overline{N}^{AiG} = \max\{\overline{N}^{iGiS}, \overline{N}^{iGjS}\}$. Denote the equilibrium expected payoff of the DM as $E(U_p(N))$, where N is the number of messages in QSE. In the appendix we solve the difference equations (11), and the expected payoffs can be computed as

$$E(U_p^{iGiS}(N)) = \frac{2}{3} - \frac{1}{6N^2} - \frac{\frac{(b_i+b_j)^2}{4}N^2 - (b_i+b_j)^2 + 3b_i^2}{6}, \; N \text{ even}, \tag{12}$$

$$\begin{aligned} E(U_p^{iGjS}(N)) &= \frac{2}{3} - \frac{(1+(b_i-b_j))\left[b_i^2+b_j^2\right] - 4b_ib_j}{12} \\ &\quad - \frac{1-(b_i-b_j)}{24}\left[\frac{(2+(b_i-b_j))^2}{N^2} + (b_i+b_j)^2N^2\right], \; N \text{ odd}. \end{aligned} \tag{13}$$

To understand the expressions of (12) and (13), note that the first term of $E(U_p)$, 2/3, is the expectation of the first order statistic of two random variables that are uniformly distributed on

[15] Again see the expression of $a_{i,n}$ in (9).

[0, 1]. That is, 2/3 is the expected payoff the DM can get if both agents fully reveal their private information. The last two terms reflect the payoff loss or inefficiency when two agents do not fully reveal information. Under asymmetric equilibria, the efficiency loss arises because two agents' adjacent intervals overlap: when two agents send two adjacent messages, the project with a lower return might get implemented. QSE, by partitioning the adjacent elements under the asymmetric equilibria more finely, highlights that the efficiency loss under QSE occurs only when the agents send the same message: if the agents send different messages, then the alternative with a higher return is implemented with probability one. However, when the agents send the same message, given that the principal will implement one given project with probability one, the project with a lower actual return might be implemented.

From the expressions of (12) and (13), it is easy to verify that the equilibrium expected payoff of the DM is increasing in N (for $N < \overline{N}^{AiG}$), and decreasing in both b_i and b_j. Intuitively, when the partition becomes finer, the probability that two agents send the same message decreases, which decreases the probability that the wrong project is implemented. If the bias b_i or b_j decreases but the number of intervals remain the same, the intervals will be of more even size. A more even partition will reduce efficiency loss, since the probability that two agents send the same message is not only increasing, but also convex in the length of the intervals.[16]

Our focus will be on the most informative equilibrium that maximizes the DM's expected payoff. The next lemma compares different types of equilibria.

Lemma 2 *(i) Giving the less biased agent, agent 1, the give-up option leads to weakly more equilibrium partitions:* $\overline{N}^{1G1S} \geq \overline{N}^{2G2S}$, $\overline{N}^{1G2S} \geq \overline{N}^{2G1S}$, *and* $\overline{N}^{A2G} \leq \overline{N}^{A1G} \leq \overline{N}^{A2G} + 1$. *(ii) For equilibria with the same number of partitions, an* $1G1S$ *equilibrium is more informative than a* $2G2S$ *equilibrium. (iii) For equilibria with the same number of partitions, a* $2G1S$ *equilibrium is more informative than an* $1G2S$ *equilibrium. (iv) If a 2G1S equilibrium with* $2N + 1$ *elements does not exist but an 1G2S equilibrium with* $2N + 1$ *elements exists, then an 1G1S equilibrium with* $2N$ *elements is more informative than an 1G2S equilibrium with* $2N + 1$ *elements.*

To understand the intuition behind Lemma 2, we compare the patterns of the equilibrium partition points between the two types of equilibria. Specifically, let $\{a_n\}$ and $\{a'_n\}$ be the sequences of partition points, and let the size of n^{th} element be $a_1 + \Delta_n$ and $a'_1 + \Delta'_n$ ($\Delta_1 = \Delta'_1 = 0$), for A1G QSE and A2G QSE, respectively. The term Δ_n can be interpreted as the incremental size of the nth partition element relative to the size of the first element. By the difference equations (11), for A1G QSE Δ_n follows the following pattern: 0, $2b_1$, $2b_1 + 2b_2$, $4b_1 + 2b_2$, $4b_1 + 4b_2$ ..., while for A2G QSE Δ'_n follows the following pattern: 0, $2b_2$, $2b_1 + 2b_2$, $2b_1 + 4b_2$, $4b_1 + 4b_2$ .... From these patterns we can see that, in A1G QSE b_1 enters into the incremental step size more often than b_2, while in A2G QSE it is the opposite. This implies that, compared to A2G QSE, in A1G QSE the partition sizes increase more slowly, which potentially allows more elements. These patterns also imply that, for odd n we have $\Delta_n = \Delta'_n$, and for even n we have $\Delta'_n - \Delta_n = 2(b_2 - b_1) > 0$.

[16] In a two-partition example, let the partition point be $a_1 \in (0, 1/2)$. The overall probability of tying is $(1 - a_1)^2 + a_1^2$, which decreases when a_1 increases (when two partitions become more even).

Now compare an 1G1S QSE and a 2G2S QSE with the same number of elements (N is even). By the fact that the total length of all intervals must be 1, we have $N(a_1 - a'_1) + \sum_{n=1}^{N}(\Delta_n - \Delta'_n) = 0$. Since N is even, the equation implies that $a_1 - a'_1 = b_2 - b_1 > 0$. For $1 < n < N$, we have

$$a_n - a'_n = n(a_1 - a'_1) + \sum_{j=1}^{n}(\Delta_j - \Delta'_j). \tag{14}$$

Using the fact that $a_1 - a'_1 = b_2 - b_1$ and the cyclical pattern of $\Delta_j - \Delta'_j$, we conclude that, for n odd $a_n > a'_n$, and for n even $a_n = a'_n$. Given this pattern, on average 1G1S QSE leads to a relatively more even partition, and a more even partition is less likely to lead to a wrong choice. Therefore, the 1G1S QSE results in a higher expected payoff for the DM than the 2G2S QSE.

Example 2 *Suppose $b_1 = 0.06$, and $b_2 = 0.08$. The most informative 1G1S QSE and 2G2S QSE are illustrated in Figure 4. Both equilibria have 4 elements. The partition points a_2 are the same under two equilibria, but a_1 and a_3 are bigger under the 1G1S QSE than those under the 2G2S QSE. Therefore, overall the partition under the 1G1S QSE is more even.*

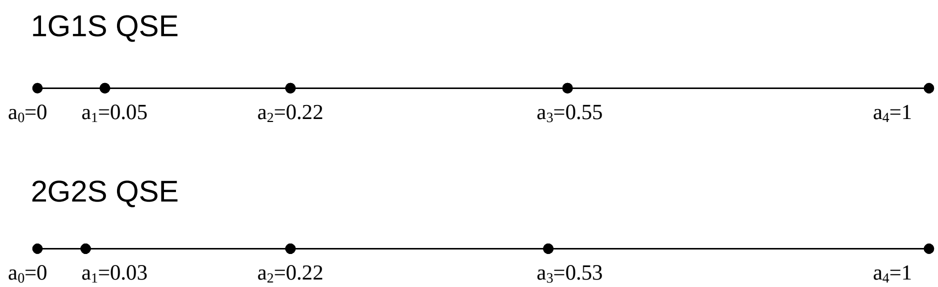

Figure 4: 1G1S QSE Have More Even Partitions

Next consider an 1G2S QSE and a 2G1S QSE with the same odd number, N, of elements. Since N is odd, $a_1 - a'_1 = (b_2 - b_1)(N - 1)/N$. By (14), we have:

$$n \text{ odd:} \qquad a_n - a'_n = n\frac{N-1}{N}(b_2 - b_1) - (n-1)(b_2 - b_1) > 0,$$
$$n \text{ even} \quad : \quad a_n - a'_n = n\frac{N-1}{N}(b_2 - b_1) - n(b_2 - b_1) < 0.$$

The above inequalities indicate the following pattern. For two adjacent elements starting with an odd element, the elements under the 1G2S QSE are more even. However, for two adjacent elements starting with an even element, the elements under the 2G1S QSE are more even. Since the total number of elements is odd, the last two elements under the 2G1S QSE are more even. And since the elements are increasing in size and the efficiency loss is increasing and convex in size, making larger elements more even is more important. Therefore, the 2G1S QSE leads to a lower efficiency loss and is more informative overall than the 1G2S QSE.

Example 3 *Suppose* $b_1 = 0.1$*, and* $b_2 = 0.16$*. The most informative 1G2S QSE and 2G1S QSE are illustrated in Figure 5. Both equilibria have 3 elements. Compared to the 1G2S QSE, for the 2G1S QSE, though the first element size is smaller, the sizes of the second and third elements are closer, which leads to more even elements overall. In particular,* $E(U_p^{2G1S}) = 0.627$ *which is greater than* $E(U_p^{1G2S}) = 0.624$*.*

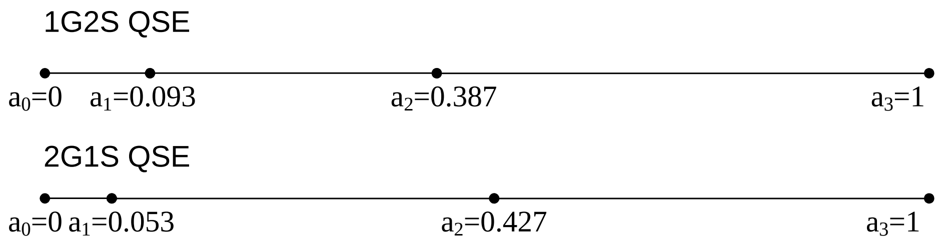

Figure 5: 1G2S QSE and 2G1S QSE

Finally, compare an 1G1S QSE with $2N$ elements and an 1G2S QSE with $2N+1$ elements, conditional on that a 2G1S QSE with $2N+1$ elements does not exist. Let $\{a_n\}$ and $\{\Delta_n\}$ be the partitions points and the incremental size of elements of the 1G1S QSE, and $\{a'_n\}$ and $\{\Delta'_n\}$ be those of the 1G2S QSE. Now compare the size difference of the largest elements. By the pattern of partition mentioned earlier, we have

$$\Delta_{2N} - \Delta'_{2N+1} = (a_1 - a'_1) - 2b_2 < 0.$$

The inequality holds because $a_1 < 2b_2$, since otherwise a 2G1S QSE with $2N+1$ elements would have existed. Therefore, the largest element of the 1G1S QSE is smaller than the largest element of the 1G2S QSE. Thus, although the 1G2S QSE has one more element (one more message), the partition is relatively more even under the 1G1S QSE. Since the efficiency loss is increasing and convex in the size of element, the second effect dominates and 1G1S leads to a lower efficiency loss despite having one fewer message.

This feature is different from standard cheap talk models, in which more messages typically mean a higher expected payoff to the DM. The underlying reason for this feature is that a change from an 1G1S QSE with $2N$ elements to an 1G2S QSE with $2N+1$ elements causes a complete reshuffling of the partition points, as the sure option is switched from agent 1 to agent 2 and agent 2 has a stronger incentive to excercise the sure option. On the other hand, a change from an 1G1S QSE with $2N$ elements to a 2G1S QSE with $2N+1$ elements is smooth in the sense that the partition points move continuously. This is because adding one additional message (the give-up option) at the bottom will not change the remaining partitions points due to the recursive structure of the partition elements. This suggests that who has the sure option is more critical than who has the give-up option in affecting the DM's payoff.

Example 4 *Suppose* $b_1 = 0.1$*, and* $b_2 = 0.21$*. The most informative 1G1S QSE and 1G2S QSE are illustrated in Figure 6. The 1G1S QSE has two elements, the 1G2S QSE has three elements, and* *the 2G1S QSE with three elements does not exist. Although the 1G2S QSE has one more element (message), the size of the largest element under the 1G1S QSE* (0.6) *is smaller than that under the 1G2S QSE* (0.68)*, which means that overall the elements are more even under the 1G1S QSE. In particular,* $E(U_p^{1G1S}(2)) = 0.62$*, which is greater than* $E(U_p^{1G2S}(3)) = 0.6113$*.*

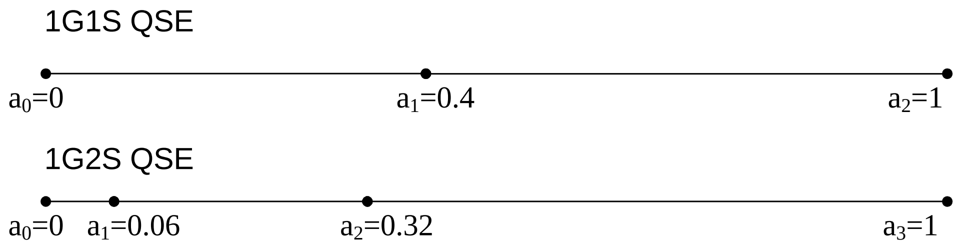

Figure 6: 1G1S QSE and 1G2S QSE

The following proposition shows that in the most informative equilibrium agent 1 always has the sure option.

Proposition 4 *(i) If* $\overline{N}^{1G1S} > \overline{N}^{2G1S}$*, then the most informative equilibrium is an 1G1S equilibrium. (ii) If* $\overline{N}^{1G1S} < \overline{N}^{2G1S}$*, then the most informative equilibrium is a 2G1S equilibrium. (iii) The most informative equilibrium will never be a 2G2S or an 1G2S equilibrium.*

Proposition 4 implies the following features in the most informative equilibrium. First, relative to the agent with a bigger bias, the agent with a smaller bias has weakly more messages.[17] Second, the agent with a smaller bias always has the sure option. Third, the give-up option could be allocated to either agent. And finally, the most informative equilibrium might not be the equilibrium that has the maximum number of messages.

The intuition for Proposition 4 is as follows. The principal would like to allocate both options to the less biased agent. By giving the give-up option to the less biased agent, the principal not only maximizes the use of the first message, as the less biased agent will be more willing to admit that his project should not be implemented, but also (indeed, because of it) maximizes the number of equilibrium messages. Conversely, by giving the sure option to the less biased agent, the principal maximizes the precision of the highest message, as the less biased agent has the least interest to guarantee the acceptance of his project. But since fixed biases imply a finite number of informative messages, sometimes giving the less biased agent both options entails the reduction of the number of messages by one, and in the equilibria that have the maximum number of messages it is infeasible for the less biased agent to have both options. That is, allocating the give-up option to agent 1 may lead to allocating the sure option to agent 2, and vice versa. Now the principal faces a tradeoff. The resolution of this tradeoff is that the less biased agent should always have the sure option, even if sometimes it means that the number of messages will be reduced by one. This is because

[17] However, in the most informative equilibrium the agent with a smaller bias could transmit less amount of information than the other agent. When the most informative equilibrium is an 2G1S equilibrium, agent 2's partitions are more even and hence he transmits more information than agent 1 (Example 1).

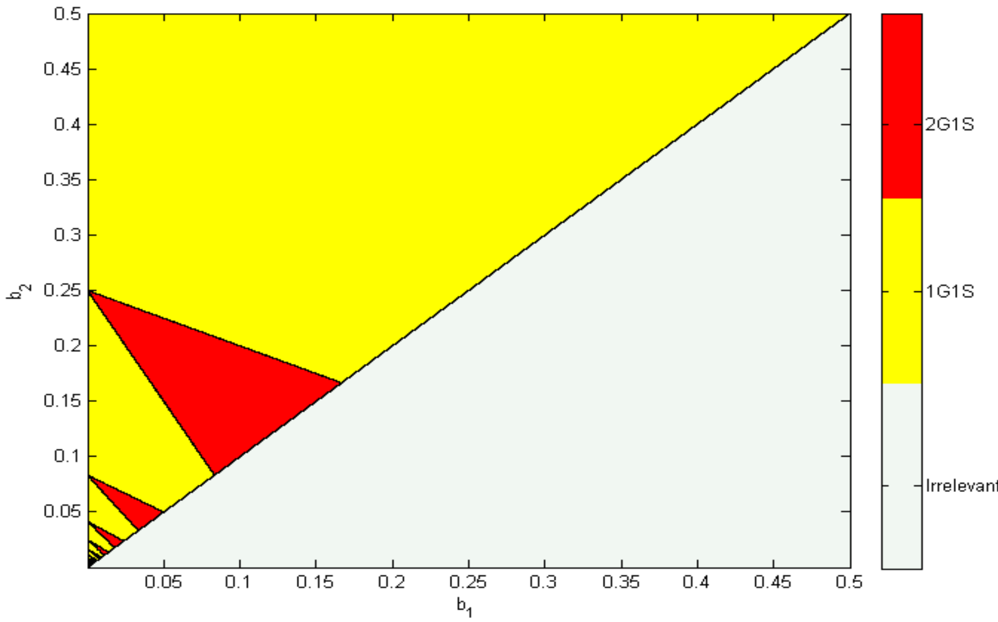

Figure 7: The Most Informative Equilibrium and the Biases

making the sure option more precise is more important in determining the overall informativeness of communication, as the efficiency loss is increasing and convex in the size of partition elements. Given that agent 1 has the sure option, who has the give-up option depends on which allocation maximizes the number of equilibrium messages.

In Figure 7, we illustrate the frequency of each type of equilibrium being the most informative equilibrium. Specifically, the yellow (red) areas are the combinations of the biases such that an 1G1S equilibrium (2G1S equilibrium) is the most informative equilibrium. Consider decreasing the biases of the two agents. The first informative equilibrium that exists is 1G1S by maximizing the number of messages. As the biases decrease, 1G2S with one more message starts to exist, but the most informative equilibrium is still 1G1S. As the biases decrease further, 1G1S is able to add a message and becomes the 2G1S equilibrium, which becomes the most informative one, and the cycle begins again.

The most informative equilibrium might not be Pareto dominant: while it is clear that the DM always prefers the most informative equilibrium, the two agents might prefer different equilibria as they also take into account the probabilities that their own projects will be implemented. The ex ante probabilities that each project will be implemented in different equilibria are characterized in the following proposition.

Proposition 5 *In any informative equilibrium, the agent with (without) the sure option has an ex ante probability strictly greater (less) than* $1/2$ *of having his own project implemented. In the most informative equilibrium, project 1 (2) will be implemented with an ex ante probability strictly greater (less) than* $1/2$.

The intuition for Proposition 5 is that having the sure option allows the agent to implement his project whenever he prefers his project over the other project, thus maximizing his expected payoff. Since in the most informative equilibrium the less biased agent always has the sure option, from the ex ante sense the project of the less biased agent is more likely to be implemented than the project of the more biased agent. Therefore, the less biased agent is always rewarded with advantage (more likely to reap private benefit) while the more biased agent is punished (less likely to reap private benefit).

3.4 Discussion

Since the most informative equilibrium might not be Pareto dominant, we cannot invoke Pareto dominance to select the most informative equilibrium. However, we argue that more informative equilibria are the more reasonable ones, based on equilibrium refinement by introducing out of equilibrium messages. The details of the equilibrium refinement can be found in the Appendix.

Among the four types of equilibria, is the DM able to implement a particular type of equilibria? The answer is yes. For example, suppose the DM wants agent 1 to have the give-up option. To achieve that, the DM can do following: if both agents send the lowest messages, then project 2 will be implemented. Knowing this, agent 1's incentive to send the lowest message is reduced (for a smaller range of θ_i) since sending this message means giving up his own project, while agent 2's incentive to send the lowest message is enhanced (for a wider range of θ_j). And this leads to $E(\theta_1|m_{1,1}) < E(\theta_2|m_{2,1})$, which means that it is optimal for the DM to implement project 2 when both agents send the lowest messages. Similarly, the DM can allocate the sure option to either agent as he wishes.

A particular feature of the equilibria described above has been that the DM always chooses one alternative over the other with probability one. In addition to these equilibria, there exists a continuum of another type of equilibria, which we label as *quasi-symmetric mixed strategy equilibria* (QSMSE). In these equilibria, the two agents continue to have the same partitions. However, when two agents send the same message, instead of always choosing one alternative over the other as in QSE, the DM will randomize between the alternatives.

While the detailed analsis can be found in the Appendix, here we report two main results. First, 1G1S QSE yields a higher expected payoff to the DM than any QSMSE with the same (even) number of partition elements. Second, 2G1S QSE dominates any QSMSE with the same (odd) number of elements if the two agents' biases are close enough. However, if the two agents' biases are too far apart, then QSMSE generates a higher expected payoff to the DM than 2G1S QSE. This implies that sometimes it is beneficial to give the more biased agent some authority/veto power through randomization.

3.5 Comparative Statics

Corollary 1 *The DM's expected payoff in the most informative equilibrium is decreasing (i) in c_i in the case of multiplicative bias, and (ii) in b_i in the case of additive bias.*

A result stronger than Corollary 1 holds: both agents will transmit more information in the most informative equilibrium if one agent's bias decreases. Thus in some sense two agents' information transmissions are strategic complements.[18] This feature is also present in the two-sender cheap talk model of McGee and Yang (2013), but for a different reason. The reason for this property to arise in the current model is again due to the competitive nature of cheap talk. Intuitively, one agent will exaggerate less and transmit more information if he has a smaller bias. But this increases the cost of exaggeration for the other agent, thus allowing him to also transmit more meaningful information.

In the rest of this subsection we study the following question: fixing the combined bias of two agents, does the DM prefer two agents having relatively equal biases or relatively unequal biases?

Proposition 6 *In the case of multiplicative bias, suppose $c_1 < c_2$. Fixing c_1c_2, in the most informative equilibrium the DM's expected payoff increases as c_1 and c_2 becomes further apart (c_1 decreases and c_2 increases).*

The proof of Proposition 6 is straightforward. By previous results, in the most informative equilibrium agent 1 always has the sure option. Observing (10), it can be easily verified that EU_p^{1S} increases when c_1c_2 remains the same but c_1 decreases. Proposition 6 indicates that, in the case of multiplicative bias the DM prefers two agents having unequal biases. To understand this result, note that agent 2's (without the sure option) equilibrium partitions only depends on the combined bias c_1c_2. However, agent 1's (who has the sure option) equilibrium partitions become more even as he becomes less biased.

In the case of additive bias, we fix $b_1 + b_2 = 2b$, and let $b_2 - b_1 = 2d$ be the difference of the biases, $0 \le d \le b$. Note that $b_2 = b + d$ and $b_1 = b - d$. As d increases, two agents' biases become further apart. We are interested in how the DM's expected payoff in the most informative equilibrium will change as d changes.

Proposition 7 *Suppose two agents' biases become further apart, or d increases. (i) The maximum number of partition elements in A1G equilibria either stays the same or increases by 1; the maximum number of partition elements in A2G equilibria either stays the same or decreases by 1. (ii) If initially the most informative equilibrium is an 1G1S equilibrium, then the DM is better off in the most informative equilibrium. (iii) If initially the most informative equilibrium is a 2G1S equilibrium, then the DM can be either better off or worse off in the most informative equilibrium.*

To understand the intuition of part (i) of Proposition 7, first consider A1G QSE. Recall that the incremental partition size Δ_n follows the following pattern: 0, $2b_1$, $2b_1+2b_2$, $4b_1+2b_2$, $4b_1+4b_2$ .... We can see that, as the two agents' biases become further apart (d increases), while the incremental partition sizes of odd number of partitions do not change, those of even number of

[18]The technical reason is that, as mentioned earlier, the incremental step size of the interior partitions for agent i is, $4(c_ic_j - 1)a_{i,n}$ in the multiplicative case, and $4b_1 + 4b_2$ in the additive case. This implies that, when one agent's bias decreases, then in the most informative equilibrium the other agent's number of partitions will weakly increase and his partitions will become more even.

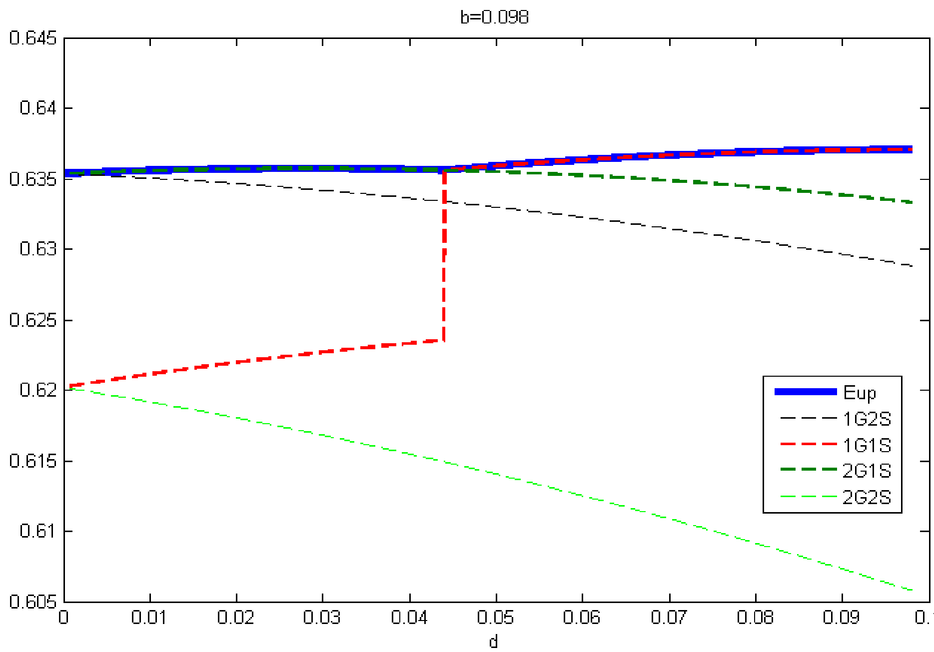

Figure 8: DM's Payoff as Biases Become More Unequal

partitions decreases since b_1 decreases. Therefore, the maximum number of partitions will either stay the same or increase by 1. Now consider A2G QSE. Recall that the incremental partition size Δ_n follows the following pattern: 0, $2b_2$, $2b_1 + 2b_2$, $2b_1 + 4b_2$, $4b_1 + 4b_2$ .... We can see that, as d increases, while the incremental partition sizes of even number of partitions do not change, those of odd number of partitions increase since b_2 increases. Therefore, the maximum number of partitions will either stay the same or decrease by 1.

The underlying reason for part (ii) of Proposition 7 is that an increase in d improves 1G1S equilibria. Recall that the DM's expected payoff increases if the two largest partitions become more even. When the total number of partitions is even, the difference between the sizes of the two largest partitions is $2b_1$. This means that an increase in d leads to overall more even partitions. However, an increase in d may improve or worsen 2G1S equilibria (fixing the number of elements).

Proposition 7 implies that in the most informative equilibrium making the two agents' biases more unequal does not always improve or reduce the DM's expected payoff: sometimes it is better for two agents to have relatively equal biases and sometimes it is the opposite. Figure 8 ($b_1+b_2 = 0.196$) illustrates the pattern. As d increases from 0 to 0.043, the most informative equilibrium is a 2G1S equilibrium, and the DM's payoff first increases then decreases. For d bigger than 0.043, the most informative equilibrium is an 1G1S equilibrium, and the DM's payoff increases with d.

Although in most cases the DM's expected payoff in the most informative equilibrium decreases with a larger total bias, it is possible that the DM's expected payoff could increase as the total bias increases, if the distribution of biases change as well. This is illustrated in the following example.

Example 5 *Suppose $b_1 = 0.151$ and $b_2 = 0.175$. The most informative equilibrium is the 1G2S*

QSE with 3 *elements (the 2G1S QSE with 3 elements does not exist), and* $E(U_p) = 0.612$*. Suppose* $b_1 = 0.154$ *and* $b_2 = 0.173$*. Note that, compared to the former case,* b_1 *increases,* b_2 *decreases, and the total bias increases. The most informative equilibrium is the 2G1S QSE with* 3 *elements (the 1G2S QSE with* 3 *elements still exists), and* $E(U_p) = 0.613$*. That is, the DM's expected payoff increases.*

4 Sequential Communication and Delegation

A specific feature of the solution derived under simultaneous communication is that the equilibrium constitutes an ex post equilibrium: neither agent wants to change their choice of message even after they learn the choice of the other agent. In this section, we will illustrate how the equilibrium outcome under simultaneous communication is equivalent to that under both sequential communication and delegation.

4.1 Sequential communication

Consider first the situation in which the two agents communicate sequentially to the DM, with the message of the first agent observed by the second agent before his choice of message. Denote agent i as the agent who moves first and agent j as the agent who moves second. A strategy for agent i specifies a message m_i for each θ_i, which is denoted as the communication rule $\mu_i(m_i|\theta_i)$. A strategy for agent j specifies a message m_j for each pair of θ_j and m_i, which is denoted $\mu_j(m_j|\theta_j, m_i)$. A strategy for the DM specifies an action d for each message pair (m_i, m_j), which is denoted as decision rule $d(m_i, m_j)$. The DM's posterior beliefs on θ_i and θ_j after hearing messages are denoted as belief functions $g_i(\theta_i|m_i)$ and $g_j(\theta_j|m_j, m_i)$.

A Perfect Bayesian Equilibrium (PBE) requires:

(i) Given the DM's decision rule $d(m_1, m_2)$ and agent j's communication rule $\mu_j(m_j|\theta_j, m_i)$, agent i's communication rule $\mu_i(m_i|\theta_i)$ is optimal.

(ii) Given the DM's decision rule $d(m_1, m_2)$, agent i's communication rule $\mu_i(m_i|\theta_i)$, and agent i message m_i, agent j's communication rule $\mu_j(m_j|\theta_j, m_i)$ is optimal.

(iii) The DM's decision rule $d(m_1, m_2)$ is optimal given beliefs $g_i(\theta_i|m_i)$ and $g_j(\theta_j|m_j, m_i)$.

(iv) The belief functions $g_i(\theta_i|m_i)$ and $g_j(\theta_j|m_j, m_i)$ are derived from the agents' communication rules $\mu_i(m_i|\theta_i)$ and $g_j(\theta_j|m_j, m_i)$ according to Bayes rule whenever possible.

Lemma 3 *In PBE the following properties hold. (i) Given any message of agent* i*,* m_i*, agent* j*, who moves second, has at most two irreducible messages. (ii) Agent* i*, who moves first, has an equilibrium strategy of interval form.*

Lemma 3 indicates that given any message sent by agent i, agent j will have at most two irreducible messages, which are equivalent to recommending his project for implementation and recommending the first project for implementation. Agent i's equilibrium strategy is still of interval form because the single-crossing condition is satisfied. In particular, compared to a lower type, a higher type of agent i will send a weakly higher message (induce a higher posterior) since with a higher type project i is more likely to be the better project.

Proposition 8 *For any equilibrium under simultaneous talk, there exists an outcome-equivalent equilibrium under sequential talk, and vice versa.*

We present the proof of Proposition 8 below. Let $\{a_{i,n}\}$ be the partition points of agent i, and he sends message $m_{i,n}$ if $\theta_i \in [a_{i,n-1}, a_{i,n}]$. Suppose that agent i has sent a message $m_{i,n}$, inducing a posterior belief $E(\theta_i|m_{i,n})$. Recall that agent j has at most two irreducible messages: recommending his own project or recommending project i. Note that agent j's message choice is outcome-relevant only if the DM listens to him. Suppose this is the case (this point will be discussed in more details later). Now, agent j will recommend his own alternative if

$$c_j\theta_j + b_j \geq E(\theta_i|m_{i,n}) \Leftrightarrow \theta_j \geq a_{j,n} = \frac{E(\theta_i|m_{i,n}) - b_j}{c_j}, \tag{15}$$

and recommend project i otherwise, where $a_{j,n}$ is the cutoff type of agent j who is indifferent between recommending two projects. Of course, if $a_{j,n} \leq 0$, then agent j essentially only has one message, which recommends project j. Similarly, if agent i sends message $m_{i,n+1}$, then agent j recommends his own project if and only if $\theta_j \geq a_{j,n+1}$. Actually, as agent i's message $m_{i,n}$ varies, $\{a_{i,n}\}$ induces a sequence of partition points $a_{j,n}$, which we label as $\{a_{j,n}\}$. Note that $\{a_{j,n}\}$ can be interpreted as unconditional partition points of agent j before agent i's choice of messages.

Next, consider agent i's incentive. Suppose agent i's type is a marginal type $a_{i,n}$. Knowing agent j's cutoff strategy, agent i anticipates that if he sends the message $m_{i,n}$, then agent j will induce the acceptance of project j if and only if $\theta_j \geq a_{j,n}$, whereas if he sends the message $m_{i,n+1}$, then agent j will induce the acceptance of project j if and only if $\theta_j \geq a_{j,n+1}$. Thus, type $a_{i,n}$'s indifference condition is given by

$$\begin{aligned} \Pr(\theta_j &\geq a_{j,n+1})E(\theta_j|\theta_j \geq a_{j,n+1}) + \Pr(\theta_j < a_{j,n+1})(c_i a_{i,n} + b_i) \\ &= \Pr(\theta_j \geq a_{j,n})E(\theta_j|\theta_j \geq a_{j,n}) + \Pr(\theta_j < a_{j,n})(c_i a_{i,n} + b_i), \end{aligned}$$

which then immediately simplifies to

$$(c_i a_{i,n} + b_i) = E(\theta_j|a_{j,n+1} > \theta_j \geq a_{j,n}) \Leftrightarrow a_{i,n} = \frac{E(\theta_j|a_{j,n+1} > \theta_j \geq a_{j,n}) - b_i}{c_i}. \tag{16}$$

Finally, given that agent j adopts cutoff strategies conditional agent i's message, $E(\theta_j|a_{j,n+1} > \theta_j \geq a_{j,n})$ is equivalent to $E(\theta_j|m_{j,n+1})$ in the case of simultaneous communication.[19] Thus, the indifference conditions of the sequential talk case, (2) and (3), are identical to the simultaneous talk case, (15) and (16), and thus we have outcome-equivalence.

[19] That is, for $\theta_j \in [a_{j,n}, \theta_{j,n+1}]$, which is the range for which agent j cares whether agent i sends $m_{i,n}$ or $m_{i,n+1}$, agent j can simply send a message $m_{j,n}$ ex ante under simultaneous talk that induces the desired outcome under sequential talk.

The difference between simultaneous talk and sequential talk is that, under sequential talk the agent who talks second can condition his message on the first agent's message, and thus only has at most two irreducible messages given the first agent's message. However, the unconditional messages of the second agent, j, under simultaneous talk, are equivalent to his conditional messages under sequential talk. Specifically, if agent i sends message $m_{i,n}$, then all messages of agent j under simultaneous talk $m_{j,k}$ ($k \leq n$) are combined to an irreducible message (recommends project i), and all messages $m_{j,k}$ ($k > n$) are combined to another irreducible message (recommends project j). This point is illustrated in the following example, which uses the same parameter values as Example 1.

Example 6 *Figure 9 illustrates an equilibrium under sequential talk (agent 1 talks first) that is equivalent to the equilibrium under simultaneous talk described in Figure 1. The dotted line indicates the posterior of* θ_1 *given agent 1's messages. When agent 1 sends the highest message, agent 2's cutoff is* a_{22} *and he recommends project 2 if and only if* $\theta_2 \geq a_{22}$*. When agent 1 sends the second highest message, agent 2's cutoff is* a_{21}*. In total, agent 2 has three unconditional messages (partitions). When agent 1 sends the highest message, agent 2's two lower unconditional messages are combined to a single conditional message inducing rejection of project 2. When agent 1 sends the second highest message, agent 2's two higher unconditional messages are combined to a single conditional message inducing acceptance.*

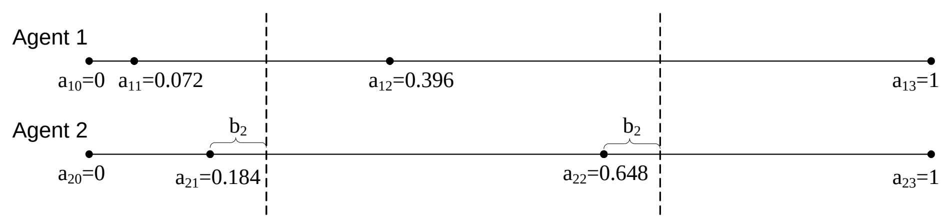

Figure 9: The Equivalence between Sequential Talk and Simultaneous Talk

The equivalence between simultaneous talk and sequential talk under competitive cheap talk is new and surprising. In other cheap talk models with multiple senders, simultaneous talk and sequential talk usually lead to different outcomes.[20]

Why the second agent's ability, under sequential talk, to condition his message on the first agent's message does not change the equilibrium outcome? The underlying reason is that, since only the comparison of two projects matters, even under simultaneous talk the agents are able to forecast when their messages will be pivotal and they thus anticipate that and choose messages accordingly. In particular, under simultaneous talk one agent's choice of two adjacent messages matters only if the other agent's message has an overall ranking lying between his two messages. In

[20] For example, in Krishna and Morgan (2001b) where two agents have symmetric opposing biases and communicate simultaneously (corresponding to open rules with heterogenous committee), full information revelation is achievable in equilibrium. However, in Krishna and Morgan (2001a) where two agents have opposing biases and communicate sequentially, full information revelation is not achievable. In a model in which agents' biases are private information, Li (2010) shows that sequential talk is superior to simultaneous communication.

other words, when one agent decides which message to send after observing his own state, he has already implicitly conditioned on that the other agent's message is pivotal or has an intermediate overall ranking between his two messages. This implies that, under sequential talk, the second agent's ability to directly condition his message on the first agent's message does not matter.

Indeed, the key feature of the equilibrium under simultaneous talk is that, *conditional on the information revealed through the messages, each agent prefers the outcome induced over any other alternative, except when they both send the their highest messages.* Because the agents can forecast the outcome, it does not matter whether they talk simultaneously or sequentially. The only source of conflict arises when both agents send their strongest recommendation in favor of their projects, and the allocation of the sure option determines which agent will have their way.

We conclude with the following observations. First, the give-up option could be allocated to either agent. Specifically, if $b_j \leq E(\theta_i|m_{i,1})$, then agent j has the give-up option. Otherwise, agent i, the first mover has the give-up option. Second, for any interior message of agent i, $m_{i,n}$, $2 \leq n \leq N-1$, agent j must have two messages and the DM always follow agent j's recommendation. To see this, suppose for message $m_{i,2}$ agent j only has one message which always leads to the acceptance of project j. Given that $E(\theta_i|m_{i,2}) > E(\theta_i|m_{i,1})$, equilibrium requires that for $m_{i,1}$ agent j only has one message as well, which always leads to the acceptance of project j. But now for agent i, messages $m_{i,1}$ and $m_{i,2}$ are outcome equivalent and can be combined to one single irreducible message. Third, if for the highest message sent by agent i, $m_{i,N}$, agent j has two messages, then agent j has the sure option.[21] Therefore, agents would like to talk second, as that gives them a higher likelihood of having their project chosen and thus a higher expected payoff. Note that organizationally, this setting is equivalent to a hierarchy. The first agent talks to the second agent, who then recommends to the principal which alternative to implement. Finally, note that while intuitively attractive, this is not the only feasible equilibrium outcome, an observation that highlights how delicately the content of communication can depend on its interpretation. Suppose that we maintain the same sequential structure, where agent i talks first to agent j, who then makes the final recommendation to the DM, but we allow for agent i access to a single message that bypasses the chain of command and is taken by the DM to be sufficiently good evidence to implement agent i's alternative, no questions asked. Now, that essentially allocates agent i the sure option, as agent j can now have his alternative implemented only when agent i chooses not to send the highest message (which in turn makes agent j send the highest message for a wider range of parameters, thus leading to the case that $E(\theta_i|m_{i,N_i}) > E\left(\theta_j|m_{j,N_j}\right) > E(\theta_i|m_{i,N_i-1})$), while agent i's incentives to send the highest message are curtailed by the fear of replacing an even better alternative by agent j, exactly as in the simultaneous talk case. Thus, it is agent i who has the sure option and the rest of the communication equilibrium adjusts accordingly. Therefore, it is fundamentally the interpretation of the messages by the DM that determines the allocation of the

[21] The DM always follows agent j's recommendation of project j if $E(\theta_i|m_{i,n}) \leq (c_j - b_j)/(2c_j - 1)$. Recall that agent j recommends his own project if and only if $\theta_j \geq a_{j,n} = \frac{E(\theta_i|m_{i,n})-b_j}{c_j}$. Now the DM's posterior about θ_j following $m_{i,n}$ and recommendation of agent j becomes $(1+a_{j,n})/2$, which is bigger than $E(\theta_i|m_{i,n})$ if and only if $E(\theta_i|m_{i,n}) \leq (c_j - b_j)/(2c_j - 1)$.

sure and give-up options, not the sequence of talk.

4.2 Delegation

We only consider the case of simple delegation (Aghion and Tirole, 1997; Dessein, 2002), where the DM delegates the decision right to one of the agents, say agent j. Since agent j cares about the quality of the project implemented, he first consults agent i regarding θ_i and then makes the decision as to which project to implement. Considering this variant of the game, we get the following proposition.

Proposition 9 *The set of equilibria under delegation is a subset of the equilibria under sequential talk. In particular, delegation to agent j is equivalent to the sequential talk equilibrium where agent j talks second and has the sure option.*

The logic behind this result is straightforward, and hinges on the binary nature of the final choice. As discussed above, under sequential talk following the message of agent i there are only two meaningful recommendations: either agent j recommends his alternative and that is implemented, or agent j recommends against his alternative, in which case the other project is implemented. In other words, because of the binary decision, following the recommendation of the agent is equivalent to fully delegating authority to that agent. This result stands in contrast to the continuous-decision setting, where the DM can continuously alter his response in relation to the preferred decision of the agent.

The reason why delegation can implement only a subset of equilibria under sequential talk is that the DM, through his interpretation of messages, can potentially lower the information revealed, while an agent cannot ignore what he already knows. For example, a DM could assume that the messages contain no information and thus choosing randomly between the two alternatives is an equilibrium outcome. In contrast, while agent j can place no weight on the messages sent to him, he cannot ignore his own private information. Thus, given authority, he will not be willing to randomize and instead will choose his own alternative when $\theta_j \geq \frac{E(\theta_i|\emptyset)-b_j}{c_j}$, where $\emptyset$ indicates babbling, and vice versa. Most importantly, this plays a role in the allocation of the give-up option. For example, the DM may allocate the give-up option to agent i simply through the interpretation of the messages, resulting in $E(\theta_i|m_{i,1}) < E(\theta_j|m_{j,1})$. However, if $\frac{E(\theta_i|m_{i,1})-b_i}{c_i} > 0$, then for $\theta_j \in [0, \frac{E(\theta_i|m_{i,1})-b_i}{c_i}]$ agent j, if granted authority, will choose to reject his alternative in favor of agent i's alternative. But when searching for the equilibrium with the highest payoff to the DM, this is irrelevant because the additional information revealed by agent j makes the delegation solution strictly dominant (intuitively, since agent j is least likely of the three parties to admit that his alternative is poor, all benefit from that revelation. Further, it helps to make rest of the communication finer).

Combining with previous results, we conclude that simultaneous talk, sequential talk, and simple delegation are essentially all equivalent, in terms of the most informative equilibrium. This result is quite surprising, as in other cheap talk models cheap talk and simple delegation in general lead to different equilibrium outcomes.[22] Moreover, the agent having the decision rights is always better off relative to the other agent. Finally, previous results imply that the DM always prefers to delegate the decision right (equivalent to giving the sure option) to the less biased agent.

5 More Than Two Agents

Now we go back to the setting of simultaneous communication, and study the situation where there are more than two agents.

5.1 K+1≥ 2 symmetric agents

Suppose there are $K+1 \geq 2$ symmetric agents with preferences $c\theta_i + b$ (so that each agent is playing against K other agents, just to simplify notation). We will focus on the symmetric communication equilibrium where the agents use the same message set and where ties are broken randomly and symmetrically among the agents. In particular, if there are k equal (and highest) claims, the DM chooses one of the alternatives with probability $1/k$. In this setting, we consider how the number of agents affects the precision of communication, with the result that increased competition increases the precision of information transmission. Thus, not only does the DM benefit from getting an additional draw from the state distribution by introducing an additional agent, but also gains in terms of the information revealed by the agents regarding the quality of their alternatives.

To derive this result, let $\{a_n\}$, $0 \leq n \leq N$, denote the cutoff points, and each agent i sends message m_n if $\theta_i \in [a_{n-1}, a_n]$. Suppose one agent's type is a_n. To construct the indifference condition for the agent in question for the choice between messages m_n and m_{n+1}, note first that the choice is again irrelevant if, among the messages sent by all other agents, there exists a message that is above m_{n+1} (no chance of acceptance), or all messages are below m_n (guaranteed acceptance). Thus, we can write the indifference condition as

$$\begin{aligned} & [E\left(\theta|m_{n+1}\right) - (ca_n + b)] \sum_{k=1}^{K} \frac{K!}{k!(K-k)!} [\Pr(m_{n+1})]^k [\Pr(m < m_{n+1})]^{K-k} \frac{1}{1+k} \\ = \quad & [(ca_n + b) - E\left(\theta|m_n\right)] \sum_{k=1}^{K} \frac{K!}{k!(K-k)!} [\Pr\left(m_n\right)]^k [\Pr\left(m < m_n\right)]^{K-k} \frac{k}{1+k}. \end{aligned} \tag{17}$$

To understand this expression, the first line is the expected cost of sending the higher message m_{n+1} (relative to sending the lower message m_n) when at least one other agent sends the higher message. In other words, by sending the lower message, the agent is guaranteed rejection, while

[22] For instance, Dessein (2002) shows that simple delegation is strictly better than cheap talk whenever informative cheap talk is feasible. In a two-sender model, which is more comparable to the current model, McGee and Yang (2013) shows that simple delegation is strictly better than simultaneous talk if two agents have like biases, and it can be better or worse than simultaneous talk if two agents have opposing biases.

sending the higher message carries a probability of $\frac{1}{1+k}$ being accepted (and thus replacing one of the other better alternatives), given a total number k of the higher messages sent. Finally, to compute the overall probability of such replacement (which is what the agent fundamentally cares about), note that the probability of having a particular set of k agents with the higher message is $[\Pr(m_{n+1})]^k[\Pr(m < m_n)]^{K-k}$, then from the set of K agents we can draw the k agents in $\frac{K!}{k!(K-k)!}$ unique combinations, and finally adding over the possible k we get the expected probability.

Similarly, the second line captures the expected gain of sending the higher message when the highest message sent by the other agents is m_n. Now, sending the higher message guarantees acceptance, while sending the lower message runs the risk of acceptance of another (worse) alternative with probability $\frac{k}{1+k}$, given a total number k of agents sending the lower message, and then adding over the possible combinations, as with the first line.

Next, simplify the notation by letting $\Phi(K, m_{n+1})$ and $\Phi(K, m_n)$ denote the expected probability of being pivotal, which allows us to write the indifference condition in a shorter form as

$$[E(\theta|m_{n+1}) - (ca_n + b)]\Phi(K, m_{n+1}) = [(ca_n + b) - E(\theta|m_n)]\Phi(K, m_n).$$

Now we make the dependence of a_n on K explicit and write a_n as $a_n(K)$. Then, note that given $a_{n+1}(K)$ and $a_{n-1}(K)$, an increase in $a_n(K)$ implies more even partitions and thus more informative communication.[23] Then, from the indifference condition it follows immediately that for two groups of agents, K and K', we have that

$$a_n(K) < a_n(K') \Leftrightarrow \frac{\Phi(K, m_n)}{\Phi(K, m_{n+1})} > \frac{\Phi(K', m_n)}{\Phi(K', m_{n+1})}.$$

In other words, an agent is more conservative in his recommendations in group K' when he is relatively more likely to be pivotal when sending the higher message. Intuitively, the agent's incentives to exaggerate are curtailed by his fear of replacing an even better alternative. The more likely such replacement becomes, relative to allowing a worse alternative being implemented, the more conservative the agent becomes.

The final step is then to consider how this expression depends on the number of participants. We can solve for the probabilities as

$$\begin{aligned}\Phi(K, m_{n+1}) &= \frac{\Pr(m \leq m_{n+1})\left[[\Pr(m \leq m_{n+1})]^K - [\Pr(m < m_{n+1})]^K\right] - K\Pr(m_{n+1})[\Pr(m < m_{n+1})]^K}{\Pr(m_{n+1})(K+1)} \\ \Phi(K, m_n) &= \frac{K\Pr(m_n)[\Pr(m \leq m_n)]^K - \Pr(m < m_n)\left[[\Pr(m \leq m_n)]^K - [\Pr(m < m_n)]^K\right]}{\Pr(m_n)(K+1)}.\end{aligned}$$

[23] In equilibrium, more even partitions implies a slower growth rate of their size and thus more even and potentially more messages.

Under the uniform distribution, these expressions further simplify to

$$\begin{aligned}\Phi(K, m_{n+1}) &= \frac{a_{n+1}\left(a_{n+1}^K - a_n^K\right) - K\left(a_{n+1} - a_n\right) a_n^K}{\left(a_{n+1} - a_n\right)(K+1)} \\ \Phi(K, m_n) &= \frac{K\left(a_n - a_{n-1}\right) a_n^K - a_{n-1}\left(a_n^K - a_{n-1}^K\right)}{\left(a_n - a_{n-1}\right)(K+1)}.\end{aligned} \tag{18}$$

And a simple manipulation of the probabilities yields the following proposition:

Proposition 10 *As the number of agents, $K+1$, increases, in symmetric equilibrium the incremental step size of partitions decreases.*

The underlying reason for Proposition 10 is as follows. Recall that the indifference condition balances the expected gain (when among the other agents the highest message is the lower message) and the expected loss (when among the other agents the highest message is the higher message) of sending the higher message (relative to sending the lower message). When the number of agents increases, other things equal, if the agent in question sends the higher message, relative to the probability of gaining (when among the other agents the highest message is the lower message), the probability of incurring loss (when among the other agents the highest message is the higher message) increases. Intuitively, more agents means it is more likely that some agents' projects are better than your own project. Therefore, increasing the number of participants increases the cost of exaggeration and thus improves the flow of information. In other words, adding an agent gives the DM a double benefit. First, it offers an additional draw from the distribution. Second, it encourages better transmission of information from the pre-existing agents.

Example 7 *Suppose $c = 0$ and $b = 0.4$. When there are two agents, the most informative symmetric equilibrium has two partitions, with partition point $a_1 = 0.1$. When there are three agents, in the two-partition equilibrium the partition point is $a_1 = 0.1572$. Clearly, when there are three agents, the incremental step size is smaller and the partitions are more even, and hence more information is transmitted by each agent.*

5.2 Asymmetric agents

Here we just briefly discuss the case of three asymmetric agents. All PBE still must be interval equilibria. In the asymmetric equilibrium, all equilibrium messages of all three agents can be ranked unambiguously according to the posteriors.[24] To make the set of messages irreducible, two messages having the consecutive overall rankings must belong to different agents. However, with three agents the messages do not need to have an alternating ranking structure (unlike in the two-agent case), whereby the overall rankings of three agents' messages have a cyclical pattern (for

[24] With three agents actively competing with each other, quasi-symmetric equilibrium defined in the two-agent case no longer exists. This is because now, at any interior partition point it is impossible to make one agent's IC binding and the other two agents indifferent at the same time by manipulating the tie-breaking rule.

example, the lowest message belongs to agent 1, the second lowest to agent 2, the third lowest to agent 3, the fourth lowest to agent 1, and so on). There are many other possibilities, as long as two messages having the consecutive overall rankings belong to different agents. For example, agent 3 babbles (only has one message), and the messages of agent 1 and agent 2 basically have an alternating ranking structure (excluding agent 3's sole message). Essentially, only agent 1 and 2 are actively competing with each other, with agent 3's project (with expected payoff 1/2) serving as an outside option. Alternatively, one can think of complicated ranking structures in which three agents' messages or partitions are intertwined. There are a few interesting questions to ask. What kind of ranking structure will emerge in the most informative equilibrium? Is it better to have only two agents competing actively or to have all three agents competing actively? To maximize the DM's payoff, should the agent who has the smallest bias continue to have the sure option? We leave this topic for future research.

6 Conclusions and Discussions

This paper studies a competitive cheap talk model in which two agents, who each is responsible for a single project, communicate with the DM before exactly one project is chosen. Both agents and the DM share some common interests, but at the same time each agent has an own project bias. We first fully characterize the equilibria under simultaneous communication. All equilibria are shown to be partition equilibria, and the partitions of two agents' are intimately related: the interior partition points of the two agents have an alternating structure and the equilibrium number of distinct messages by the two agents are either the same or differ by one. Letting the agent with the smaller bias to have the give-up option leads to more equilibrium messages while letting the agent with the smaller bias to have the sure option optimizes the exercise of veto power at the top. Thus, ideally, the principal would like to allocate both to the less biased agent. But when the communication equilibrium has only a finite number of elements, in the equilibria having maximum number of messages allocating the give-up option to one agent may necessitate allocating the sure option to the other agent, and allocating both options to the less biased agent might reduce the number of messages by one. Then, the tension is resolved as follows. The less biased agent should always have the sure option since it is more important in determining the overall informativeness of communication, and the give-up option could be allocated to either agent to maximize the number of equilibrium messages. Suprisingly, sometimes the most informative equilibrium might not be the one with the maximum number of messages. We also show that, fixing the total bias of two agents, making the biases more unequal could increase or decrease the DM's payoff in the most informative equilibrium.

We then study sequential communication and delegation and illustrate how these versions of the game are essentially outcome-equivalent to the simultaneous-talk case, in that all games have the same most informative equilibrium. The equivalence between sequential communication and delegation follows from the result that, due to the binary nature of the final decisions, there exists an equilibrium under sequential communication where the DM always follows the recommendation of the second-mover, which is equivalent to delegation. The equivalence between simultaneous and sequential talk follows from the result that even under simultaneous talk, the agents need to predict when their messages will be pivotal and condition their strategy on that, which makes the ability to see the other agent's message worthless. We also show that when the number of agents increases, the amount of information transmitted by each agent increases in the symmetric equilibrium.

Throughout the paper we have assumed that the return of each project is uniformly distributed. With more general distributions, the difference equations will not have analytical solutions, which would complicate the analysis. However, we think that majority of the results of our paper will hold qualitatively under more general distributions. Recall that, due to technical difficulty, we did not explicitly solve the general case with hybrid biases. But we believe that the general case is qualitatively similar to the case of additive bias, as in both cases there is no point of congruence and thus the number of equilibrium messages is finite and the give-up option has bite. It is also encouraging that the basic conclusions following from the additive and multiplicative cases are qualitatively very similar.

In the paper we also assumed that exactly one project will be implemented. In some situations, it is reasonable to think that there is an outside option under which neither project is implemented. If the DM chooses the outside option, then neither agent gets private benefit. Depending on the return to the outside option, an agent may either always prefer to implement his project over the outside option, or prefer the outside option for sufficently low values, which will influence the information content of the lowest messages sent by each agent. Apart from the lowest message, if either agent sends higher messages then the DM will definitely not adopt the outside option. In other words, starting from the second lowest messages two agents are competing with each other to have his own project implemented, which is essentially the same as the basic model. From this discussion, we can see that adding an outside option would not qualitatively change the existing results much. The main effect will be (if the acceptance constraint is binding under the original equilibrium) to worsen the amount of information transmission by making the initial messages less precise. Finally, it is also interesting to study the case in which two projects are asymmetric or their returns have different distributions. We leave this for future research.

Appendix

Proof of Proposition 1.

Proof. Consider any decision rule $\Pr(d = i|m_{i,n}, m_{j,n'})$. Then, knowing the decision rule and the communication strategy of the other agent j, agent i's expected payoff, given his type θ_i and message $m_{i,n}$, can be written as

$$E_{\theta_j}[U_i|\theta_i, m_{i,n}] = \Pr(d = i|m_{i,n})(c_i\theta_i + b_i) + \Pr(d = j|m_{i,n})E\left(\theta_j|d = j, m_{i,n}\right).$$

Order the messages so that $n > n' \Leftrightarrow \Pr(d = i|m_{i,n}) > \Pr(d = i|m_{i,n'})$. That is, a higher message of agent i means project i will be implemented with a higher probability. Then, we have that

$$\frac{d\left(E_{\theta_j}[U_i|\theta_i, m_{i,n}] - E_{\theta_j}[U_i|\theta_i, m_{i,n'}]\right)}{d\theta_i} = \left[\Pr(d = i|m_{i,n}) - \Pr(d = i|m_{i,n'})\right] c_i > 0.$$

This implies that a higher type of θ_i has an incentive to send a higher message, or the single-crossing condition is satisfied. Therefore, the only feasible equilibria are interval equilibria. ■

Characterization of QSE.

For iGiS QSE (note that the number of partitions N is even), solving the difference equations of (11), we get

$$a_1 = \frac{1}{N} - \frac{N}{2}b_i - (\frac{N}{2} - 1)b_j.$$

Then $\overline{N}^{iGiS}$ is the largest even integer N that satisfies the following inequality:

$$\frac{N^2}{2}b_i + N(\frac{N}{2} - 1)b_j < 1. \tag{19}$$

Similarly, for iGjS QSE (note that N is odd) the difference equations of (11) yield

$$a_1 = \frac{2 - (N-1)(N+1)b_i - (N-1)^2 b_j}{2N}. \tag{20}$$

Then $\overline{N}^{iGjS}$ is the largest odd integer N that satisfies the following inequality

$$\frac{(N-1)(N+1)}{2}b_i + \frac{(N-1)^2}{2}b_j < 1. \tag{21}$$

The equilibrium expected payoff of the DM, $E(U_p(N))$, can be written as:

$$E(U_p^{AiG}(N)) = \frac{1}{2}\sum_{n=1}^{N}[(a_n - a_{n-1})a_n(a_n + a_{n-1}) + (a_n - a_{n-1})(1 - a_n^2)],$$

which can be explicitly calculated as (12) and (13).

Proof of Lemma 2.

Proof. We prove the results in terms of QSE, and let N be the number of partition elements.

Part (i). Inspecting (19), for the same even N we can see that the LHS of the inequality is larger under an A2G QSE than under an A1G QSE, since $b_1 < b_2$. Thus, $\overline{N}^{1G1S} \geq \overline{N}^{2G2S}$. By (21), the same pattern holds for odd N, and hence $\overline{N}^{1G2S} \geq \overline{N}^{2G1S}$. Therefore, we must have $\overline{N}^{A2G} \leq \overline{N}^{A1G}$. To show that $\overline{N}^{A1G} \leq \overline{N}^{A2G} + 1$, first consider the case that N is even. Note that the LHS of (19) of an 1G1S equilibrium with N is larger than the LHS of (21) of a 2G1S equilibrium with $N-1$. Thus, $\overline{N}^{1G1S} \leq \overline{N}^{2G1S} + 1$. When N is odd, it can be verified that the LHS of (21) of an 1G2S equilibrium with N is larger than the LHS of (19) of a 2G2S equilibrium with $N-1$. Thus, $\overline{N}^{1G2S} \leq \overline{N}^{2G2S} + 1$. Combine the above results, we have $\overline{N}^{A1G} \leq \overline{N}^{A2G} + 1$.

Part (ii). Consider an 1G1S QSE and a 2G2S QSE with the same even N. By (12), $E(U_p^{1G1S}(N)) - E(U_p^{2G2S}(N)) = (b_2^2 - b_1^2) > 0$. This implies that the 1G1S QSE is more informative than the 2G2S QSE.

Part (iii). Consider an 1G2S QSE and a 2G1S QSE with the same odd N. By (13), we have

$$\begin{aligned} E(U_p^{1G2S}(N)) - E(U_p^{2G1S}(N)) &\propto 2[(b_2^3 - b_1^3) + b_1 b_2 (b_1 - b_2)] + \frac{(b_1 - b_2)^3}{N^2} + (b_1 - b_2)(b_1 + b_2)^2 N^2 \\ &< 2(b_2^3 - b_1^3) + (b_1 - b_2)(b_1 + b_2)^2 N^2 < 0, \end{aligned}$$

where the last inequality uses the fact that $N \geq 3$ (informative equilibrium). Therefore, the 2G1S QSE is more informative than the 1G2S QSE.

Part (iv). First, consider the boundary case in which b_1 and b_2 are such that the size of the first element, Δ_1, in an 1G1S equilibrium with $2N$ elements, exactly equals to $2b_2$. This implies that a 2G1S equilibrium with $2N+1$ elements barely exists, in which $\Delta_1 = 0$. Note that in this boundary case, the 1G1S equilibrium with $2N$ elements is equivalent to the 2G1S equilibrium with $2N+1$ elements. Moreover, by part (i) an 1G2S equilibrium with $2N+1$ elements exists. Following part (iii), the 2G1S equilibrium with $2N+1$ elements is more informative than the 1G2S equilibrium with $2N+1$ elements, we conclude that in this boundary case the 1G1S equilibrium with $2N$ elements is more informative than the 1G2S equilibrium with $2N+1$ elements.

Second, we compute the difference in expected payoffs between an 1G1S equilibrium with $2N$ elements and an 1G2S equilibrium with $2N+1$ elements. By (12) and (13), we have

$$\begin{aligned} &E(U_p^{1G1S}(2N)) - E(U_p^{1G2S}(2N+1)) \propto \\ &8N^3 (N+1) \left(b_2^2 - b_1^2\right) + N^2 b_1 \left(2(1+4N) - 8(N+1) b_1 + 8(N+1) b_2\right) \\ &+2N(1+N)(1+4N) b_2 - 4N - 1. \end{aligned} \tag{22}$$

Our goal is to show (22) is always strictly positive. Recall that in the first step we have shown that it holds for the boundary case. Now to show this also holds for generic case in which an 1G2S equilibrium with $2N+1$ elements exists but a 2G1S equilibrium with $2N+1$ elements does not, it is sufficient to show that (22) is increasing in b_1 and b_2.

It is obvious that (22) is increasing in b_2. To show it is increasing in b_1, we take the derivative of (22) with respect to b_1. The derivative is proportional to

$$(2+8N) - 16(N+1)b_1 + 8\,(N+1)\,b_2 - 16N(N+1)b_1.$$

The above expression is greater than 0 if

$$2 + 8N - 8(N+1)b_1 - 16N(N+1)b_1 \geq 0. \tag{23}$$

The following condition is sufficient for (23) to hold: $1 \geq 2(N+1)2b_1$. Note that, by the fact that an 1G2S equilibrium with $2N+1$ elements exists, and $b_2 \geq b_1$, $1 \geq N(2N+1)2b_1$. For $N \geq 2$, $N(2N+1) > 2(N+1)$. Thus (23) holds for $N \geq 2$. Now consider the case that $N = 1$. Now (23) becomes: $10 - 48b_1 \geq 0$. By the fact that an 1G2S equilibrium with 3 elements exists, $1 \geq 6b_1$, which implies that $10 - 48b_1 \geq 0$. Thus (23) holds for $N = 1$ as well. ∎

Proof of Proposition 4.

Proof. Part (i). Given the condition $\overline{N}^{1G1S} > \overline{N}^{2G1S}$, 2G1S equilibria cannot be the most informative equilibrium. By part (i) of Lemma 2, we have $\overline{N}^{1G1S} \geq \overline{N}^{2G2S}$. Following part (ii) of Lemma 2, the most informative 1G1S equilibrium is more informative than the most informative 2G2S equilibrium. Since $\overline{N}^{1G1S} + 1 \geq \overline{N}^{1G2S}$, by part (iv) of Lemma 2, the most informative 1G1S equilibrium is more informative than the most informative 1G2S equilibrium. Therefore, the most informative 1G1S equilibrium is the most informative equilibrium.

Part (ii). Given the condition $\overline{N}^{2G1S} > \overline{N}^{1G1S}$, 1G1S equilibria cannot be the most informative equilibrium. This implies that 2G2S equilibria cannot be the the most informative equilibrium either, as they are dominated by the most informative 1G1S equilibrium. Since $\overline{N}^{2G1S} > \overline{N}^{1G1S}$, by part (i) of Lemma 2, we must have $\overline{N}^{2G1S} = \overline{N}^{1G2S}$. Now by part (iii) of Lemma 2, the most informative 2G1S equilibrium is more informative than 1G2S equilibria. Therefore, the most informative equilibrium is a 2G1S equilibrium.

Part (iii). Parts (i) and (ii) exhaust all the possibilities, hence the most informative equilibrium cannot be a 2G2S or an 1G2S equilibrium. ∎

Proof of Proposition 5.

Proof. Since the situations of the other two equilibria are similar, we only prove the claims for 1G1S and 1G2S equilibria.

1G1S equilibria. Consider an 1G1S QSE with an even number (say $2N$) of partitions. Since the returns of the two projects have the same distribution, the probability that θ_1 lies in a higher partition than θ_2 is the same as the probability that θ_2 lies in a higher partition than θ_1. Therefore, we only need to consider the situations that both θ_1 and θ_2 lie in the same partition (or ties). Recall that the alternating tie-breaking rule favors agent 2 for $(2n-1)th$ partition, and favors agent 1 for $(2n)th$ partition. Given that in total there are $2N$ partitions, we can group all $2N$ partitions into N pairs, with each pair containing two adjacent partitions: $(2n-1)th$ partition and $(2n)th$ partition. Since the partition sizes are increasing, ties for higher partitions are more likely. This implies that for each pair of partitions, project 1 is more likely to be implemented than project 2. Therefore, overall project 1 (2) will be implemented with a probability strictly greater (less) than 1/2.

1G2S equilibria. Consider an 1G2S QSE with an odd number (say $2N+1$) of partitions. The proof is similar to that for 1G1S equilibria. The only difference is that we need to use different grouping. Given that in total there are $2N+1$ partitions, we can group the $2N$ highest partitions into N pairs, with each pair containing two adjacent partitions: $(2n)th$ partition and $(2n+1)th$ partition. Since the partition sizes are increasing, ties for higher partitions are more likely. This implies that for each pair of partitions, project 2 is more likely to be implemented than project 1. Moreover, in the 1st partition project 2 is favored. Therefore, overall project 2 (1) will be implemented with a probability strictly greater (less) than 1/2. ■

Proof of Corollary 1.

Proof. Part (i). Suppose $c_1 \leq c_2$, or agent 1 is the less biased agent. By previous results, in the most informative equilibrium agent 1 has the sure option. Observing (10), it can be easily verified that EU_p^{1S} increases as either c_1 or c_2 decreases.

Part (ii). Suppose b_2 decreases to $b_2' < b_2$. It is enough to show that the DM's payoff in the most informative 1G1S equilibrium and that in the most informative 2G1S equilibrium both increase. Consider 1G1S equilibria first. Since $b_2' < b_2$, by previous results $\overline{N}^{1G1S} \leq \overline{N}'^{1G1S}$. If $\overline{N}^{1G1S} < \overline{N}'^{1G1S}$, then in the most informative equilibrium the DM's payoff must be higher under b_2'. If $\overline{N}^{1G1S} = \overline{N}'^{1G1S}$, by (12), again in the most informative equilibrium the DM's payoff is higher under b_2'. Similarly, one can show that the DM's payoff in the most informative 2G1S equilibrium is higher under b_2'. ■

Proof of Proposition 7.

Proof. Let $d' > d$. And we use superscript $'$ to denote the endogenous variables under d'.

Part (i). Rearrange the inequalities regarding the number of partitions of A1G QSE, (19) and (21), we get

$$\begin{aligned}(N^2-N)b - Nd &< 1 \text{ for even } N,\\ [(N-1)^2+(N-1)]b-(N-1)d &< 1 \text{ for odd } N.\end{aligned}$$

Since the LHS of the above inequalities is decreasing in d, it follows that $\overline{N}^{A1G'} \geq \overline{N}^{A1G}$. Since $d \leq b$, $\overline{N}^{A1G'} \leq \overline{N}^{A1G}+1$. Therefore, either $\overline{N}^{A1G'} = \overline{N}^{A1G}$, or $\overline{N}^{A1G'} = \overline{N}^{A1G}+1$. In a similar fashion, we can show that, for A2G QSE, either $\overline{N}^{A2G'} = \overline{N}^{A2G}$ or $\overline{N}^{A2G'} = \overline{N}^{A2G}-1$.

Part (ii). Since initially the most informative equilibrium is an 1G1S equilibrium, we have $\overline{N}^{1G1S} > \overline{N}^{2G1S}$. By part (i), the following relationships hold: $\overline{N}^{1G1S'} \geq \overline{N}^{1G1S} > \overline{N}^{2G1S} \geq$

$\overline{N}^{2G1S'}$. Therefore, the most informative equilibrium is still the most informative 1G1S equilibrium. If $\overline{N}^{1G1S'} > \overline{N}^{1G1S}$, then it is obvious that $E(U_p^{1G1S'}) > E(U_p^{1G1S})$. If $\overline{N}^{1G1S'} = \overline{N}^{1G1S}$, then by (12) the only term in $E(U_p^{1G1S})$ that depends on d is $-(b-d)^2/2$, which is increasing in d. Thus, $E(U_p^{1G1S'}) > E(U_p^{1G1S})$.

Part (iii). Since initially the most informative equilibrium is a 2G1S equilibrium, we have $\overline{N}^{1G1S} < \overline{N}^{2G1S}$. By part (i), we have several cases to consider. In the first case, $\overline{N}^{1G1S'} > \overline{N}^{2G1S}$. In this case under d' the the most informative equilibrium is an 1G1S equilibrium, which improves upon the initial most informative equilibrium. In the second case, $\overline{N}^{2G1S} > \overline{N}^{1G1S'} = \overline{N}^{1G1S} > \overline{N}^{2G1S'}$. In this case under d' the the most informative equilibrium is an 1G1S equilibrium, which is worse than the initial most informative equilibrium. In the third case $\overline{N}^{2G1S} = \overline{N}^{2G1S'} > \overline{N}^{1G1S'} = \overline{N}^{1G1S}$. In this case under d' the the most informative equilibrium is a 2G1S equilibrium, with the same number of elements as before. By (13) the only term in $E(U_p^{2G1S})$ that depends on d is as follows:

$$E(U^{A2G}(N)) \propto d(2b^2N^2 - 3d - 2d^2).$$

By the above expression, $E(U_p^{2G1S})$ increases in d if and only $2b^2(\overline{N}^{2G1S})^2 - 6d - 6d^2 > 0$. But the sign of this inequality cannot be determined. ■

Equilibrium selection.

Consider the following equilibrium refinement. In an AiG equilibrium suppose $a_{i,1} > 2b_j$ (or $E(\theta_i|m_{i,1}) > b_j$). Note that in equilibrium, if agent i sends the lowest message $m_{i,1}$ then project j is implemented for sure. Now suppose the realized return of project j is very low: $\theta_j \in [0, a_{i,1} - 2b_j)$. In this case, both agent j and the DM would prefer project i being implemented, given agent i strategy. To achieve that, agent j could send an out of equilibrium message, say "θ_j is very low" or "do not implement my project j," and the DM would listen to it and implement project i. This shows that an AiG equilibrium with $a_{i,1} > 2b_j$ is not stable or reasonable; and for an AiG equilibrium to be stable it must be the case that $a_{i,1} \leq 2b_j$.

Lemma 4 *(i) Suppose an 1G1S (1G2S, 2G1S, 2G2S) equilibrium is not the most informative 1G1S (1G2S, 2G1S, 2G2S) equilibrium, then it is not stable. (ii) The most informative A1G equilibrium must be stable.*

Proof. Part (i). We only prove the case for 1G1S equilibria, since the proof for other equilibria is similar. Consider an 1G1S QSE with N (even) partitions. Since it is not the most informative 1G1S QSE, an 1G1S QSE with $N+2$ partitions exists. We want to show that $a_{1,1}(N) > 2b_2$. Given that a QSE with $N+2$ partitions exists, by (21) we have $\frac{(N+1)(N+3)}{2}b_1 + \frac{(N+1)^2}{2}b_2 < 1$. More explicitly, by (20),

$$a_{1,1}(N) - 2b_2 \propto 2 - (N-1)(N+1)b_1 - (N+1)^2 b_2 > 0,$$

where the inequality follows the previous one.

Part (ii). Let $\overline{N}$ be the number of partitions in the most informative A1G QSE. We only prove the case that $\overline{N}$ is odd. We need to show that $a_{1,1}(\overline{N}) \leq 2b_2$. Suppose to the contrary $a_{1,1}(\overline{N}) > 2b_2$. By (20), it implies that

$$2 - (\overline{N}-1)(\overline{N}+1)b_1 - (\overline{N}+1)^2 b_2 > 0.$$

But given that $b_2 > b_1$, the above inequality implies that

$$2 - (\overline{N}-1)(\overline{N}+1)b_2 - (\overline{N}+1)^2 b_1 > 0,$$

which, by (21), implies that an A1G QSE with $\overline{N}+1$ partitions exists. This contradicts the fact that the most informative A1G QSE has $\overline{N}$ partitions. ■

The results of Lemma 4 are intuitive. If an equilibrium with more partitions exists, it implies that the first partition in the equilibrium of fewer partitions is large relative to the biases, which further means that the equilibrium with fewer partitions is not stable. Although Lemma 4 does not establish that the most informative equilibrium must be stable and any equilibrium that is not the most informative one is not stable, it suggests that only the more informative equilibria can be potentially stable and thus are the more reasonable ones.

Quasi-symmetric mixed strategy equilibrium.

Here we study QSMSE in detail. In QSMSE, the two agents continue to have the same partitions and so it continues to be the case that $E(\theta_i|m_n) = E(\theta_j|m_n)$, where $\{m_n\}$ is the common message set for two agents. And the DM will randomize between the alternatives, with $\lambda_n \in (0,1)$ as the probability that the DM will choose agent 1's alternative when two agents send the same message m_n. Such an equilibrium is characterized by N, $\{\lambda_n\}$, and $\{a_n\}$.

Note that under QSMSE, all the messages of each agent are irreducible, as they will induce different acceptance probabilities. Thus, both agents' indifference conditions have to be satisfied at each interior partition point. For $\{\lambda_n\}$ and $\{a_n\}$ to be an equilibrium, the indifference conditions at a_n for agent 1 and agent 2 can be written as

$$\lambda_{n+1}\Pr(m_{n+1})[E(\theta_2|m_{n+1}) - (c_1 a_n + b_1)] = (1-\lambda_n)\Pr(m_n)[(c_1 a_n + b_1) - E(\theta_2|m_n)], \quad (24)$$

$$(1-\lambda_{n+1})\Pr(m_{n+1})[E(\theta_1|m_{n+1}) - (c_2 a_n + b_2)] = \lambda_n \Pr(m_n)[(c_2 a_n + b_2) - E(\theta_1|m_n)]. \quad (25)$$

To understand equations (24) and (25), observe that agent 1's messages m_n and m_{n+1} are outcome-relevant only if agent 2's messages are either m_n or m_{n+1}. The LHS of (24) is type a_n of agent 1's expected cost of sending the higher message m_{n+1}, while the RHS is his expected benefit of sending the higher message.

In total, we have $2N-1$ endogenous variables $(a_1, ..., a_{N-1}; \lambda_1, ..., \lambda_N)$, but we only have $2N-2$ equations (2 equations for each a_n, $1 \leq n \leq N-1$). Thus, there is one degree of freedom. The reason for this degree of freedom arises from the fact that the indifference condition is influenced by the *relative* attractiveness of the higher and the lower message. In particular, for agent 1, (the inverse of) the relative attractiveness of sending the higher message is $\frac{\lambda_{n+1}}{1-\lambda_n}$, which is the ratio between the likelihood of replacing a more attractive alternative when tied at m_{n+1} versus the likelihood of allowing the implementation of a worse alternative when tied at m_n. Similarly, for agent 2, the relative attractiveness of sending the higher message is related to $\frac{1-\lambda_{n+1}}{\lambda_n}$. To induce the same posterior beliefs, if agent 2 has stronger incentives to exaggerate, we need to counter that by having stronger consequences of exaggeration for agent 2. In other words, for $E(\theta_2|m_n) = E(\theta_1|m_n)$ to arise, we have that if $(c_2a_n + b_2) > (c_1a_n + b_1)$, then $\frac{1-\lambda_{n+1}}{\lambda_n} > \frac{\lambda_{n+1}}{1-\lambda_n}$. The key, however, is that it is the relative attractiveness of the two messages that matters. We can achieve this both by increasing $(1 - \lambda_{n+1})$, i.e. the likelihood that we choose the alternative of the more biased agent when he sends the higher message, and by decreasing λ_n, the likelihood that the lower message leads to the implementation of the alternative of the less biased agent. As a result, there will be a continuum of equilibria that can be sustained through the appropriate sequence of mixing probabilities.

The general case turns out to be too hard to solve, as the difference equations of (24) and (25) are too complicated. From now on we restrict our attention to the additive case: $c_1 = c_2 = 0$, $0 < b_1 < b_2 < 1/2$. Solving the difference equations of (24) and (25), we get the following relationships between $\{\lambda_n\}$ and the partitions

$$\lambda_{n+1} = \frac{(\Delta_n + 2b_1)(-\Delta_{n+1} + \Delta_n + 2b_2)}{2\Delta_{n+1}(b_2 - b_1)}, \tag{26}$$

$$\lambda_n = \frac{(\Delta_{n+1} - 2b_2)(\Delta_{n+1} - \Delta_n - 2b_1)}{2\Delta_n(b_2 - b_1)}, \tag{27}$$

where $\Delta_{n+1} = (a_{n+1} - a_n)$ is the length of the partition element. Now, using (26) and (27), we get

$$\lambda_n = \frac{(\Delta_{n+1} - 2b_2)(\Delta_{n+1} - \Delta_n - 2b_1)}{2\Delta_n(b_2 - b_1)} = \lambda_n = \frac{(\Delta_{n-1} + 2b_1)(-\Delta_n + \Delta_{n-1} + 2b_2)}{2\Delta_n(b_2 - b_1)},$$

which can be simplified as

$$\Delta_{n+1} = \Delta_{n-1} + 2\left(b_1 + b_2\right). \tag{28}$$

Equation (28) tells us that if the first two elements are determined, then all the later elements are determined recursively as well.

Intuitively, any QSMSE is mixture of the A1G and A2G QSE with the same number of elements N. As λ_{2k+1} increases and λ_{2k+2} decreases, QSMSE put more weights on A2G QSE (a_n puts more weight on (24) and less weight on (25)). Notice that for a QSMSE to exist, the following condition $\Delta_2 > 2b_2$ must hold. This is because for type a_1 of agent 2 to be indifferent between sending messages m_1 and m_2, if the size of the second element Δ_2 were smaller than $2b_2$, then agent 2 will always send m_2. This can be seen from the expression of (27), where $\Delta_2 \leq 2b_2$ implies that $\lambda_1 \leq 0$. This implies that QSMSE only exists in the neighborhood of A2G QSE, which gurantees

$\Delta_2 > 2b_2$, and QSMSE might not exist in the neighborhood of A1G QSE, which only guranttees $\Delta_2 > 2b_1$.

The total number of partition elements is even. Lets first consider the case that the total number of elements, $2N$, is even. Given the recursive structure (28), we have

$$N(\Delta_1 + \Delta_2) + 2N(N-1)(b_1 + b_2) = 1.$$

Thus $\Delta_1 + \Delta_2$ does not depend on the mixing probabilities $\{\lambda_n\}$. By the recursive structure, it means that $\Delta_{2n+1} + \Delta_{2n+2}$, and hence a_{2k}, do not depend on $\{\lambda_n\}$ either. However, a_{2n+1} will depend on the mixing probabilities. Given this feature, a bigger a_{2n+1} means that the partition is more even, as Δ_{2n+2} is always bigger than Δ_{2n+1}. More explicitly, the DM's expected payoff can be computed as

$$E(U_p) = \frac{2}{3} - \sum_{n=1}^{2N} \frac{\Delta_n^3}{6}, \tag{29}$$

where $\sum_{n=1}^{2N} \frac{\Delta_n^3}{6}$ is the efficiency loss. Since $\Delta_{2n+1} + \Delta_{2n+2}$ is fixed, by (29), reducing the difference between Δ_{2n+1} and Δ_{2n+2} decreases the efficiency loss.

Now consider a grouped partition $\Delta_{2n+1} + \Delta_{2n+2} = a_{2k+2} - a_{2k}$. By (26), we have

$$2(b_2 - b_1)\lambda_{2n+1} = \frac{(a_{2n+2} - 2a_{2n+1} + a_{2n} - 2b_1)(a_{2n+2} - a_{2n+1} - 2b_2)}{a_{2n+1} - a_{2n}}. \tag{30}$$

By (25), we have $a_{2n+2} - a_{2n+1} - 2b_2 > 0$, which implies that $a_{2n+2} - 2a_{2n+1} + a_{2n} - 2b_1 > 0$ by (30). Then it is obvious that the RHS of (30) is decreasing in a_{2n+1}. Therefore, a_{2n+1} is decreasing in λ_{2n+1}. This means that a_{2n+1} is the biggest when $\lambda_{2n+1} = 0$ (which implies that $\lambda_{2n+2} = 1$), or under 1G1S QSE. Thus, 1G1S QSE leads to the most even partition and a higher expected payoff than any QSMSE.

Since a_1 is decreasing in λ_1, if a QSMSE with $2N$ elements exists then an 1G1S QSE with $2N$ elements ($\lambda_1 = 0$) must exist. Therefore, any QSMSE with $2N$ elements is worse than an 1G1S QSE with $2N$ elements. It follows that, if $\overline{N}^{1G1S} > \overline{N}^{1G2S}$, then an 1G1S QSE with $\overline{N}^{1G1S}$ elements yields a higher payoff to the DM than any QSMSE.

The total number of partition elements is odd. Now consider the case that the number of partition elements $2N+1$ is odd. By the recursive structure (28), we have

$$(N+1)\Delta_1 + N\Delta_2 + 2N^2(b_1 + b_2) = 1. \tag{31}$$

Solving for Δ_2 in (31) and using it in (27), we get

$$\begin{aligned} \lambda_1 &= \frac{(\Delta_2 - 2b_2)(\Delta_2 - \Delta_1 - 2b_1)}{2\Delta_1(b_2 - b_1)} \\ &= \frac{(\frac{1-(N+1)\Delta_1 - 2(b_1+b_2)N^2}{N} - 2b_2)(\frac{1-(N+1)\Delta_1 - 2(b_1+b_2)N^2}{N} - \Delta_1 - 2b_1)}{2\Delta_1(b_2 - b_1)}. \end{aligned} \tag{32}$$

It is easy to verify that the RHS of (32) is decreasing in Δ_1. Therefore, Δ_1 is decreasing in λ_1. Given the partition pattern, we conclude that as λ_1 increases, the size of odd number elements, Δ_{2k+1} decreases, and the size of even number elements, Δ_{2k+2}, increases.

Now we compute the efficiency loss, EL, under QSMSE. By (29) and the recursive structure of partition,

$$EL = \sum_{n=1}^{N+1} \frac{[\Delta_1 + 2(n-1)(b_1 + b_2)]^3}{6} + \sum_{n=1}^{N} \frac{[\Delta_2 + 2(n-1)(b_1 + b_2)]^3}{6}.$$

Taking the derivative of EL with respect to Δ_1 and simplifying yield

$$\frac{\partial EL}{\partial \Delta_1} \propto 3\Delta_1^2 - 3\Delta_2^2 + 6N(b_1 + b_2)\Delta_1 - 6(b_1 + b_2)(N-1)\Delta_2 + 2(4N-1)(b_1 + b_2)^2. \tag{33}$$

It can be readily seen from (33) that $\frac{\partial EL}{\partial \Delta_1}$ is increasing in Δ_1 (Δ_2 is decreasing in Δ_1). Recall that QSMSE is a mixture of 2G1S QSE and 1G2S QSE, and Δ_1 is minimized in 2G1S QSE. Therefore, $\frac{\partial EL}{\partial \Delta_1}$ is always positive if (33) is positive evaluated at 2G1S QSE, which means that 2G1S QSE yields a higher payoff than any QSMSE.

Lets evaluate (33) at 2G1S QSE: $\Delta_2 = \Delta_1 + 2b_2$ and, by (31), $(2N+1)\Delta_1 + 2Nb_2 + 2N^2(b_1+b_2) = 1$. The algebra yields

$$\frac{\partial EL}{\partial \Delta_1}|_{2G1S} = 6b_2b_1 + 3(b_1 - b_2) + \left[2N^2 + 2N - 1\right](b_1 + b_2)^2. \tag{34}$$

Inspecting the expression of (34), we can see that $\frac{\partial EL}{\partial \Delta_1}|_{2G1S} \geq 0$ if $|b_1 - b_2|$ is small enough, or two agents' biases are not too far apart. Thus, we conclude that if two agents' biases are not too far apart, then 2G1S QSE dominates any QSMSE with the same number of elements $2N+1$.

However, if two agents' biases are too far apart, then it is possible that $\frac{\partial EL}{\partial \Delta_1}|_{2G1S} < 0$. Since 2G1S QSE dominates 1G2S QSE, it means that QSMSE can improve upon QSE. This point is illustrated in the following example.

Example 8 *Suppose* $b_1 = 0.04$ *and* $b_2 = 0.2$. *The maximum number of partition elements is 3. In 2G1S QSE,* $\Delta_1 = 0.04$, $\Delta_2 = 0.44$, *and* $\Delta_3 = 0.52$. *The efficiency loss under the QSE is* $EL_{2G1S} = 0.03764$. *In a QSMSE with* $\lambda_1 = 0.3625$ *(*$\lambda_2 = 0.029$, $\lambda_3 = 0.855$*),* $\Delta_1 = 0.05$, $\Delta_2 = 0.42$, *and* $\Delta_3 = 0.53$. *The efficiency loss under the QSMSE is* $EL_{QSMSE} = 0.03718 < EL_{2G1S}$. *Thus the QSMSE yields a higher payoff than the 2G1S QSE. The optimal QSMSE has* $\lambda_1 \to 0$ *(*$\lambda_2 = 0.066$,

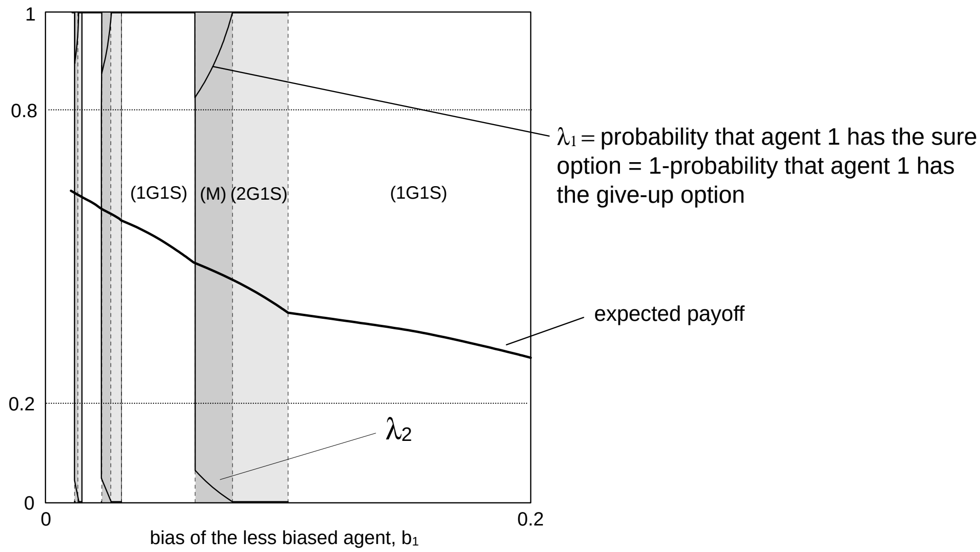

Figure 10: An example of the optimality of mixed equilibrium, with $b_2 = 2b_1$

$\lambda_3 = 0.722$*),* $\Delta_1 \to 0.06$*,* $\Delta_2 \to 0.4$*, and* $\Delta_3 \to 0.54$*, which yields an efficiency loss* 0.03695.[25]

This result shows that sometimes randomization or giving the more biased agent some authority/veto power (in the sense of the sure option) is beneficial. In the example, in the optimal QSMSE agent 2 has the sure option with probability 0.278. To understand this result, note that compared to QSMSE, in 2G1S QSE the size of the largest element (Δ_{2N+1}) is minimized, but the size of the second largest element (Δ_{2N}) is maximized. When b_1 is significantly smaller than b_2, Δ_{2N} (Δ_{2n}) is of similar size to Δ_{2N+1} (Δ_{2n+1}). Now consider increasing Δ_1 by $\varepsilon > 0$ (reducing λ_1 and λ_{2N+1} from 1). By the recursive structure, it means that Δ_{2N+1} (Δ_{2n+1}) increases by ε, but Δ_{2N} (Δ_{2n}) decreases by $\frac{N+1}{N}\varepsilon$, which is more than ε (in the example with 3 elements, Δ_2 decreases by 2ε). Given that Δ_{2N} (Δ_{2n}) is of similar size to Δ_{2N+1} (Δ_{2n+1}), this change could reduce the total efficiency loss, which is increasing and convex in the element size. In short, when b_1 is significantly smaller than b_2, in 2G1S QSE the even number elements are relatively large. In this case, introducing randomization would reduce the sizes of the even number elements, and make the partition overall more even.

In the following figure, with $b_2 = 2b_1$, we illustrate the regions of b_1 in which QSMSE is optimal.

Proof of Lemma 3.

Proof. Part (i). It is enough to rule out the case that agent j has three irreducible messages for some m_i, since the argument to rule out more than three irreducible messages is similar. Suppose, given m_i, agent j has three irreducible messages: l, m, and h. Let the probability that project j is implemented given m_j, $j = l, m, h$, be p_j. Since the messages are irreducible, these probabilities must be different. Without loss of generality, suppose $p_l < p_m < p_h$. It follows that $p_m \in (0,1)$. Denote $\overline{m}_i = E(\theta_i | m_i)$. Now consider agent j's incentive. For all types of $\theta_j > (\overline{m}_i - b_j)/c_j$, agent

[25] There is no QSMSE for $\Delta_1 \in (0.06, 0.147)$, where in the 1G2S QSE $\Delta_1 = 0.147$.

j strictly prefers sending message h; for all types of $\theta_j < (\overline{m}_i - b_j)/c_j$, agent j strictly prefers sending message l; for type $\theta_j = (\overline{m}_i - b_j)/c_j$, agent j is indifferent among all three messages. Thus, message m can only be sent by the type of $(\overline{m}_i - b_j)/c_j$ of agent j. But, then from the DM's point of view, after hearing message m from agent j he should implement project i with probability 1. This contradicts the presumption that $p_m \in (0,1)$. Therefore, agent j can have at most two messages for any given m_i.

Part (ii). This is similar to the proof of Proposition 1. Let $m_j(\theta_j, m_{i,n})$ be agent j's communication strategy, and DM's decision rule be $\Pr(d = i|m_{i,n}, m_j(\theta_j, m_{i,n}))$. Correspondingly, $\Pr(d = i|m_{i,n})$ and $E(\theta_j|d = j, m_{i,n})$ are modified as:

$$\begin{aligned}\Pr(d &= i|m_{i,n}) = \int_0^1 \Pr(d = i|m_{i,n}, m_j(\theta_j, m_{i,n}))d\theta_j, \\ E(\theta_j|d = j, m_{i,n}) &= \int_{\{d=j|m_{i,n}, m_j(\theta_j, m_{i,n})\}} \theta_j d\theta_j.\end{aligned}$$

Agent i's expected payoff, given his type θ_i and message $m_{i,n}$, can still be written as

$$E_{\theta_j}[U_i|\theta_i, m_{i,n}] = \Pr(d = i|m_{i,n})(c_i\theta_i + b_i) + \Pr(d = j|m_{i,n})E(\theta_j|d = j, m_{i,n}).$$

The rest of the proof is exactly the same as that of Proposition 1. ■

Proof of Proposition 10.

Proof. By previous analysis, we only need to show $\frac{\Phi(K-1,m_n)}{\Phi(K-1,m_{n+1})} > \frac{\Phi(K,m_n)}{\Phi(K,m_{n+1})}$.

By (18), the above inequality is equivalent to $(a_n^K - a_{n+1}^K)(a_{n+1} - a_{n-1})(a_n^K - a_{n-1}^K) + (a_{n+1} - a_n)(a_n - a_{n-1})(a_{n+1}^K - a_{n-1}^K)a_n^{K-1}K > 0$. Given that $a_{n-1} < a_n < a_{n+1}$, this inequality is equivalent to

$$-(\sum_{i=0}^{K-1} a_{n+1}^{K-i-1}a_n^i)(\sum_{i=0}^{K-1} a_n^{K-i-1}a_{n-1}^i) + (\sum_{i=0}^{K-1} a_{n+1}^{K-i-1}a_{n-1}^i)a_n^{K-1}K > 0. \tag{35}$$

We show that inequality (35) holds by induction. Let $A_K \equiv \sum_{i=0}^{K-1} a_{n+1}^{K-i-1}a_n^i$, $B_K \equiv \sum_{i=0}^{K-1} a_{n+1}^{K-i-1}a_{n-1}^i$ and $C_k \equiv (\sum_{i=0}^{K-1} a_n^{K-i-1}a_{n-1}^i)$. For $K = 2$, inequality (35) becomes $(a_{n+1} - a_n)(a_n - a_{n-1}) > 0$, which obviously holds. Now suppose inequality (35) holds for K, that is, $A_K C_K < B_K a_n^{K-1}K$. We want to show inequality (35) holds for $K+1$, that is, $A_{K+1}C_{K+1} < B_{K+1}a_n^K(K+1)$. This inequality can be expanded as

$$a_n a_{n+1}(A_K C_K - B_K a_n^K K) + a_n^K(-a_{n-1}^K K + a_n C_K) + a_{n+1}(-B_K a_n^K + A_K a_{n-1}^K) < 0.$$

Given that $A_K C_K < B_K a_n^{K-1}K$, it is enough to show that $a_n^K(a_{n-1}^K K - a_n C_K) + a_{n+1}(B_K a_n^K -$

$A_K a_{n-1}^K) > 0$. Specifically,

$$
\begin{aligned}
& a_n^K(a_{n-1}^K K - a_n C_K) + a_{n+1}(B_K a_n^K - A_K a_{n-1}^K) \\
= & \sum_{i=0}^{K-1} \{a_n^K a_{n-1}^i (a_{n-1}^{K-i} - a_n^{K-i}) + a_{n+1}^{i+1}(a_n^K a_{n-1}^{K-i-1} - a_n^{K-i-1} a_{n-1}^K)\} \\
> & \sum_{i=0}^{K-1} \{a_n^K a_{n-1}^i (a_{n-1}^{K-i} - a_n^{K-i}) + a_n^{i+1}(a_n^K a_{n-1}^{K-i-1} - a_n^{K-i-1} a_{n-1}^K)\} \\
= & \sum_{i=0}^{K-1} a_n^K[-a_{n-1}^i a_n^{K-i} + a_n^{i+1} a_{n-1}^{K-i-1}] = 0.
\end{aligned}
$$

■

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