Cheap Talk with Two Senders and Complementary Information

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Abstract

This paper studies a cheap talk model in which two senders having partial and non-overlapping private information simultaneously communicate with an uninformed receiver. The sensitivity of the receiver's ideal action to one sender's private information depends on the other sender's private information. When senders have type-independent biases, their information transmissions exhibit strategic complementarity: more information transmitted by one sender leads to more information being transmitted by the other sender. When senders have type-dependent biases, their information transmissions can exhibit strategic substitutability. We also study delegation when senders have type-independent biases. When the two senders have like-biases, it is always optimal for the receiver to delegate decision rights to the sender with the smaller bias. When the senders have opposing biases, simultaneous communication is more likely to dominate delegation.

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1 Introduction

Decision makers often seek advice from multiple experts. Specifically, consider a firm with two functional divisions: a marketing division and a production division. The firm's CEO must choose the size of a new factory to produce a new product. The optimal size of the new factory depends on the profitability of the new product, which further depends on the demand for and cost of production of the new product. Due to functional specialization, the marketing division manager knows only the demand for the product while the production division manager knows only the production cost. Thus the CEO must consult both managers regarding his decision, but the managers' interests may not be perfectly aligned with the CEO's interests. In particular, a

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manager might prefer a smaller or a bigger factory relative to the CEO's ideal size. Alternatively, a manager may wish to choose the factory size to maximize the performance of his own division rather than the firm's overall performance.

The above example has three distinguishing features: (i) a decision maker consults two experts regarding relevant information before making a decision; (ii) the experts' interests are not perfectly aligned with the decision maker's; and (iii) the experts observe different aspects of the information that is relevant for the decision (i.e., the experts observe *non-overlapping information*). The purpose of the paper is to study communication or information transmission in the above setting, with communication modeled as cheap talk (Crawford and Sobel, 1982, CS hereafter). While the first two features are standard in cheap talk models, the third feature—which is unique to this paper—is understudied in the literature despite its obvious relevance in the real world. That different experts observe non-overlapping information occurs in many situations as a result of specialization. In organizations such as firms and governments, different divisions specialize in different functional areas. At the individual-level, experts often specialize and only have expertise in one field.

Real world situations sharing the above three features abound. For instance, a president deciding on a bailout plan for banks must determine the optimal size of the bailout, which depends on how deep the banking crisis is and the constraints of the federal budget. The president consults a banking expert who knows only how serious the banking crisis is and a budget expert from OMB who knows only the availability of bailout funds. Alternatively, a military leader who must decide how many troops to send into combat consults with an intelligence expert and a field commander. The optimal number of troops depends on both the strength of the enemy and the strength of his own army. While the intelligence expert may only know the strength of the enemy, the field commander may only know the strength of his own forces. Finally, a dean must decide how much to invest in a new interdisciplinary center involving both economics and psychology, the optimal size of which depends on the local conditions in the economics and psychology departments. The dean consults with the chairs of both departments who only know the conditions of their own departments.

We develop a cheap talk model that captures all three of the above features. To model senders (experts) having partial and non-overlapping private information, we assume that the state of the world has two dimensions, θ_1 and θ_2. Each expert i perfectly observes the realized state in dimension i (θ_i) but does not observe the realized state in dimension j (θ_j). The receiver (decision maker) takes a single action and observes neither θ_1 nor θ_2. The receiver's ideal action is a function of the realized states, $y^*(\theta_1, \theta_2)$. We focus on the case in which *the marginal impact* *of information in dimension i on the ideal decision depends on the realized state in dimension j* (i.e., $\partial^2 y^*(\theta_1, \theta_2)/\partial\theta_1\partial\theta_2 \neq 0$). In our leading example, $y^*(\theta_1, \theta_2) = \theta_1\theta_2$. This formulation has an intuitive interpretation: the larger the realized state in one dimension, the more *sensitive* the ideal action is to the information in the other dimension.[1] Consider the CEO example in which θ_1 is the demand size and θ_2 is the efficiency of production. In our leading example, the optimal factory size is more sensitive to production efficiency when the market size is larger and vice versa.[2] In the Appendix, we show that under monopoly pricing the optimal factory size (and, hence, output level) can indeed be expressed as $\theta_1\theta_2$.

Given multiple senders and multi-dimensional states, there are several ways to model experts' biases. In the basic model we study type-independent biases, in which each expert's ideal action differs from the receiver's ideal action by a constant independent of realized states. Of course, two experts can have different biases. Adopting terminology from Krishna and Morgan (2001a, KM hereafter), we say that two experts have *opposing biases* when one expert wants to pull the decision to the left and the other to the right. Alternatively, two experts can have *like biases* if both want to pull the decision in the same direction, but possibly to different degrees. Type-independent biases are realistic in situations where biases are generated by different ideologies: both experts agree with the receiver about how the optimal action depends on realized states, but the experts prefer different actions due to their different ideologies. In the bailout example, for instance, a Republican expert who believes in limited government may always prefer a smaller bailout plan than a Democratic expert. In the troop deployment example, a dovish expert may always prefer to deploy fewer troops than an hawkish expert.

We also study the case in which the experts' biases are type-dependent. In particular, expert i's ideal action may only depend on his own information θ_i. These preferences are realistic in multidivisional organizations. In the CEO example, for instance, each division manager may only care about the performance of his own division and thus prefer an action suited to his division's local conditions only. In the dean example, each department chair may only care about the welfare of his own department.

We first study the basic model with independent biases in which the two experts send messages simultaneously. Equilibria are shown to be partition equilibrium in which each sender indicates only to which interval the realized state that he observes belongs as in standard CS cheap talk models. We focus on the most informative equilibrium. Interestingly, the two senders'

[1] If $\partial^2 y^*(\theta_1, \theta_2)/\partial\theta_1\partial\theta_2 = 0$, say $y^*(\theta_1, \theta_2) = \theta_1 + \theta_2$, then the cheap talk game with two-senders is qualitatively similar to standard cheap talk game with one sender. See Section 3.2 for a more detailed discussion.

[2] In the military example, let θ_1 be the weakness of one's own army and θ_2 be the strength of the enemy. In our leading example, the weaker one's own army is, the more sensitive is the optimal number of troops to the strength of the enemy and vice versa.

information transmissions exhibit strategic complementarity: the more information that one sender transmits, the less the incentive the other sender has to distort his report in the direction of his bias, and, hence, the more information he will transmit. As a result, a reduction in one sender's bias leads not only to more information being transmitted by himself but also induces the other agent to transmit more information. The underlying reason for strategic complementarity is a *"variance" effect*. Intuitively, since the ideal action is multiplicative in states, in higher states of θ_2 the receiver's action is more sensitive to sender 1's report and sender 1's information distortion will be amplified more. Thus, relative to sender 1's ideal action, in general there is over-distortion of information in higher states and under-distortion in lower states of θ_2. As sender 2 transmits more information the sensitivity of the receiver's action to sender 1's report will vary more, and the same amount of information distortion by sender 1 now leads to bigger over-distortion in higher states of θ_2 and bigger under-distortion in lower states of θ_2. By reducing the amount of distortion a little bit, sender 1 can reduce the amount of over-distortion in higher states significantly without increasing the amount of under-distortion in lower states of θ_2 by much. Therefore, sender 1 will have an incentive to reduce his amount of information distortion when sender 2 transmits more information. We also show that the equilibrium information transmissions depend only on the absolute value of the senders' biases, implying that whether the experts have like biases or opposing biases does not matter.

We then study simultaneous communication with type-dependent biases. It turns out that the information transmissions of the two senders can exhibit strategic substitutability: the more information that one sender transmits, the more the incentive the other sender has to distort his report and, hence, the less information he will transmit. That said, it can also be the case that the information transmissions of the two senders exhibit strategic complementarity when little information is transmitted, but the senders' information transmissions become strategic substitutes when a lot of information is transmitted. Which case applies depends on whether one sender has an incentive to under-report or over-report when the other sender transmits no information. The general pattern is that when sender i transmits more information, sender j will tend to under-report more (or over-report less). The underlying force behind those results is again the variance effect.

We next study the possibility of delegation in the basic model with type-independent biases. Specifically, the receiver delegates his decision rights to one of the senders. The agent to whom the decision rights are delegated first consults the other sender regarding his private information and then makes a decision. We show that it is always better for the receiver to delegate decision rights to the expert with a smaller bias in absolute value. The underlying reason for this result is that the effectiveness of communication between the two senders does not depend on which expert has the decision rights. Therefore, the decision rights should be delegated to the expert with the smaller bias to minimize the loss of control experienced by the receiver.

Finally, we compare delegation to simultaneous communication. Interestingly, whether delegation is optimal for the receiver depends on whether the experts have like biases or opposing biases. When the experts have like biases and communication is informative under simultaneous communication, delegation dominates simultaneous communication for the receiver. This result still holds when the experts have opposing biases and the absolute value of the smaller bias is small enough. On the other hand, simultaneous communication dominates delegation if the two experts have opposing biases and the absolute value of the smaller bias is big enough. Intuitively, when the experts have like (opposing) biases the communication between the experts is more (less) informative than that between experts and the receiver because the effective bias between two experts is smaller (bigger) than that between the experts and the receiver. These results imply that we are more likely to observe delegation when the experts are biased in the same direction relative to the principal. In a political context, delegation is more likely when either both experts are more liberal or both experts are more conservative than the principal.

The rest of the paper is organized as follows. The next subsection discusses related literature. Section 2 lays out the basic model with type-independent biases. The equilibrium of this model with simultaneous communication is characterized and the robustness of the results is examined in Section 3. In Section 4 we study type-dependent biases and summarize the strategic interactions between the experts' information transmissions under different conditions. In Section 5 we consider delegation and compare it to simultaneous communication. Section 6 offers conclusions and discussions. All of the missing proofs in the text can be found in the Appendix.

1.1 Related Literature

Following the original work of CS on cheap talk, there is a growing literature on cheap talk with multiple senders. Gilligan and Krehbiel (1989) study a model in which two experts with symmetric opposing biases simultaneously communicate by submitting bills to a decision-making legislature. They show that the restrictive "closed rule," in which amendments to bills are not permitted, is informationally superior to the "open rule" in which bills are freely amendable. Krishna and Morgan (2001b) reexamine the model and derive different results. Epstein (1998) generalizes the model of Gilligan and Krehbiel to the case where two experts have asymmetric opposing biases. KM study a more general model in which the two experts, who can have like or opposing biases, communicate sequentially. Gick (2009) considers the model of KM with like biases by adding the twist that the receiver is able to commit to not best responding to the second sender. A common feature of these models is that the state space is one dimensional and both senders perfectly observe the same realized state. In contrast, in our model the two senders have partial and non-overlapping private information.[3]

Battaglini (2002) studies a multidimensional cheap talk model with multiple senders. He concludes that—in contrast to one-dimensional cheap talk models with one sender—generically information can be fully revealed in equilibrium communication, and Ambrus and Takahashi (2008) provide further conditions under which fully revealing equilibria are possible in this setting. Our model differs from these multidimensional cheap talk models in two regards. First, in our model each sender only observes the realized state in his own dimension, while in their models each sender observes the realized states in all dimensions. Second, in our model the decision is a one-dimensional variable while in theirs the decision is a two-dimensional vector. Based on these differences, fully revealing equilibria are impossible in our model.

Austen-Smith (1993) considers a two-sender model with two experts imperfectly informed about the state. In his model each expert receives a noisy (binary) signal about the state, which is also binary. Morgan and Stocken (2008) model information aggregation in polls as a cheap talk game with multiple senders, with senders (who are polled) receiving imperfect, conditionally independent, binary signals regarding the state. The differences between our model and their models will be further discussed in Section 3.[4]

Our paper is also related to Alonso et. al (2008, ADM hereafter), who study strategic communication between a CEO and two division managers. Each manager has private information regarding the local conditions of his own division, and a decision needs to be made for each division. Furthermore, the decisions of the two divisions need to be coordinated, and each manager has a bias toward maximizing the profit of his own division. They compare two communication modes: vertical communication (centralization) versus horizontal communication (decentralization).[5] Our study differs from theirs in that in our model there is only one decision to make instead of two. Furthermore, the need to communicate in their model results from the need to coordinate two decisions, while the need to communicate arises in our model because the optimal decision for the receiver depends on the private information of both experts. These differences

[3]For cheap talk models with one sender and multiple receivers, see Farrell and Gibbons (1989) and Goltsman and Pavlov (2009).

[4]Li (2007) studies a model in which two experts perfectly observe the realized state, but each expert's bias is his own private information. Again in his model the number of states and signals is finite.

[5]Under centralization, each manager communicates with the CEO simultaneously, and then the CEO makes decisions for each division. Under decentralization, the two managers simultaneously communicate with each other and then each manager makes the decision for his own division. They show that even when the need for coordination is large, decentralization can be superior to centralization from the CEO's perspective.

affect the experts' incentives to communicate. Another paper related to ours is Martimort and Semenov (2008). They study an informational lobbying game in which two interest groups influence the legislature by communicating private information regarding their preferences. Instead of modeling communication as cheap talk, they consider the situation in which the legislature is able to commit to a decision rule and use the mechanism design approach.

With respect to delegation our paper is related to Dessein (2002), who compares delegation and cheap talk in a one-sender model as in CS. Alonso and Matouschek (2007) endogenize the commitment power of the principal by developing an infinitely repeated delegation game. Our paper differs from these studies in that we compare *delegation with cheap talk* to a cheap talk model with two senders (see Section 5 for more details). Harris and Raviv (2005), McGee (2008) and Chen (2009) study cheap talk models when both the receiver and sender have private information, and the possibility of delegation is considered in Harris and Raviv (2005) and McGee (2008). In all of these models, however, there is only one sender.

Since the work of CS, it is well-known that cheap talk models have multiple equilibria. There have been efforts made on equilibrium refinement (Matthews et. al, 1991; Chen et. al, 2008). We will follow a common practice in cheap talk models: whenever there are multiple equilibria, we will focus on the most informative equilibrium because it is usually ex ante Pareto dominant.

2 The Basic Model

To formalize the examples in the introduction, we provide a stylized model that can be applied to a broad range of institutional settings. Consider a decision maker (DM) who consults two experts $i = 1, 2$. Both experts and the DM are expected utility maximizers. The DM takes an action $y \in R$, and his utility depends on some underlying states of nature θ_1 and θ_2. Each θ_i is distributed on $[0, A_i]$ with density $f(\theta_i)$, and θ_1 and θ_2 are independent from each other. The DM does not observe the realization of either θ_1 or θ_2, and expert i observes only the realized value of θ_i. This captures the fact that each expert is knowledgeable only in his own field. Note that both experts have private information, yet this private information is not overlapping in the sense that θ_1 and θ_2 are independent.

Expert i offers advice to the DM by sending message m_i. Unless stated otherwise, we focus on the case of simultaneous communication in which the two experts send messages simultaneously. After receiving messages m_1 and m_2, the DM takes an action $y(m_1, m_2)$.

Given realized states θ_1 and θ_2, the ideal action for the DM is $y^*(\theta_1, \theta_2)$. The utility function for the DM is

$$U^P(y,\theta_1,\theta_2) = -(y - y^*(\theta_1,\theta_2))^2.$$

In the basic model we assume that expert i's ideal action is $y^*(\theta_1,\theta_2)+b_i$, where the constant b_i is expert i's bias relative to the DM. Bias b_i, which can be positive or negative, measures the degree to which the DM's and expert i's interests are aligned. Specifically, the utility function for expert i is

$$U^{A_i}(y,\theta_1,\theta_2,b_i) = -[y - (y^*(\theta_1,\theta_2)+b_i)]^2.$$

The biases are common knowledge. When b_1 and b_2 have the same sign, we say that the experts have like biases; otherwise, we say that they have opposing biases. Note that in the current setup the difference between agent i's ideal action and the DM's ideal action, b_i, is independent of the realized states θ_1 and θ_2. We call this the case of type-independent biases. As mentioned in the introduction, type-independent biases are realistic in situations where biases result from differences in ideologies. In a majority of the papers mentioned in the literature review studying cheap talk with multiple senders, the senders' biases are type-independent.

Under simultaneous communication, a strategy for expert i specifies a message m_i for each θ_i, which is denoted by the communication rule $\mu_i(m_i|\theta_i)$. A strategy for the DM specifies an action y for each message pair (m_1,m_2), which is denoted by the decision rule $y(m_1,m_2)$. Let the belief function $g(\theta_1,\theta_2|m_1,m_2)$ be the DM's posterior beliefs on θ_1 and θ_2 after hearing messages m_1 and m_2. Since θ_1 and θ_2 are independent and expert i observes only θ_i, the belief function can be decomposed into distinct belief functions $g_1(\theta_1|m_1)$ and $g_2(\theta_2|m_2)$.

Our solution concept is Perfect Bayesian Equilibrium (PBE), which requires:

(i) Given the DM's decision rule $y(m_1,m_2)$ and expert j's communication rule $\mu_j(m_j|\theta_j)$, for each i, expert i's communication rule $\mu_i(m_i|\theta_i)$ is optimal.

(ii) The DM's decision rule $y(m_1,m_2)$ is optimal given beliefs $g_1(\theta_1|m_1)$ and $g_2(\theta_2|m_2)$.

(iii) The belief functions $g_i(\theta_i|m_i)$ are derived from the agents' communication rules $\mu_i(m_i|\theta_i)$ according to Bayes rule whenever possible.

We first derive the DM's optimal decision rule $\overline{y}(m_1,m_2)$. Given m_1 and m_2, $\overline{y}(m_1,m_2)$ maximizes $-E[(y-y^*(\theta_1,\theta_2))^2|m_1,m_2]$. Thus, $\overline{y}(m_1,m_2) = E[y^*(\theta_1,\theta_2)|m_1,m_2]$. Denote the partial derivative of y^* with respect to θ_1 as y_1^* (other partial derivatives are denoted accordingly). The following lemma specifies a set of sufficient conditions under which all PBE are interval equilibria. That is, the state space $[0,A_i]$ is partitioned into intervals and expert i only reveals to which interval θ_i belongs.

Lemma 1 *Suppose the following assumptions hold: (i) $y_1^* > 0$ (or < 0), $y_2^* > 0$ (or < 0) for all (θ_1, θ_2); and (ii) both $|y_{11}^*/y_1^*|$ and $|y_{22}^*/y_2^*|$ are small enough for all (θ_1, θ_2). Then all PBE in the communication game must be interval equilibria.*

Define $\overline{m}_i$ as the posterior of state θ_i given message m_i; that is, $E[\theta_i|m_i] \equiv \overline{m}_i$. Before we proceed, we first prove a useful result.

Claim 1 $E[\theta_i \overline{m}_i] = E[\overline{m}_i^2]$.

Proof. Note that $\overline{m}_i = E[\theta_i|m_i]$ and m_i is coarser than θ_i. Therefore,

$$E[\theta_i \overline{m}_i] = E[\theta_i E[\theta_i|m_i]] = E\{E[\theta_i E[\theta_i|m_i]]|m_i\} = E\{E[\theta_i|m_i]E[\theta_i|m_i]\} = E[\overline{m}_i^2].$$

■

3 Simultaneous Communication with Type-independent Biases

To facilitate analysis, for most of the paper we will focus on a specific functional form of the DM's ideal action: $y^*(\theta_1, \theta_2) = \theta_1\theta_2$.[6] The robustness of our results with more general functional forms will be discussed later. Note that $\frac{\partial^2 y^*(\theta_1,\theta_2)}{\partial\theta_1\partial\theta_2} > 0$. As mentioned in the introduction, this condition implies that the marginal impact of state θ_i on the ideal action depends on the realized state θ_j. This relationship leads to strategic interactions between the experts' information transmissions as we show later. For simplicity, we assume that each θ_i is uniformly distributed on $[0, A_i]$ with density $1/A_i$. It can be readily seen that Lemma 1 applies to the current setup. Therefore, all PBE must be interval equilibria.

3.1 Equilibrium and equilibrium properties

Because θ_1 and θ_2 are independent and $\theta_1|m_1$ and $\theta_2|m_2$ are independent, the DM's optimal action is given by

$$\overline{y}(m_1, m_2) = E[\theta_1|m_1]E[\theta_2|m_2] = \overline{m}_1\overline{m}_2. \quad (1)$$

Having established that all PBE must be interval equilibria, we now characterize them. Let N_i be the number of partition elements in the partition (or the size of the partition) of agent i's information space, and $(a_{i,0}, a_{i,1}, ..., a_{i,n}, ..., a_{i,N_i}) \equiv a_i$ be the partition points with $a_{i,0} = 0$ and $a_{i,N_i} = A_i$. Define $\overline{m}_{i,n}$ as the receiver's posterior of θ_i after receiving a message $m_{i,n} \in (a_{i,n-1}, a_{i,n})$. It follows that $\overline{m}_{i,n} = (a_{i,n-1} + a_{i,n})/2$. In state $\theta_1 = a_{1,n}$, agent 1 should be

[6] The same formulation is adopted in McGee (2008).

indifferent between sending a message that induces a posterior $\overline{m}_{1,n}$ and a posterior $\overline{m}_{1,n+1}$, that is, $E_{\theta_2}[U^{A_1}|\overline{m}_{1,n}, a_{1,n}] = E_{\theta_2}[U^{A_1}|\overline{m}_{1,n+1}, a_{1,n}]$.

More explicitly, the indifference condition can be written as

$$E_{\theta_2}[\{\overline{m}_2 \frac{a_{1,n} + a_{1,n-1}}{2} - (\theta_2 a_{1,n} + b_1)\}^2] = E_{\theta_2}[\{\overline{m}_2 \frac{a_{1,n} + a_{1,n+1}}{2} - (\theta_2 a_{1,n} + b_1)\}^2].$$

Using the fact that $E[\theta_i \overline{m}_i] = E[\overline{m}_i^2]$, we can simplify the above indifference condition further as

$$(a_{1,n+1} - a_{1,n}) - (a_{1,n} - a_{1,n-1}) = \frac{E(\theta_2)}{E(\overline{m}_2^2)} 4b_1. \tag{2}$$

Similarly, the cutoff points $a_{2,n}$ characterizing agent 2's partition equilibrium satisfy the indifference condition:

$$(a_{2,n+1} - a_{2,n}) - (a_{2,n} - a_{2,n-1}) = \frac{E(\theta_1)}{E(\overline{m}_1^2)} 4b_2. \tag{3}$$

Inspecting indifference conditions (2) and (3), we see that there is strategic interaction between the senders' information transmissions in equilibrium as the term $\frac{E(\theta_j)}{E(\overline{m}_j^2)}$ appears in the condition that determines sender i's cutoff points. In the quadratic-uniform case of CS's one-sender model, the indifference condition for cutoff points implies that the difference between the lengths of any two adjacent intervals, the incremental step size, is always $4b$. Conditions (2) and (3) show that the strategic interaction between the two senders changes the incremental step sizes: the effective incremental step size is $\frac{E(\theta_j)}{E(\overline{m}_j^2)} 4b_i$. Note that

$$E(\overline{m}_j^2) = (E(\overline{m}_j))^2 + var(\overline{m}_j) = (E(\theta_j))^2 + var(\overline{m}_j).$$

That is, $E(\overline{m}_j^2)$ is a constant plus the variance of posterior mean. Since a higher variance of posterior mean implies that more information is transmitted,[7] a bigger $E(\overline{m}_j^2)$ means more information is transmitted by sender j. As one agent transmits more information, however, the effective incremental step size for the other agent decreases, which leads to more information being transmitted by the other agent. Therefore, the agents' information transmissions exhibit strategic complementarity.

An equilibrium is characterized by two sequences of partition points, (a_1, a_2), that satisfy the indifference conditions (2) and (3) and the boundary conditions $a_{i,0} = 0$ and $a_{i,N_i} = A_i$. Let

[7] If no information is transmitted by sender j, then the variance of the posterior mean is 0. If information is fully transmitted by sender j, then the variance of the posterior mean reaches its maximum, the variance of θ_i.

$\frac{E(\theta_i)}{E(\overline{m}_i^2)} \equiv x_i$. Given N_i and x_j and using the boundary conditions $a_{i,0} = 0$ and $a_{1,N_i} = A_i$, we can solve for the solutions for difference equations (2) and (3):

$$a_{i,n} = A_i \frac{n}{N_i} + 2b_i x_j n(n - N_i). \tag{4}$$

In equilibrium, the following inequality should be satisfied:

$$2|b_i|x_j N_i(N_i - 1) < A_i. \tag{5}$$

That is, the total length of the partition for each agent $2|b_i|x_j N_i(N_i - 1)$ should be less than the length of the support of θ_i, A_i.

Lemma 2 *(i)* $E(\overline{m}_i^2) = \frac{A_i^2}{3} - \frac{A_i^2}{12N_i^2} - \frac{b_i^2 x_j^2 (N_i^2-1)}{3}$*; (ii)* $E(\overline{m}_i^2)$ *is strictly increasing in* N_i*, strictly decreasing in* x_j *and* b_i*, and strictly increasing in* $E(\overline{m}_j^2)$*; (iii)* $\frac{A_i^2}{4} \leq E(\overline{m}_i^2) \leq \frac{A_i^2}{3}$.

Intuition Lemma 2 formally shows that the agents' equilibrium information transmissions exhibit strategic complementarity: $E(\overline{m}_i^2)$ is increasing in $E(\overline{m}_j^2)$ and vice versa. This result comes from the fact that the DM's ideal decision, $\theta_1\theta_2$, is multiplicative in two states. To understand the intuition for this result, consider the incentives of agent 1 to misrepresent his information. For this purpose, suppose that agent 1 can credibly misrepresent his information by reporting $\widehat{\theta}_1$ and that agent 2 sends a message according to $\mu_2(m_2|\theta_2)$.[8] Then the optimal report of agent 1, $\widehat{\theta}_1$, minimizes

$$E_{\theta_2}\{[\widehat{\theta}_1 E_{\theta_2}(\theta_2|m_2) - \theta_1\theta_2 - b_1]^2\}.$$

The optimal "message distortion," which is measured by $\widehat{\theta}_1 - \theta_1$, is given by

$$\widehat{\theta}_1 - \theta_1 = \frac{E(\theta_2)b_1}{E[E(\theta_2|m_2)^2]} = \frac{E(\theta_2)b_1}{(E(\theta_2))^2 + var[\overline{m}_2]}. \tag{6}$$

From (6), we see that agent 1's incentive to distort information depends on $var[\overline{m}_2]$, or the informativeness of agent 2's communication. In particular, as agent 2 transmits more information (i.e., $var[\overline{m}_2]$ increases), according to (6) agent 1's incentive to distort information, measured by $\widehat{\theta}_1 - \theta_1$, decreases.[9]

To understand the result, note that agent 1 always wants to distort the DM's decision (relative to the DM's ideal action) by b_1. Agent 1, however, can only distort his own report

[8] ADM perform a similar exercise to study agents' incentives to misrepresent their information.

[9] When agent 2 transmits no information, the RHS of (6) becomes $b_1/E(\theta_2)$. When agent 2 transmits full information, the RHS of (6) becomes $E(\theta_2)b_1/E(\theta_2^2)$, which is less than $b_1/E(\theta_2)$.

to achieve this end. Given that the ideal decision is multiplicative, the impact of agent 1's distortion on the DM's action depends on agent 2's report. In particular, if agent 1 expects a higher (lower) average θ_2, then the DM's expected action becomes more (less) sensitive to the report concerning θ_1, and any given distortion by agent 1 will be amplified (dampened). As a result, agent 1 will have an incentive to reduce (increase) his information distortion.

The above discussion describes a "mean" effect: when the mean of θ_2 increases, agent 1 has an incentive to reduce his own information distortion. This mean effect, however, is exogenous in the sense that it does not depend on how much information is transmitted by agent 2 because the mean effect works through agent 1's ex ante expectation about the DM's posterior regarding θ_2, $\overline{m}_2$. No matter whether agent 2 constantly overstates or understates θ_2, he cannot successfully mislead the DM in shifting the mean, as the DM takes into account his incentive to distort information. Therefore, the expectation about the DM's posterior is always the unconditional mean, which is unbiased.[10] As a result, the mean effect does not depend on the agents' information transmission strategies.

Agent 2's information transmission strategy, however, will in general affect the variance of $\overline{m}_2$, and this in turn affects agent 1's incentive to distort information. To illustrate this "variance" effect, compare the cases in which agent 2 transmits no information (i.e., sends only one message m_{20}) and in which agent 2 has two partition elements with two potential messages m_{21} and m_{22}, $\overline{m}_{21} < \overline{m}_{22}$. Note that in both cases $E(\overline{m}_2) = E(\theta_2)$, and $\overline{m}_{21} < \overline{m}_{20} = E(\theta_2) < \overline{m}_{22}$. In other words, the $\overline{m}_2$ in the second case is a mean preserving spread of the $\overline{m}_2$ in the first case. In the first case, agent 1's optimal message distortion is $b_1/E(\theta_2)$. Now consider the second case, and suppose agent 1 still distorts his message by $b_1/E(\theta_2)$. Because $\overline{m}_{21} < E(\theta_2) < \overline{m}_{22}$, relative to his ideal action $(\theta_1\overline{m}_{2j}+b_1)$ agent 1 under-distorts information when agent 2's message is m_{21} and over-distorts information when agent 2's message is m_{22}. Now overall agent 1 has an incentive to reduce his information distortion slightly (by, say, $\varepsilon > 0$) from the initial amount $b_1/E(\theta_2)$. To see this, note that such a reduction in information distortion brings the DM's expected action closer to agent 1's ideal action when agent 2's message is m_{22} but pulls the DM's expected action further away from his ideal action when agent 2's message is m_{21}. However, the benefit of the first effect is bigger than the loss resulting from the second effect because, given $\overline{m}_{22} > \overline{m}_{21}$, the reduction in over-distortion $(\overline{m}_{22}\varepsilon)$ is bigger than the increase in under-distortion $(\overline{m}_{21}\varepsilon)$.[11] To summarize, the distance between the DM's action and agent 1's ideal action in the second

[10] Formally, this is due to the law of iterative expectation: $E[E(\theta_2|m_2)] = E(\theta_2)$.

[11] To see that more formally, let α $(1-\alpha)$ be the probability that agent 2 sends message m_{21} (m_{22}). Since $E(\overline{m}_2) = E(\theta_2)$, we have $\alpha = \frac{\overline{m}_{22}-E(\theta_2)}{\overline{m}_{22}-\overline{m}_{21}}$. Suppose agent 1 distorts his information by $b_1(\frac{1}{E(\theta_2)} - \varepsilon)$, with $\varepsilon > 0$ where ε is very small. That is, agent 1 reduces the amount of information distortion slightly. Now agent 1's

case is a mean preserving spread of that in the first case, leading to over-distortion in higher states (of θ_2) and under-distortion in lower states. Given that the DM's expected action is more sensitive to agent 1's report at higher states of θ_2, if agent 1 reduces the amount of information distortion the reduction in over-distortion at higher states of θ_2 will be bigger than the increase in under-distortion at lower states of θ_2, thus overall bringing the DM's expected action closer to agent 1's ideal action. Therefore, agent 1 will distort his information by less in the second case than in the first case.

Generally, as agent 2 transmits more information and $\overline{m}_2$ has a larger variance, the sensitivity of the DM's action to agent 1's report will vary more. More precisely, the new $\overline{m}_2$ is a mean preserving spread of the old $\overline{m}_2$. As a result, with the same amount of information distortion by agent 1 the distance between the DM's action and agent 1's ideal action under the new $\overline{m}_2$ is a mean preserving spread of that under the old $\overline{m}_2$, leading to bigger over-distortions at higher states of θ_2 and bigger under-distortions at lower states of θ_2. Agent 1, however, can exploit the bigger variance in the sensitivity of the DM's action to his own report. By reducing the amount of distortion a little bit, agent 1 can reduce the amount of over-distortion in higher states significantly without increasing the amount of under-distortion in lower states by as much. Therefore, agent 1 will have an incentive to reduce his information distortion when agent 2 transmits more information.

We want to emphasize that—in contrast to the exogenous mean effect—the variance effect is endogenous in the sense that it depends on how much information is transmitted by the other agent as the other agent chooses how much information to transmit. Therefore it is the variance effect that generates the strategic complementarity between the agents' information transmissions. Note that the assumption that the ideal decision is multiplicative in the states is mainly driving this result. When the two states are additive (e.g., $y^*(\theta_1, \theta_2) = \theta_1 + \theta_2$) such

expected payoff (minus some constant) $U^{A_1}(\varepsilon)$ can be written as:

$$U^{A_1}(\varepsilon) = -b_1^2\{\alpha[1 - \frac{\overline{m}_{21}}{E(\theta_2)} + \overline{m}_{21}\varepsilon]^2 + (1-\alpha)[\frac{\overline{m}_{22}}{E(\theta_2)} - \overline{m}_{22}\varepsilon - 1]^2\}.$$

The derivative of U^{A_1} with respect to ε evaluated at $\varepsilon = 0$ can be calculated as:

$$\begin{aligned}\frac{dU^{A_1}}{d\varepsilon}(\varepsilon &= 0) \sim \alpha[1 - \frac{\overline{m}_{21}}{E(\theta_2)}](-\overline{m}_{21}) + (1-\alpha)[\frac{\overline{m}_{22}}{E(\theta_2)} - 1]\overline{m}_{22} \\ &= \alpha[1 - \frac{\overline{m}_{21}}{E(\theta_2)}](\overline{m}_{22} - \overline{m}_{21}) > 0,\end{aligned}$$

where the last equality uses the fact that $\alpha = \frac{\overline{m}_{22} - E(\theta_2)}{\overline{m}_{22} - \overline{m}_{21}}$. Therefore, at the optimal amount of information distortion in the first case, agent 1 will benefit from reducing his amount of information distortion in the second case.

that $\frac{\partial^2 y^*(\theta_1,\theta_2)}{\partial\theta_1\partial\theta_2} = 0$, this strategic interaction between the agents is absent.[12]

Comparison to literature The property that the agents' information transmissions are strategic complements in our model is quite different from results in previous papers. In ADM, the agents' information transmissions are strategically independent in the sense that the amount of information one agent transmits does not depend on how much information the other agent transmits. This is because the ideal decisions in their model are additive in the private signals (the local conditions).

In Austen-Smith (1993), the two agents' information transmissions are strategic substitutes: informative communication of one agent becomes more difficult when the other agent transmits more information. Morgan and Stocken (2008) have the same feature that agents' truthful reports are strategic subsitututes: as the number of agents increases, each agent has less incentive to report truthfully. Note that in both models the agents' biases are type-independent and thus their results are in direct contrast to our result. The strategic substitutability of information transmissions in their models is due to the fact that in both models the state of the world is one dimensional and agents receive conditionally independent signals (i.e., the agents' private information is overlapping). Under this information structure, as other agents transmit more information the receiver's decision becomes less sensitive to the report of any individual agent (an information congestion effect), and thus individual agents have less incentive to transmit information. In contrast, in our model agents have non-overlapping private information. Thus the information congestion effect in their models is absent in our model. Instead, the strategic interaction between the agents operates through the variance effect when the ideal decision is multiplicative in the states.

For PBE of the overall communication game, a babbling equilibrium always exists in which $N_1 = N_2 = 1$ and the DM ignores the messages. Thus we do not need to worry about the existence of PBE. Straightforward calculation shows that the ex ante equilibrium payoffs for the DM, U^P_{ST}, and for agent i, $U^{A_i}_{ST}$ (where the subscripts denote simultaneous talk), are given by:

$$\begin{aligned} U^P_{ST} &= -E[(\overline{m}_1\overline{m}_2 - \theta_1\theta_2)^2] = -E(\theta_1^2)E(\theta_2^2) + E(\overline{m}_1^2)E(\overline{m}_2^2). \qquad (7) \\ U^{A_i}_{ST} &= -E(\theta_1^2)E(\theta_2^2) + E(\overline{m}_1^2)E(\overline{m}_2^2) - b_i^2 \end{aligned}$$

[12] When $y^*(\theta_1,\theta_2) = \theta_1 + \theta_2$, the indifference condition for agent i's partition points $a_{i,n}$ is given by

$$(a_{i,n+1} - a_{i,n}) - (a_{i,n} - a_{i,n-1}) = 4b_i.$$

It is clear that each agent's information transmission is independent from the other's, and the two-sender model collapses to a one-sender model.

By inspection, the most informative equilibrium on which we focus is also ex ante Pareto dominant.

Proposition 1 *(i) Any equilibrium is characterized by a pair of numbers of partition elements* (N_1, N_2) *that satisfy*

$$E(\overline{m}_1^2) = \frac{E(\theta_1)}{x_1} \Leftrightarrow \frac{A_1^2}{3} - \frac{A_1^2}{12N_1^2} - \frac{b_1^2 x_2^2 (N_1^2 - 1)}{3} = \frac{A_1}{2x_1}, \tag{8}$$

$$E(\overline{m}_2^2) = \frac{E(\theta_2)}{x_2} \Leftrightarrow \frac{A_2^2}{3} - \frac{A_2^2}{12N_2^2} - \frac{b_2^2 x_1^2 (N_2^2 - 1)}{3} = \frac{A_2}{2x_2}; \tag{9}$$

and the two inequalities

$$2|b_1|x_2N_1(N_1 - 1) < A_1; \quad 2|b_2|x_1N_2(N_2 - 1) < A_2. \tag{10}$$

(ii) Both N_1 *and* N_2 *are finite. (iii) In the most informative equilibrium, the partition elements* (N_1^*, N_2^*) *are the largest* N_1 *and* N_2 *that satisfy (8), (9) and (10), and their lower and upper bounds are given by*

$$\langle -\frac{1}{2} + \frac{1}{2}(1 + \frac{A_1A_2}{|b_i|})^{1/2}\rangle \leq N_i^* \leq \langle -\frac{1}{2} + \frac{1}{2}(1 + \frac{4A_1A_2}{3|b_i|})^{1/2}\rangle \tag{11}$$

(iv) For (N_1, N_2) *such that* $1 \leq N_1 \leq N_1^*$ *and* $1 \leq N_2 \leq N_2^*$*, the existence of equilibrium is not guaranteed. If there is an equilibrium with* (N_1, N_2) *and* $1 < N_i \leq 8$ *for both* $i = 1, 2$*, then an equilibrium with* $(N_1 - 1, N_2 - 1)$ *exists as well.*

Proof. Part (i) follows immediately from previous analysis. In particular, given N_1 and N_2, x_1 and x_2 are determined from equations (8) and (9). If N_1, N_2, x_1 and x_2 satisfy the inequalities (10), then there is an equilibrium associated with N_1 and N_2, with the partition elements being characterized by (4). To show part (ii), note that by Lemma 2 x_i has a lower bound $\frac{3}{2A_i} > 0$. Now inspecting the inequalities (10), we can see that as N_i goes to infinity one of the inequalities must be violated. Therefore, both N_1 and N_2 must be finite.

As to part (iii), we see that by (7) $E(\overline{m}_1^2)$ and $E(\overline{m}_2^2)$ should be maximized in the most informative equilibrium. By part (ii) of Lemma 2, $E(\overline{m}_i^2)$ is increasing in N_i. Moreover, $E(\overline{m}_i^2)$ is decreasing in x_j and x_j is decreasing in $E(\overline{m}_j^2)$. Thus $E(\overline{m}_i^2)$ is increasing in N_j as well. Therefore, N_1 and N_2 are maximized in the most informative equilibrium subject to condition (10). The bounds of N_i^* come from part (iii) of Lemma 2. In particular, (11) follows from the fact that $x_i \in [\frac{3}{2A_i}, \frac{2}{A_i}]$.

Regarding part (iv), we see that the ratio of the upper bound of x_i to its lower bound is 4/3. On the other hand, the ratio of $\frac{(N_i-1)(N_i-2)}{N_i(N_i-1)} = (N_i - 2)/N_i$, and this ratio is less than 3/4 if N_i

is less than 8. Therefore, if conditions (10) are satisfied with (N_1, N_2) and $1 < N_i \leq 8$ for both $i = 1, 2$, they must be satisfied with $(N_1 - 1, N_2 - 1)$. Hence an equilibrium with $(N_1 - 1, N_2 - 1)$ exists. ∎

In standard CS cheap talk models, if the number of partitions in the most informative equilibrium is N^*, then for all N such that $1 \leq N < N^*$, there is a corresponding equilibrium. Similar results do not hold in our model, as illustrated by part (iv) of Proposition 1.[13] This is due to the fact that in our model the two agents' information transmissions are strategic complements. If the number of partition elements in one agent's partition is reduced and, hence, less information is transmitted, the other agent will have less incentive to transmit information. As a result, the original equilibrium number of partition elements for the second agent might no longer be sustainable.[14]

Corollary 1 *In the most informative equilibrium, a decrease in b_i results in not only an increase in $E(\overline{m}_i^2)$ but also an increase in $E(\overline{m}_j^2)$.*

Proof. By Proposition 1, N_1^* and N_2^* are nonincreasing in b_i. The rest follows Lemma 2. ∎

Corollary 1 can potentially be empirically tested: when one agent is replaced by a new agent whose interests are more aligned with those of the DM, more information will be transmitted by the other agent. This is illustrated in the following example.

Example 1 *Suppose $A_1 = 10$ and $A_2 = 4$. Agent 1 has bias $b_1 = 2$, and agent 2 has bias $b_2 = 1.15$. Under simultaneous communication, $N_1^* = 2$, $N_2^* = 2$, $E(\overline{m}_1^2) = 30.6009$, and $E(\overline{m}_2^2) = 4.9646$. When b_2 decreases to $\frac{3}{4}$, $N_1^* = 2$, $N_2^* = 3$, $E(\overline{m}_1^2) = 30.6456$, and $E(\overline{m}_2^2) = 5.1453$. Note that $E(\overline{m}_1^2)$ increases as b_2 decreases.*

Corollary 2 *(i) The DM's equilibrium payoff when consulting two agents is higher than that when only one agent is consulted. (ii) Fix all the other parameter values but change b_i to $-b_i$. Then (N_1^*, N_2^*), $E(\overline{m}_i^2)$ and $E(\overline{m}_2^2)$ remain the same, and each player's ex ante equilibrium payoff is unchanged.*

[13] Specifically, if $N_i > 8$ for both $i = 1, 2$ then the following situation could arise: an equilibrium with (N_1, N_2) exists while an equilibrium with $(N_1 - 1, N_2 - 1)$ does not exist.

[14] For example, with $(N_1^* - 1, N_2^*)$ an equilibrium might not exist. When the number of elements in agent 1's partition is reduced to $N_1^* - 1$, the amount of information transmitted by agent 1 decreases and thus x_1 increases. With this increase in the incremental step size for agent 2, the inequality of (10) for agent 2 might be violated with N_2^*.

Proof. Suppose the DM only consults one agent, say agent i. Then no information is transmitted by agent j. $E(\overline{m}_j^2)$ reaches its lower bound $\frac{A_j^2}{4}$, and x_j reaches its upper bound $\frac{2}{A_j}$. It follows that the effective incremental step size for agent i is (weakly) bigger than when two agents are consulted. By Lemma 2, this implies that both $E(\overline{m}_i^2)$ and $E(\overline{m}_1^2)E(\overline{m}_2^2)$ are smaller when only agent i is consulted. This proves part (i).

Observing (8) and (9), we see that, fixing N_i, $E(\overline{m}_i^2)$ remains the same when b_i is replaced by $-b_i$. This means that $E(\overline{m}_j^2)$ remains the same as well. Also note that condition (10) remains the same when the sign of b_i changes. Therefore, N_1^* and N_2^* will not change either. This proves part (ii). ■

As Corollary 2 indicates, in our model equilibrium information transmission does not depend on whether the two agents have opposing or like biases, and it is always better for the DM to consult two agents instead of one.[15] Whether two agents have like or opposing biases does not matter in our model because the interaction between the two agents' communication occurs only through the terms $E(\overline{m}_1^2)$ and $E(\overline{m}_2^2)$. When b_i changes sign, only the direction of the partition of m_i reverses; the same amount of information is transmitted in equilibrium. One may still wonder whether two agents having like or opposing biases matters based on the following logic. Suppose both agents initially have positive biases. One may think that the agents have less incentive to overstate their information because each agent expects the other to overstate his information so he needs to overstate his own information less to induce his ideal action. After agent 1's bias changes to $-b_1$, one might think that agent 2 has a stronger incentive to overstate his own information as he expects agent 1 to understate θ_1. This is not, however, what occurs in equilibrium. In equilibrium, the principal is not fooled, which means that neither agent can successfully understate or overstate his information. Anticipating this, whether one agent has an incentive to understate or overstate his information (as long as the absolute value of the bias is the same) will not affect the other agent's incentive to misrepresent his own information.

3.2 Robustness of the results

Negative Domain So far we have assumed that the support of θ_i is $[0, A_i]$. In the following proposition we study what happens if the support of θ_i is extended into the negative domain.

[15] These results are different from those in KM, where equilibrium information transmission depends on whether the two agents have opposing or like biases. Specifically, if the two agents have like biases, then there is no strict benefit from consulting the second agent for the DM. The results in KM are mainly due to sequential communication. In Krishna and Morgan (2001b) where communication is simultaneous, having a second expert is always valuable because it allows the DM to achieve full information revelation.

Proposition 2 *Suppose $y^*(\theta_1, \theta_2) = \theta_1\theta_2$ and for $i = 1, 2$, θ_i is uniformly distributed on $[-B_i, A_i]$, with $A_i > 0$ and $B_i > 0$. If $B_i = A_i$ and agent i's communication is informative, then in the most informative equilibrium agent j reveals his information fully. If $B_i \neq A_i$, then the results in Proposition 1 hold qualitatively, and any equilibrium involves finite partition elements for agent j.*

Proposition 2 shows that in a knife-edge case in which the support of an agent's information is symmetric with respect to zero, the other agent might reveal information fully in equilibrium. This is because the exogenous mean effect is subtle when the support is extended to the negative domain. To understand the intuition, suppose $b_1 > 0$ and $E(\theta_2) = 0$. If agent 1 overstates his information, he can potentially pull the DM's decision to the right if the posterior regarding θ_2 is positive, but the decision will be pulled to the left if the posterior regarding θ_2 is negative. When $E(\theta_2) = 0$, the posterior regarding θ_2 is equally likely to be positive or negative, implying that agent 1 can gain nothing by overstating or understating his information. As a result, he has no incentive to distort his information and reveals his information fully in equilibrium.

General distributions Now we discuss how our results will change if we generalize the distribution of the underlying states. For $i = 1, 2$, suppose θ_i is distributed on $[0, A_i]$ with density function $f_i(\theta_i)$ and cumulative distribution function $F_i(\theta_i)$, and $f_i(\theta_i) > 0$ holds everywhere on the support $[0, A_i]$. By Lemma 1, all equilibria must be of partition form. Let the sequence of partition points be $a_{i,n}$, and $\overline{m}_{i,n}$ be the DM's posterior regarding θ_i after receiving a message $m_{i,n} \in (a_{i,n-1}, a_{i,n})$. Now $\overline{m}_{i,n} = \int_{a_{i,n-1}}^{a_{i,n}} \frac{\theta_i f_i(\theta_i) d\theta_i}{F_i(a_{i,n}) - F_i(a_{i,n-1})}$. The indifference condition that characterizes $a_{1,n}$ now is written as:

$$\int_{a_{1,n-1}}^{a_{1,n}} \frac{\theta_1 f_1(\theta_1) d\theta_1}{F_1(a_{1,n}) - F_1(a_{1,n-1})} + \int_{a_{1,n}}^{a_{1,n+1}} \frac{\theta_1 f_1(\theta_1) d\theta_1}{F_1(a_{1,n+1}) - F_1(a_{1,n})} - 2a_{1,n} = \frac{E(\theta_2)}{E(\overline{m}_2^2)} 2b_1. \qquad (12)$$

Inspecting (12), we see that the effective bias of agent 1, $b_1' \equiv \frac{E(\theta_2)}{E(\overline{m}_2^2)} 2b_1$, decreases as $E(\overline{m}_2^2)$ increases or agent 2 transmits more information. To show that agent 1 transmits more information when his effective bias decreases under general distributions, we need to impose a monotonicity condition on the solutions to the difference equation system (Assumption M in Section 5 of CS), which roughly says that the solutions must all move up or down together.[16] As long as this monotonicity condition holds, the agents' information transmissions are still

[16] Formally, Assumption M is as follows. Let a_1 and $\widetilde{a}_1$ be two forward solutions ($a_{1,n} < a_{1,n+1}$) of the difference equation system with N_1 partition elements each. For a given bias of b_1', if $a_{1,1} > \widetilde{a}_{1,1}$, then $a_{1,n} > \widetilde{a}_{1,n}$ for any $2 \leq n < N_1$. In Theorem 2 of CS, they provide a sufficient condition for Assumption M to hold. The sufficient condition depends on the density function of the distribution.

strategic complements under general distributions: as agent 2 transmits more information and $E(\overline{m}_2^2)$ increases, b_1' decreases and agent 1 transmits more information as well.

More general functions for the ideal decision It would be desirable to consider a general function for the ideal decision $y^*(\theta_1, \theta_2)$ that is monotonic insofar as $y_1^*(\theta_1, \theta_2) > 0$ (or $y_1^*(\theta_1, \theta_2) < 0$) and $y_2^*(\theta_1, \theta_2) > 0$ (or $y_2^*(\theta_1, \theta_2) < 0$) and not purely additive (i.e., either $y_{12}^*(\theta_1, \theta_2) > 0$ or $y_{12}^*(\theta_1, \theta_2) < 0$). It is, however, hard to derive analytical results without further specifying the functional form of $y^*(\theta_1, \theta_2)$. We focus on the case in which $y^*(\theta_1, \theta_2) = s(\theta_1)t(\theta_2)$, with both $s(\cdot)$ and $t(\cdot)$ being monotonic. In this case, the DM's optimal action is given by $E[s(\theta_1)|m_1]E[t(\theta_2)|m_2]$. We examine agent 1's incentive to distort information. Suppose agent 2 sends a message according to $\mu_2(m_2)$, and the DM believes that agent 1 tells the truth. The optimal report for agent 1, $\widehat{\theta}_1$, minimizes

$$E_{\theta_2}\{[s(\widehat{\theta}_1)E_{\theta_2}(t(\theta_2)|m_2) - s(\theta_1)t(\theta_2) - b_1]^2\}.$$

The optimal "message distortion" is given by

$$s(\widehat{\theta}_1) - s(\theta_1) = \frac{E[E(t(\theta_2)|m_2)]b_1}{E[(E(t(\theta_2)|m_2))^2]} = \frac{E(t(\theta_2))b_1}{[E(t(\theta_2))]^2 + var[E(t(\theta_2)|m_2)]}. \tag{13}$$

Since $s(\cdot)$ is monotonic, the LHS of the above equation (13) measures agent 1's incentive to distort information (whether s is monotonically increasing or decreasing, a bigger $s(\widehat{\theta}_1) - s(\theta_1)$ implies a bigger $\widehat{\theta}_1 - \theta_1$). As the amount of information transmitted by agent 2 increases (i.e., $var[E(t(\theta_2)|m_2)]$ increases), the RHS of (13) decreases. In particular, $var[E(t(\theta_2)|m_2)] = 0$ when agent 2 transmits no information, and $var[E(t(\theta_2)|m_2)]$ reaches its maximum $var(t(\theta_2))$ when agent 2 transmits full information. Equation (13) shows that agent 1's incentive to distort information decreases when agent 2 transmits more information, and agent 1 will transmit more information as well. Therefore, the agents' information transmissions are strategic complements.[17]

Note that $y^*(\theta_1, \theta_2) = (\theta_1)^k(\theta_2)^l$, $k \neq 0$ and $l \neq 0$, is a special case of the case we studied. To see how things work in more detail, consider the special case in which $y^*(\theta_1, \theta_2) = \theta_1/\theta_2$. Now the agents' optimal message distortions are given by:

$$\begin{aligned} \widehat{\theta}_1 - \theta_1 &= \frac{E(\frac{1}{\theta_2})b_1}{[E(\frac{1}{\theta_2})]^2 + var[E(\frac{1}{\theta_2}|m_2)]}; \\ \frac{1}{\widehat{\theta}_2} - \frac{1}{\theta_2} &= \frac{E(\theta_1)b_2}{[E(\theta_1)]^2 + var[E(\theta_1|m_1)]}. \end{aligned}$$

[17] Switching $y^*(\theta_1, \theta_2)$ to $-y^*(\theta_1, \theta_2)$ only changes the sign of each expert's effective bias and would not change the results qualitatively.

For either agent, the optimal "message distortion" decreases when the other agent transmits more information. When agent 2 reports a higher (lower) message, the DM's posterior $E(\frac{1}{\theta_2}|m_2)$ is smaller (bigger), and agent 1's distortion of information is dampened (amplified)—meaning that the "mean effect" works in the opposite way that it does in our leading examply by making the DM's expected action less sensitive to agent 1's report at high values of θ_2. The variance effect, however, still works in the same way. As agent 2 transmits more information, the new $E(\frac{1}{\theta_2}|m_2)$ becomes a mean-preserving spread of the old one. By reducing the amount of distortion a little bit, agent 1 can reduce the amount of over-distortion in lower states of θ_2 significantly without increasing the amount of under-distortion in higher states by as much. Therefore, agent 1 will have an incentive to reduce the amount of his information distortion when agent 2 transmits more information.

Although we are not able to derive analytical results for more general functional forms, we expect our main intuition to more or less hold with type-independent biases. Essentially, the driving force behind the result that the agents' information transmissions are strategic complements is that the sensitivity of the DM's action to one agent's report depends on the other agent's report. This feature of changing sensitivity will be present as long as $\partial^2 y^*(\theta_1, \theta_2)/\partial\theta_1\partial\theta_2 \neq 0$. When agent 2 transmits more information, the variance of the DM's sensitivity to agent 1's report increases. This leads to the variance effect and makes the agents' information transmissions strategic complements.

3.3 Optimal assignment of agents

In some environments, the DM may have the freedom to change the assignment of agents to observe information in different dimensions. Suppose the two information dimensions have different underlying uncertainty, and the agents have different biases. Without loss of generality, suppose $A_1 > A_2$ and $|b_1| = r|b|$ and $b_2 = b$ with $r > 1$; that is, θ_1 has a bigger variance (and a bigger mean) and agent 1 has a bigger bias. The question naturally arises: to induce more effective overall communication, should the DM assign the agent with the smaller or larger bias to observe the dimension with more uncertainty (and a bigger mean)? We refer to assigning agent 2 to observe θ_1 and agent 1 to observe θ_2 as positive assortative (PA) assignment and the reverse assignment as negative assortative (NA) assignment.

Without the strategic interactions between the agents' information transmissions, compared to NA assignment, PA assignment will lead to more information transmitted regarding θ_1 (about which there is more underlying uncertainty), but less information transmitted regarding θ_2

(about which there is less underlying uncertainty).[18] Different assignments will also affect the overall information transmission through the strategic complementarity between two agents' information transmission. Thus it is not obvious which assignment will lead to more overall information transmission. The following proposition identifies the conditions under which assignments do not matter.

Proposition 3 *(i) If r is big enough such that agent 1's communication is uninformative under both assignments, then both assignments yield the same ex ante payoff for the principal. (ii) If $b \to 0$ but $r|b| > 0$, then both assignments yield the same ex ante payoff. (iii) Suppose both agents' communications are informative under either assignment, and let N_1^* and N_2^* ($N_1^{*'}$ and $N_2^{*'}$) be the equilibrium numbers of partitions under NA (PA) assignment. If $N_1^* = N_2^{*'}$ and $N_2^* = N_1^{*'}$, then both assignments yield the same ex ante payoff.*

Parts (i) and (ii) of Proposition 3 show that assignment does not matter if one agent's bias is big enough such that his communication is always uninformative, or if one agent's bias is arbitrarily small such that his communication will be fully informative. Part (iii) considers the scenario when communication is informative but not fully revealing in both dimensions. It shows that assignments do not matter as long as the number of partitions just flips when we change the assignments. Though the latter condition cannot be proved for all parameter values, it holds for most cases. To see this, first note that by (11) the lower and upper bounds of N_i^* under the NA assignment are the same as those of $N_j^{*'}$ under PA assignment. Second, whenever communication is informative for agent i, the equilibrium number of partitions for this agent is very likely to reach the upper bound in (11). This is illustrated by the following numerical example.

Example 2 *Suppose $A_1 = 7$ and $A_2 = 5$ and that agent 1 has bias $b_1 = 1.91$ and agent 2 has bias $b_2 = 1.89$. Under NA assignment, the lower and upper bounds for N_1^* are 1 and 2 (2.03433), respectively, and those for N_2^* are 1 and 2 (2.02115), respectively. The equilibrium N_1^* and N_2^* are both 2, the upper bound. Note that the upper bound seems hard to achieve (2.03433 is only slightly above 2): for N_1^* to be 2, $E(\overline{m}_2^2)$ has to be very close to its upper bound (when the message is fully revealing). The upper bound for N_2^*, however, is 2 as well. This means that even with just two partitions, $E(\overline{m}_2^2)$ is already very close to its upper bound. This is indeed the*

[18] Due to the exogenous mean effect, other things being equal the agent assigned to θ_1 tends to distort information more than the agent assigned to θ_2 because the mean of θ_1 is bigger than that of θ_2. In this sense, PA assignment tends to equalize the effective biases between two agents compared to NA assignment.

case. The lower and upper bound for $E(\overline{m}_2^2)$ *are* 6.25 *and* 8.33 *respectively, while* $E(\overline{m}_2^2)$ *with two partitions is* 7.6159*.*

The example above illustrates that even when an agent i's equilibrium partition has only two elements, the conditional variance of θ_i given m_i is close to its upper bound. This implies that the strategic complementarity in communication is already very strong and makes it very likely that in equilibrium agent j's communication will achieve the upper bound for the number of partition elements N_j^*. Given that these upper bounds on the number of partition elements are very likely to be achieved, changing assignments will most likely just result in the number of partition elements switching (in which case part (iii) of Proposition 2 applies). Therefore, we conclude that in most cases assignments do not matter for overall information transmission.

4 Type-dependent Biases

In this section we consider an alternative model in which agents' biases are type dependent. In particular, we assume that each agent cares only about his own state. That is, the agents' utility functions are given by:

$$U^{A_1} = -(y-\theta_1)^2; U^{A_2} = -(y-\theta_2)^2.$$

On the other hand, the DM's utility function is still given by

$$U^P = -[y - y^*(\theta_1,\theta_2)]^2.$$

Note that the difference between agent i's ideal action and the DM's is $\theta_i - y^*(\theta_1,\theta_2)$, which in general depends on the realization of states. These preferences are realistic in multidivisional organizations as each division manager may care only (or more) about the performance of his own division. Similar biases appear in ADM and Martimort and Semenov (2008). In particular, in ADM each division manager cares more about the performance of his own division than that of the other division, while in Martimort and Semenov each interest group only cares about the local conditions of its own group. Again, θ_i is assumed to be uniformly distributed on $[0, A_i]$.

4.1 DM's ideal action is additive in states

First consider the case in which $y^*(\theta_1,\theta_2) = (\theta_1+\theta_2)/2$, or the DM's ideal action is the average of the two realized states.[19] In this setup, the DM's optimal action is $\frac{1}{2}(E[\theta_1|m_1] + E[\theta_2|m_2])$.

[19] The same setting is studied in Martimort and Semenov (2008), though they use a mechanism design approach.

Now we examine agent 1's incentive to distort information. Suppose agent 2 sends message according to $\mu_2(m_2)$, and the DM believes that agent 1 tells the truth. Then the optimal report of agent 1, $\widehat{\theta}_1$, minimizes

$$E_{\theta_2}\{[\frac{1}{2}(\widehat{\theta}_1 + E[\theta_2|m_2]) - \theta_1)]^2\}.$$

The optimal message distortion is given by:

$$\widehat{\theta}_1 - \theta_1 = \theta_1 - E(\theta_2).$$

Observing the above equation, we can see that agent 1's incentive to distort information is independent of agent 2's reporting stratgey. Hence the agents' information transmissions are strategically independent.[20]

4.2 DM's ideal action is multiplicative in states

Now consider the case in which $y^*(\theta_1, \theta_2) = \theta_1\theta_2$, our leading case. Note that the difference between agent i's ideal action and the DM's is $\theta_i(1 - \theta_j)$. To ensure that the DM's ideal action and agent i's ideal action coincide in expectation, we assume that for $i = 1, 2$ $A_i = 2$. Thus $E(\theta_1) = E(\theta_2) = 1$. As in the basic model, the DM's optimal action is $E[\theta_1|m_1]E[\theta_2|m_2]$ or $\overline{m}_1\overline{m}_2$.

By a similar proof to Lemma 1, we can show that all PBE must be interval equilibria.[21] To formally characterize the equilibria, let N_i be the size of the partition of agent i, and $(a_{i,0}, a_{i,1}, ..., a_{i,n}, ..., a_{i,N_i}) \equiv a_i$ be the partition points with $a_{i,0} = 0$ and $a_{i,N_i} = 2$. When the realized state happens to be any interior partition point $a_{1,n}$, agent 1 should be indifferent between sending two adjacent messages. More explicitly, the indifference condition can be written as

$$E_{\theta_2}[\{\overline{m}_2\frac{a_{1,n} + a_{1,n-1}}{2} - a_{1,n}\}^2] = E_{\theta_2}[\{\overline{m}_2\frac{a_{1,n} + a_{1,n+1}}{2} - a_{1,n}\}^2],$$

which can be simplified as

$$(a_{1,n+1} - a_{1,n}) - (a_{1,n} - a_{1,n-1}) = [\frac{E(\theta_2)}{E(\overline{m}_2^2)} - 1]4a_{1,n}. \tag{14}$$

[20] The above setup is actually a special case of the model in ADM under centralization in which the need for coordination goes to infinity and the own division bias is 1. The details of the equilibrium characterization can be found in ADM.

[21] In particular, let ν_1 be the DM's posterior belief regarding θ_1. The expected utility of agent 1 given θ_1 and induced belief ν_1 is given by

$$E_{\theta_2}[U_1|\theta_1, \nu_1] = -E_{\theta_2}[(\nu_1\overline{m}_2 - \theta_1)^2]$$

It can be easily verified that $(\partial^2/\partial^2\theta_1)E_{\theta_2}[U_1|\theta_1, \nu_1] < 0$ and $(\partial^2/\partial\theta_1\partial\nu_1)E_{\theta_2}[U_1|\theta_1, \nu_1] > 0$. These two conditions ensure that all PBE must be interval equilibria.

Similarly, the cutoff points $a_{2,n}$ characterizing agent 2's partition equilibrium satisfy the indifference condition:

$$(a_{2,n+1} - a_{2,n}) - (a_{2,n} - a_{2,n-1}) = [\frac{E(\theta_1)}{E(\overline{m}_1^2)} - 1]4a_{2,n}. \tag{15}$$

Define $b_i \equiv \frac{E(\theta_j)}{E(\overline{m}_j^2)} - 1$. Since $[E(\theta_j)]^2 \leq E(\overline{m}_j^2) \leq E(\theta_j^2)$, $E(\overline{m}_j^2) \in [1, \frac{4}{3}]$. Thus, $b_i \in [-\frac{1}{4}, 0]$. We first treat b_i as exogenous when we solve the difference equations (14) and (15). Specifically, the solution to (14) and (15) has the following form: $a_{i,n} = K_i \cos(\theta_i n + \omega_i)$, where K_i and ω_i are two constants, and θ_i satisfies

$$\cos(\theta_i) = 1 + 2b_i \text{ or } \theta_i = \arccos(1 + 2b_i). \tag{16}$$

By (16), θ_i is decreasing in b_i and $\theta_i \in [0, \frac{\pi}{3}]$ because $b_i \in [-\frac{1}{4}, 0]$. From the initial condition $a_{i,0} = 0$, we get $\omega_i = -\pi/2$. From the ending condition $a_{i,N_i} = 2$, we get $K_i = 2/\sin(\theta_i N_i)$. Let $\overline{N}_i$ be the upper bound of N_i. Because $a_{i,n}$ is increasing in n, $\cos(\theta_i n - \frac{\pi}{2}) = \sin(\theta_i n)$ must be increasing in n, or $\theta_i N_i \leq \pi/2$. Therefore, $\overline{N}_i$ is determined by the following inequalities:

$$\theta_i \overline{N}_i \leq \pi/2 < \theta_i(\overline{N}_i + 1). \tag{17}$$

Note that $\overline{N}_i$ is finite unless $b_i = 0$. To summarize, the solution to (14) and (15) with N_i partitions is

$$a_{i,n} = \frac{2}{\sin(\theta_i N_i)} \sin(\theta_i n), \tag{18}$$

where θ_i is given by (16) and the upper bound $\overline{N}_i$ is given by (17).

The difference equations (14) and (15) are actually almost the same as those in ADM except that b_i is negative in our model but positive in theirs. Moreover, 0 is in the support of θ_i in both models. However, the solution in our model is quite different from theirs. Specifically, in ADM the number of partitions has no upper bound, while in our model generically the upper bound is finite. This is because with a positive b_i the incremental step size is increasing. Moreover, the incremental step size goes to zero when θ_i is close to zero (the incremental step size is $4b_i\theta_i$). Thus an infinity of partition elements is possible in their model with many fine partitions around zero. In our model with a negative b_i, however, the incremental step size is decreasing. As a result, the partition including $\theta_i = 0$ is coarser than other partitions, and in general the number of partition elements is finite.

Lemma 3 *(i)*

$$E(\overline{m}_i^2) = \frac{4(1 + b_i)}{3 + 4b_i}(1 + \frac{b_i}{(\sin(N_i\theta_i))^2}), \tag{19}$$

where $\theta_i = \arccos(1 + 2b_i)$*; (ii)* $E(\overline{m}_i^2)$ *is increasing in* N_i *and decreasing in* $|b_i|$*; (iii)* $E(\overline{m}_i^2)$ *is decreasing in* $E(\overline{m}_j^2)$*.*

Intuition Lemma 3 shows that the agents' information transmissions are strategic substitutes: when agent i transmits more information ($E(\overline{m}_i^2)$ increases), agent j will transmits less information ($E(\overline{m}_j^2)$ decreases). To understand the intuition, again consider agent 1's incentive to distort information. Suppose agent 2 sends message according to $\mu_2(m_2)$, and the DM believes that agent 1 tells the truth. Then agent 1's optimal report, $\widehat{\theta}_1$, minimizes $E_{\theta_2}[(\widehat{\theta}_1\overline{m}_2 - \theta_1)^2]$. The optimal message distortion is given by:

$$\widehat{\theta}_1 - \theta_1 = [\frac{E(\theta_2)}{E(\overline{m}_2^2)} - 1]\theta_1 = [\frac{E(\theta_2)}{[E(\theta_2)]^2 + var(\overline{m}_2)} - 1]\theta_1. \tag{20}$$

Inspecting (20), we see that the term $var(\overline{m}_2)$ is present. Recall that $E(\theta_2) = 1$. When agent 2 transmits no information ($var(\overline{m}_2) = 0$), the RHS of (20) is 0. Thus agent 1 has no incentive to distort information and (in the most informative equilibrium) his information will be fully revealed. On the other hand, if agent 2 transmits some information ($var(\overline{m}_2) > 0$), then the RHS of (20) is strictly less than 0. This implies that agent 1 has an incentive to under-report and his information cannot be fully revealed. More generally, when agent 2 transmits more information ($var(\overline{m}_2)$ increases), the absolute value of $\frac{E(\theta_2)}{[E(\theta_2)]^2+var(\overline{m}_2)} - 1$ increases or the effective bias becomes more negative, and agent 1 has a stronger incentive to under-report and less information will be transmitted by agent 1. Therefore, the agents' information transmissions are strategic substitutes.

This result is the opposite of what we find when the agents' biases are type-independent, where the agents' information transmissions are strategic complements. The underlying intuition, however, is very similar. To illustrate the idea, we again compare the cases in which agent 2 transmits no information (i.e., sends only one message m_{20}) and in which agent 2 has two partition elements with two potential messages m_{21} and m_{22} such that $\overline{m}_{21} < \overline{m}_{22}$. Note that in both cases $E(\overline{m}_2) = E(\theta_2) = 1$, $\overline{m}_{21} < \overline{m}_{20} = 1 < \overline{m}_{22}$, and the $\overline{m}_2$ in the second case is a mean preserving spread of the $\overline{m}_2$ in the first case. In the first case, agent 1's optimal message distortion is 0 (report truthfully) since $\overline{m}_{20} = 1$. Now consider the second case and suppose agent 1 still reports truthfully. Because $\overline{m}_{21} < 1 < \overline{m}_{22}$, relative to his ideal action (θ_1) agent 1 under-reports information when agent 2's message is m_{21} and over-reports information when agent 2's message is m_{22}. We argue that agent 1 has an incentive to under-report slightly. This is because the sensitivity of the DM's expected action to agent 1's report is higher when agent 2 sends the high message m_{22}. If agent 1 under-reports a little bit, the reduction in over-reporting when the high message m_{22} is sent will be bigger than the increase in under-reporting when the low message m_{21} is sent, thus overall bringing the DM's expected action closer to agent 1's ideal action. Therefore, agent 1 will under-report in the second case relative to the first case.[22]

Generally, as agent 2 transmits more information (i.e., $\overline{m}_2$ has a larger variance), the marginal impact of agent 1's report on the DM's expected action will vary more. More precisely, the new $\overline{m}_2$ is a mean preserving spread of the old $\overline{m}_2$. As a result, with the same information transmission strategy by agent 1 the distance between the DM's action and agent 1's ideal action under the new $\overline{m}_2$ is a mean-preserving spread of that under the old $\overline{m}_2$, leading to bigger over-reporting in higher states of θ_2 and bigger under-reporting in lower states of θ_2. By under-reporting a little bit more, agent 1 can reduce the amount of over-report in higher states significantly without increasing the amount of under-report in lower states by much. Therefore, agent 1 will have an incentive to under-report a little bit more when agent 2 transmits more information.

The following proposition charaterizes the possible equilibria.

Proposition 4 *Any equilibrium is characterized by a pair of numbers of partition elements* (N_1, N_2) *such that*

$$E(\overline{m}_i^2) = \frac{4(1+b_i)}{3+4b_i}(1+\frac{b_i}{(\sin(N_i\theta_i))^2}),\ b_i \equiv \frac{1}{E(\overline{m}_j^2)} - 1,\ \textit{and}\ \theta_i = \arccos(1+2b_i),$$

and the upper bound of N_i *is given by (17).*

We do not need to worry about the existence of equilibrium as the following (very asymmetric) equilibrium always exists: one agent fully reveals his information and the other agent reveals no information. To see that this is an equilibrium, note that when agent j transmits no information, $b_i = 0$ and agent i will fully reveal his information. On the other hand, given agent i fully reveals his information, $b_j = -1/4$ and $\theta_j = \pi/3$, hence $\overline{N}_j = 1$ and agent j will reveal no information. Therefore, it is an equilibrium.

[22] To see this more formally, let α $(1-\alpha)$ be the probability that agent 2 sends message m_{21} (m_{22}). Since $E(\overline{m}_2) = E(\theta_2) = 1$, we have $\alpha = \frac{\overline{m}_{22}-1}{\overline{m}_{22}-\overline{m}_{21}}$. Suppose agent 1 under-reports his information by $\theta_1\varepsilon$, with $\varepsilon > 0$ where ε is very small. Agent 1's expected payoff (minus some constant) $U^{A_1}(\varepsilon)$ can be written as:

$$U^{A_1}(\varepsilon) = -\theta_1^2\{\alpha[1-\overline{m}_{21}+\overline{m}_{21}\varepsilon]^2 + (1-\alpha)[\overline{m}_{22}-\overline{m}_{22}\varepsilon-1]^2\}.$$

The derivative of U^{A_1} with respect to ε evaluated at $\varepsilon = 0$ can be calculated as:

$$\begin{aligned}\frac{dU^{A_1}}{d\varepsilon}(\varepsilon &= 0) \sim \alpha[1-\overline{m}_{21}](-\overline{m}_{21}) + (1-\alpha)[\overline{m}_{22}-1]\overline{m}_{22} \\ &= \alpha[1-\overline{m}_{21}](\overline{m}_{22}-\overline{m}_{21}) > 0,\end{aligned}$$

where the last equality uses the fact that $\alpha = \frac{\overline{m}_{22}-1}{\overline{m}_{22}-\overline{m}_{21}}$. Therefore, at the optimal amount of information distortion in the first case, agent 1 will benefit from under-reporting in the second case.

As in the basic model, the DM's expected utility is still given by $U_{ST}^P = -E(\theta_1^2)E(\theta_2^2) + E(\overline{m}_1^2)E(\overline{m}_2^2)$, which is strictly increasing in $E(\overline{m}_1^2)$ and $E(\overline{m}_2^2)$. Agent i's expected uility now is given by

$$U_{ST}^{A_i} = -E(\theta_i^2) + E(\overline{m}_i^2)[2E(\theta_j) - E(\overline{m}_j^2)]. \quad (21)$$

Inspecting (21), we see that $U_{ST}^{A_i}$ is strictly increasing in $E(\overline{m}_i^2)$ since $E(\theta_j) = 1$ and $E(\overline{m}_j^2) \leq 4/3$. Moreover, agent i's expected utility is decreasing in $E(\overline{m}_j^2)$. This implies two things. First, unlike in the basic model with type-independent biases where the overall most informative equilibrium in which $E(\overline{m}_1^2)E(\overline{m}_2^2)$ is maximized is Pareto dominant, in the current setting the agents and the DM have incentives to select different equilibria. Hence there is no Pareto dominant equilibrium. Second, because the agents' information transmissions are strategic subsitututes, by increasing $E(\overline{m}_i^2)$ agent i can decrease $E(\overline{m}_j^2)$, which will increase his utility indirectly. Therefore, each agent has an incentive to choose the equilibrium involving the most informative communication between himself and the DM. Due to strategic substitutability, there are multiple equilibria ranging from very asymmetric information transmissions (the equilibrium mentioned above is the extreme example in this regard) to rather symmetric ones. Because the agents have similar incentives, symmetric equilibria with both agents transmitting similar, medium amounts of information are more plausible than other equilibria.

4.3 Summary

One common feature under both the type-independent and type-dependent biases is that, when agent 2 transmits more information, the sensitivity of the DM's action to agent 1's report will vary more, and overall agent 1 has a stronger incentive to reduce the utility loss in higher states of θ_2. In the case of type-independent biases, agent 1 always wants to distort the DM's action by b_1. Thus agent 1 will always distort his information in the direction of his own bias; the question is by how much. When agent 2 transmits more information, any distortion in agent 1's report in the direction of his bias will be amplified more in higher states of θ_2 relative to the case in which agent 2 transmits less information. As a result, overall agent 1 has an incentive to reduce his information distortion. This implies that the agents' information transmissions are strategic complements. In the case of type-dependent biases, agent 1 wants to distort the DM's action by $\theta_1(1 - \theta_2)$. Given that $E(\theta_2) = 1$, agent 1 will report truthfully when agent 2 transmits no information. When agent 2 transmits some information, relative to agent 1's ideal action this truthful reporting would result in over-reporting at higher states of θ_2 and under-reporting at lower states of θ_2. When agent 2 transmits more information, the over-reporting will be amplified more in higher states of θ_2. As a result, overall agent 1 has an incentive to under-report a little bit more, implying that the agents' information transmissions are strategic substitutes.

The above discussion indicates that in the case of type-dependent biases, when agent 2 transmits more information agent 1 always has an incentive to under-report a little or reduce the amount of his over-reporting a little. Therefore, whether the agents' information transmissions are strategic substitutes or complements depends on the initial condition (whether agent 1 has an incentive to under-report or over-report when agent 2 transmits no information) and the ending condition (whether agent 1 has an incentive to under-report or over-report when agent 2 transmits full information). In the case we presented in which $E(\theta_2) = 1$, agent 1 has no incentive to distort information when agent 2 transmits no information. When agent 2 transmits more information, agent 1 will always under-report a little more. Thus the agents' information transmissions are strategic substitutes. However, if $E(\theta_2) < 1$, then agent 1 will over-report if agent 2 transmits no information. Now as agent 2 transmits more information, agent 1 has an incentive to reduce the amount of over-reporting, leading to more information information being transmitted by agent 1. Thus the agents' information transmissions can be strategic complements with type-dependent biases. More precisely, if $E(\theta_2^2) < E(\theta_2) < 1$, then agent 1 still has an incentive to over-report even when agent 2 transmits full information.[23] Therefore, the agents' information transmissions are always strategic complements. If $E(\theta_2) < 1$ and $E(\theta_2) < E(\theta_2^2)$, however, then agent 1 has an incentive to under-report even when agent 2 transmits full information. This implies the following pattern. Initially agent 1's incentive to over-report is reduced when agent 2 transmits more information. At some point as agent 2 transmits more information, agent 1 will report truthfully. After this point as agent 2 transmits more information, agent 1's incentive to under-report continues to increase.[24] Therefore, the agents' information transmissions are strategic complements when little information is being transmitted, but they become strategic substitutes when a lot of information is being transmitted.

The following table summarizes and provides a taxomony for the cases we considered.

Table 1: Taxomony of the strategic interaction between the agents' information transmissions

[23] More formally, note that $E(\overline{m}_2^2) \leq E(\theta_2^2)$. If $E(\theta_2^2) < E(\theta_2) < 1$, then the RHS of (20) is always positive and monotonically decreasing in $var(\overline{m}_2)$.

[24] More formally, if $E(\theta_2) < 1$ and $E(\theta_2) < E(\theta_2^2)$, then there is a cutoff $var(\overline{m}_2)$ such that the RHS of (20) is 0. When $var(\overline{m}_2)$ is smaller than this cutoff, the RHS is positive and monotonically decreasing in $var(\overline{m}_2)$. When $var(\overline{m}_2)$ is bigger than the cutoff, the RHS is negative and monotonically decreasing (the absolute value is montonically increasing) in $var(\overline{m}_2)$.

Biases	DM's ideal action Additive in states	Multiplicative in states
Type-independent	Independent	Strategic complements
Type-dependent	Independent	Strategic substitues, strategic complements, or*

*: Strategic complements (substitutes) when a small (large) amount of information is transmitted.

5 Delegation

Though the DM has formal authority to make the decision, he may find it optimal to delegate decision rights to one of the agents. Given that there are two agents, two delegation arrangements need to be considered: delegating decision rights to agent 1 (D1 delegation) or delegating decision rights to agent 2 (D2 delegation). Under either delegation arrangement, the agent to whom decision rights are delegated first consults the other agent and then makes the decision. In the CEO example, if the marketing manager is given the decision rights, he first consults the production manager regarding production efficiency and then combines this information with his own information on market demand to decide on the new plant size.[25] We answer two questions in this section. First, which delegation arrangement (D1 or D2 delegation) is optimal? Second, when does the DM have an incentive to delegate his decision rights? We only consider the basic model with type-independent biases.

5.1 Optimal delegation

First consider D1 delegation. In this case, agent 2 sends message m_2 to agent 1, and then agent 1 makes the decision $y(\theta_1, m_2)$. Agent 1's optimal decision rule is $\overline{y}(\theta_1, m_2) = \theta_1 \overline{m}_2 + b_1$. Now the communication game between agent 2 and agent 1 is a one-sender cheap talk game with the receiver having private information, the setting studied by McGee (2008). It can be shown that all PBE are partition equilibria (see McGee for details). Let N_2 be the number of partition elements and $\{a_{2,0}, ..., a_{2,n}, ..., a_{2,N_2}\}$ be the cutoff points. In particular, given $\overline{y}(\theta_1, m_2)$, when $\theta_2 = a_{2,n}$ agent 2 should be indifferent between sending a message immediately to the left of $a_{2,n}$ and a message immediately to the right of $a_{2,n}$. This indifference condition can be explicitly written as:

[25] Here we consider full delegation to a single agent. Full delegation in general is not optimal. In a setting with a single agent, Melumad and Shibano (1991) show that optimal delegation involves delegation with a restricted action space. Alonso and Matouschek (2008) provide further generalizations.

$$E[\{\theta_1 \frac{a_{2,n} + a_{2,n-1}}{2} + b_1 - (\theta_1 a_{2,n} + b_2)\}^2] = E[\{\theta_1 \frac{a_{2,n} + a_{2,n+1}}{2} + b_1 - (\theta_1 a_{2,n} + b_2)\}^2]$$
$$\Leftrightarrow \quad (a_{2,n+1} - a_{2,n}) - (a_{2,n} - a_{2,n-1}) = \frac{4E(\theta_1)}{E(\theta_1^2)}(b_1 - b_2) = \frac{3}{2A_1}4(b_1 - b_2)$$

In the most informative equilibrium, the (largest) number of partition elements is $N_2^* = \langle -\frac{1}{2} + \frac{1}{2}(1 + \frac{4A_2A_1}{3|b_1-b_2|})^{1/2} \rangle$, and

$$E(\overline{m}_2^2) = \frac{A_2^2}{3} - \frac{A_2^2}{12N_2^2} - \frac{(b_1 - b_2)^2(\frac{3}{2A_1})^2(N_2^2 - 1)}{3} \tag{22}$$

Note that under D1 delegation, the incremental step size of agent 2's equilibrium partition and hence N_2^*, $E(\overline{m}_2^2)$ and the equilibrium information transmission depend only on the magnitude of the difference in biases ($|b_1 - b_2|$). The principal's equilibrium payoff under D1 delegation, U_{D1}^P, is

$$U_{D1}^P = -E[(\theta_1 \overline{m}_2 + b_1 - \theta_1\theta_2)^2] = -E(\theta_1^2)E(\theta_2^2) + E(\theta_1^2)E(\overline{m}_2^2) - b_1^2 \tag{23}$$

Now consider D2 delegation. In this case, agent 1 first sends message m_1 to agent 2, and then agent 2 makes the decision $y(\theta_2, m_1)$. Agent 2's optimal decision rule is $\overline{y}(\theta_2, m_1) = \theta_2\overline{m}_1 + b_2$. Let N_1 be the number of partition elements and $\{a_{1,0}, ..., a_{1,n}, ..., a_{1,N_1}\}$ be the cutoff points of agent 1's equilibrium communication rule. In particular, $a_{1,n}$ is characterized by

$$(a_{1,n+1} - a_{1,n}) - (a_{1,n} - a_{1,n-1}) = \frac{4E(\theta_2)}{E(\theta_2^2)}(b_2 - b_1) = \frac{3}{2A_2}4(b_2 - b_1).$$

In the most informative equilibrium, $N_1^* = \langle -\frac{1}{2} + \frac{1}{2}(1 + \frac{4A_2A_1}{3|b_1-b_2|})^{1/2} \rangle$, and

$$E(\overline{m}_1^2) = \frac{A_1^2}{3} - \frac{A_1^2}{12N_1^2} - \frac{(b_1 - b_2)^2(\frac{3}{2A_2})^2(N_1^2 - 1)}{3} \tag{24}$$

Comparing the expressions for N_1^* and N_2^*, we see that the number of partition elements is the same under D1 and D2 delegation. The principal's equilibrium payoff U_{D2}^P under D2 delegation is

$$U_{D2}^P = -E[(\theta_2 \overline{m}_1 + b_2 - \theta_1\theta_2)^2] = -E(\theta_1^2)E(\theta_2^2) + E(\theta_2^2)E(\overline{m}_1^2) - b_2^2 \tag{25}$$

Proposition 5 *Between D1 and D2 delegation, it is always optimal for the DM to delegate decision rights to the agent with the smaller bias in absolute value.*

Proof. Without loss of generality, suppose agent 1 has a smaller bias, $|b_1| < |b_2|$. We want to show that D1 delegation is better for the DM. From previous derivations, it is clear that N_1^* under D2 delegation is the same as N_2^* under D1 delegation. Let $N^* = N_1^* = N_2^*$. From (23) and (25),

$$U_{D1}^P - U_{D2}^P = E(\theta_1^2)E(\overline{m}_2^2) - E(\theta_2^2)E(\overline{m}_1^2) + (b_2^2 - b_1^2).$$

By (22) and (24),

$$\begin{aligned} & U_{D1}^P - U_{D2}^P = \frac{A_1^2}{3}[\frac{A_2^2}{3} - \frac{A_2^2}{12N^{*2}} - \frac{3(b_1-b_2)^2(N^{*2}-1)}{4A_1^2}] \\ & -\frac{A_2^2}{3}[\frac{A_1^2}{3} - \frac{A_1^2}{12N^{*2}} - \frac{3(b_1-b_2)^2(N^{*2}-1)}{4A_2^2}] + (b_2^2 - b_1^2) \\ = \quad & b_2^2 - b_1^2 > 0. \end{aligned}$$

Therefore, D1 delegation yields a higher ex ante payoff. ■

Proposition 5 indicates that decision rights should be delegated to the agent with the smaller bias. Note that this does not depend on which agent's private information has more underlying uncertainty. Intuitively, under either delegation arrangement the agent with the decision rights ends up after communication with the same amount of information to utilize because the equilibrium information transmission depends only on the difference between the biases $|b_1 - b_2|$ (i.e., $E(\theta_1^2)E(\overline{m}_2^2)$ under D1 delegation equals $E(\theta_2^2)E(\overline{m}_1^2)$ under D2 delegation). Thus decision rights should be delegated to the agent with the smaller bias to minimize the loss of control.

5.2 Comparison between delegation and simultaneous communication

Now without loss of generality, suppose agent 1 has a smaller bias, $|b_1| < |b_2|$. Between the two delegation arrangements, D1 delegation is optimal. We are interested in identifying the conditions under which the principal has an incentive to delegate instead of retaining decision rights and engaging in simultaneous cheap talk with both agents.

Proposition 6 *When two agents have like biases and informative communication is feasible from agent 1 under simultaneous communication, then the DM prefers D1 delegation to simultaneous cheap talk.*

Proposition 6 is related to Dessein (2002), who shows that in a one-sender cheap talk model the principal prefers delegation whenever cheap talk is informative. In our two-sender setting, the choice between delegation and communication becomes more complex. To understand Proposition 6, consider the four effects of delegation relative to simultaneous communication, where the sign in parentheses indicates whether the effect makes delegation more or less attractive to the DM.

(i) Delegation leads to a loss of control, measured by b_1^2. (-)

(ii) Delegation always leads to more information being utilized in the dimension of θ_1 because $E(\theta_1^2) \geq E(\overline{m}_1^2)$. (+)

(iii) $E(\theta_1^2) \geq E(\overline{m}_1^2)$ means $x_{2D} \leq x_2$. The strategic complementarity of the agents' information transmissions will lead to more information being transmitted by agent 2 under delegation. (+)

(iv) Agent 2's effective bias changes from $|b_2|$ under simultaneous communication to $|b_2 - b_1|$ under delegation, which affects agent 2's equilibrium information transmission. (?)

The last three effects measure the potential informational gain under delegation relative to simultaneous communication.[26] While effects (ii) and (iii) always favor delegation, effect (iv) depends on whether the two agents have like or opposing biases. When the two agents have like biases, optimal delegation leads to a smaller effective bias in communication, $|b_2 - b_1| < |b_2|$. Thus both effects (iii) and (iv) work in the same direction: more information is transmitted by agent 2 under delegation. This additional informational gain—absent in Dessein's (2002) one-sender cheap talk model—makes the overall informational gain under delegation even larger when the agents have like biases. Thus delegation is preferred by the DM.

Example 3 *Suppose* $A_1 = 5$*,* $A_2 = 7$*,* $b_1 = \frac{1}{8}$ *and* $b_2 = \frac{1}{2}$*. Under simultaneous communication,* $N_1^* = 9$*,* $N_2^* = 4$*,* $E(\overline{m}_1^2) = 8.28759$*, and* $E(\overline{m}_2^2) = 15.9644$*. Under D1 delegation, the effective bias is* $|b_2 - b_1| = 3/8$*. By the inequality* $N_{2D}(N_{2D} - 1) < \frac{A_1 A_2}{3|b_2 - b_1|}$*,* $N_{2D}^* = 5$*. The difference between the principal's ex ante payoffs can be expressed as*

$$U_{D1}^P - U_{ST}^P = \left[(\frac{25}{3})(16.0688) - \frac{1}{64} \right] - (8.28759)(15.9644) = 1.58446.$$

Delegation dominates simultaneous communication.

The agents need not have like biases for delegation to dominate simultaneous communication. Even if the agents have opposing biases such that effect (iv) works against delegation, effects (ii) and (iii) can still outweigh effect (iv), meaning that delegation leads to an informational gain. The following example illustrates this.

[26] Dessein (2002) compares the information gain from the second effect to the loss of control in the first effect.

Example 4 *Suppose* $A_1 = 5$*,* $A_2 = 7$*,* $b_1 = \frac{1}{8}$ *and* $b_2 = -\frac{1}{4}$*. Under D1 delegation, the effective bias is again* $|b_2 - b_1| = \frac{3}{8}$*. Because* $|b_2 - b_1| = \frac{3}{8}$*,* U_{D1}^P *is the same as in the previous example. Under simultaneous communication,* $N_1^* = 9$*,* $N_2^* = 4$*,* $E(\overline{m}_1^2) = 8.28805$*, and* $E(\overline{m}_2^2) = 16.1536$*.*

$$U_{D1}^P - U_{ST}^P = \left[(\frac{25}{3})(16.0688) - \frac{1}{64} \right] - (8.28805)(16.1536) = 0.00904693.$$

Again delegation dominates simultaneous communication.

The proof of Proposition 6 indicates that as long as informative communication is feasible for agent 1 under simultaneous communication and more information is transmitted by agent 2 under D1 delegation than under simultaneous communication ($E(\overline{m}_{2D}^2) \geq E(\overline{m}_2^2)$), the principal prefers delegation.[27] Thus we have the following corollary.

Corollary 3 *Suppose the agents have opposing biases. As long as* $|b_1|/|b_2|$ *is small enough such that* $E(\overline{m}_{2D}^2) \geq E(\overline{m}_2^2)$ *and informative communication is feasible for agent 1 under simultaneous communication, D1 delegation leads to a higher ex ante payoff for the principal.*

Of course, when the agents have opposing biases and $|b_1|$ is big enough, then effect (iv) might outweigh effects (ii) and (iii), leading to a small information gain or even an information loss under D1 delegation relative to simultaneous communication. In this case, simultaneous communication dominates delegation. The following example illustrates this possibility.

Example 5 *Suppose* $A_1 = 5$*,* $A_2 = 7$*,* $b_1 = \frac{1}{8}$ *and* $b_2 = -\frac{1}{2}$*. By Corollary 2, under simultaneous communication the principal's expected utility is the same as in example 4. Under D1 delegation, it can be verified that* $N_{2D}^* = 2$*. Because* $|b_2 - b_1| = \frac{5}{8}$*,* $E(\bar{m}_{2D}^2) = 15.7859$*. Thus*

$$U_{D1}^P - U_{ST}^P = \left[(\frac{25}{3})(15.7859) - \frac{1}{64} \right] - (8.28759)(15.9644) = -0.772796.$$

Simultaneous communication dominates delegation.

[27] Proposition 6 specifies a sufficient condition such that D1 delegation is optimal that might not be necessary. To prove the analytical result, we need informative communication to be feasible under simultaneous communication because only with this condition are we able to show unambiguously that the magnitude of the loss of control is always smaller than that of the informational gain with delegation. If both senders have almost identical but very large biases such that no information is revealed under simultaneous communication, delegation might dominate simultaneous communication as a lot of information will be revealed from agent 2 to agent 1 under delegation. No unambiguous results, however, can be derived because the loss of control will be very large as well due to the very large biases.

Given that our model has four parameter values, it is hard to provide general and clean conditions under which simultaneous communication dominates delegation. So we focus on the symmetric case.

Corollary 4 *Suppose $A_1 = A_2 = A$, $b_1 = b$, and $b_2 = -b$ with $b > 0$. If $A^2/36 < b < A^2/24$, then simultaneous communication leads to a higher ex ante payoff for the principal than D1 delegation.*

Proposition 6 and Corollaries 3 and 4 generate some interesting empirical implications. First, between two functionally parallel agents (e.g., division managers), it is possible that decision rights will be delegated to one of the two agents. In the example of a CEO choosing a plant size, for instance, it is possible that the CEO will delegate decision rights to the production manager if he has a smaller bias than the marketing manager. Second, we are more likely to observe delegation when the agents' preferences are biased in the same direction relative to the principal. In the context of politics, delegation is more likely to be observed when both experts are either both more liberal than the DM or both more conservative than the DM. On the other hand, delegation is less likely to be observed if one expert is more liberal and the other more conservative than the DM. In the context of firm organization, if both division managers are biased toward choosing a bigger factory size (e.g., empire building) relative to the CEO, then delegation is more likely—again because the agent with the decision rights is able to extract more information from communication with the other agent than the DM would be able to through simultaneous communication.

6 Conclusion and Discussion

We study a two-sender cheap talk model in which two experts have partial and non-overlapping information regarding the state of the world and communicate to the receiver simultaneously. The receiver's ideal action is multiplicative in the experts' private information, meaning that the sensitivity of the receiver's ideal action to one expert's information depends on the other expert's private information. When the experts' biases are type-independent, we show that information transmission displays strategic complementarities in that more informative communication from one expert induces more informative communication from the other. Interestingly, the informativeness of communication from both experts in equilibrium does not depend on whether they have like or opposing biases, but only depends on the magnitudes of these biases. When the decision-maker can assign the experts to the different dimensions of the state space, we show that under a broad range of circumstances this assignment will not affect the decision-maker's expected utility.

When experts' biases are type-dependent, the information transmissions could be strategic subsitutes in that more informative communication from one expert induces less informative communication from the other. The information transmissions could also be strategic complements with type-dependent biases, or the information transmissions could exihibit strategic complementarity when little information is transmitted and strategic substitutability when a lot of information is transmitted. The driving force behind the strategic interaction between the experts' information transmissions is a variance effect.

We then study delegation when the decision rights are delegated to one of the two experts when the biases are type-independent. We show that the decision-maker, if he ever delegates, always prefers to delegate decision rights to the expert with the smaller bias. Comparing delegation to simultaneous communication, we demonstrate that when two experts have like biases, delegation is always superior for the decision-maker whenever informative communication between the decision-maker and the experts is possible. On the other hand, simultaneous communication dominates delegation when the experts have opposing biases and the smaller bias is big enough.

Unlike previous models of strategic communication with multiple experts in which the experts observe basically the same realized state of the world, our model highlights how the relationship between the experts' private information influences their communication to the decision-maker. We emphasize that there could be strategic interactions between the experts' information transmissions insofar as how much information one expert transmits may depend on how much information the other expert transmits.

There are several ways to extend our analysis. The first is to consider sequential communication. Specifically, one expert sends a message first, and then the other expert sends a message after observing the first expert's message.[28] A number of questions regarding sequential communication suggest themselves. First, which agent should communicate first in sequential communication to maximize the receiver's payoff?[29] Second, how does sequential communication compare to simultaneous communication and delegation in terms of the receiver's payoff? Another possible extension is to consider the case in which the two experts' pieces of private information are correlated. In our paper, for simplicity, they are assumed to be independent. With correlated private information, on top of the variance effect identified in this paper, there

[28] Under sequential communication, the second sender's equilibrium strategy—a partition equilibrium as in CS—is easy to derive, but characterizing the first sender's equilibrium strategy is difficult. By changing his report, the first sender can potentially induce different partitions from the second sender.

[29] Ottaviani and Sorensen (2001) show that in committee debate the order of speech affects information transmission and thus matters.

could be an information congestion effect as in Austen-Smith (1993) and Morgan and Stocken (2008): when one expert transmits more information, the receiver's action will be less sensitive to the other expert's report, and the other expert might then have a stronger incentive to distort his information in the direction of his own bias. As long as the correlation is not too strong, however, we expect the variance effect to continue to dominate and the strategic interaction between the experts' information transmissions to exibit similar patterns to those in the current paper. We leave the complete analysis for future research.

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Appendix

Examples in which the ideal action is multiplicative in two states.

Consider the CEO example in which θ_1, observed only by the marketing manager, is the demand size and θ_2, observed only by the production manager, is the efficiency of production. Denote the output (size) of the new factory as Q, and suppose the firm is a monopoly. The demand faced by the firm is $Q = \theta_1(K - P)$, where P is the price and K some known constant. $K - P$ can be interpreted as the demand curve for individual consumers, while θ_1 is the number of consumers or market size. The marginal cost of production is c. We define the the efficiency of production as $\theta_2 \equiv (K - c)/2$. In the current setup, the marginal revenue is given by $MR(Q) = K - 2Q/\theta_1$. Setting the marginal revenue equal to the marginal cost, we derive the optimal (profit maximizing) output Q^* as follows: $Q^* = \theta_1(K - c)/2 = \theta_1\theta_2$. That is, the CEO's ideal action is multiplicative in the two states.

In the previous example the individual demand curve is linear. Now consider demand curves with constant elasticities. In particular, suppose $Q = \theta_1 P^{-\epsilon}$, where the elasticity parameter ε (> 0) is common knowledge. We define the efficiency of production as $\theta_2 \equiv (\frac{\epsilon}{\epsilon-1})^{-\epsilon}c^{-\epsilon}$. In this situation, the marginal revenue is given by $MR(Q) = \frac{\epsilon-1}{\epsilon}\theta_1^{1/\varepsilon}Q^{-1/\epsilon}$. The profit maximizing output Q^* satisfies $Q^* = (\frac{\epsilon}{\epsilon-1})^{-\epsilon}c^{-\epsilon}\theta_1 = \theta_1\theta_2$. Again, the CEO's ideal action is multiplicative in the two states.

Proof of Lemma 1:

Proof. We first show that given any communication rule for agent 2, $\mu_2(\cdot)$, agent 1's optimal communication rule is of the interval form. Suppose the DM holds a posterior belief v_1 regarding θ_1. Then agent 1's expected utility is

$$E_{\theta_2}\left[U^{A_1}|v_1,\theta_1\right] = -E_{\theta_2}\left[\{y^*(v_1, E\left[\theta_2|\mu_2(\cdot)\right]) - y^*(\theta_1,\theta_2) - b_1\}^2\right]. \tag{26}$$

Given that y_1^* is monotonic, it can be readily seen that $\frac{\partial^2}{\partial\theta_1\partial v_1}E_{\theta_2}[U^{A_1}|v_1,\theta_1] > 0$. Now consider the second partial derivative

$$\frac{\partial^2}{\partial\theta_1^2}E_{\theta_2}\left[U^{A_1}|v_1,\theta_1\right] = -E_{\theta_2}[2y_{11}^*(\theta_1,\theta_2)\{y^*(\theta_1,\theta_2) - y^*(v_1,\overline{m}_2) + b_1\} + 2(y_1^*(\theta_1,\theta_2))^2]. \tag{27}$$

Observing (27), we can see that $\frac{\partial^2}{\partial\theta_1^2}E_{\theta_2}\left[U^{A_1}|v_1,\theta_1\right] < 0$ if $|y_{11}^*/y_1^*|$ is small enough.

The fact that $\frac{\partial^2}{\partial\theta_1\partial v_1}E_{\theta_2}[U^{A_1}|v_1,\theta_1] > 0$ and $\frac{\partial^2}{\partial\theta_1^2}E_{\theta_2}[U^{A_1}|v_1,\theta_1] < 0$ imply that for any two different posterior beliefs of the DM, say $\underline{v}_1 < \overline{v}_1$, there is at most one type of agent 1 that is indifferent between both. Now suppose that contrary to interval equilibria, there are two states

$\underline{\theta}_1 < \overline{\theta}_1$ such that $E_{\theta_2}[U^{A_1}|\overline{v}_1, \underline{\theta}_1] \geq E_{\theta_2}[U^{A_1}|\underline{v}_1, \underline{\theta}_1]$ and $E_{\theta_2}[U^{A_1}|\underline{v}_1, \overline{\theta}_1] > E_{\theta_2}[U^{A_1}|\overline{v}_1, \overline{\theta}_1]$. Then $E_{\theta_2}[U^{A_1}|\overline{v}_1, \overline{\theta}_1] - E_{\theta_2}[U^{A_1}|\underline{v}_1, \overline{\theta}_1] < E_{\theta_2}[U^{A_1}|\overline{v}_1, \underline{\theta}_1] - E_{\theta_2}[U^{A_1}|\underline{v}_1, \underline{\theta}_1]$, which contradicts $\frac{\partial^2}{\partial\theta_1 \partial v_1}[U^{A_1}|v_1, \theta_1] > 0$.

The same argument can be applied to agent 2 given any communication rule $\mu_1(\cdot)$ for agent 1. Therefore, all PBE of the communication game must be interval equilibria. ■

Proof of Lemma 2.

Proof. We prove the results for $E(\overline{m}_1^2)$. The results for $E(\overline{m}_2^2)$ can be proved similarly. From (4), we have

$$\begin{aligned} a_{1,n} - a_{1,n-1} &= \frac{A_1}{N_1} + 2b_1x_2(2n - N - 1), \\ a_{1,n} + a_{1,n-1} &= \frac{A_1}{N_1}(2n-1) + 2b_1x_2[2n^2 - (2n-1)(N+1)]. \end{aligned}$$

By definition,

$$\begin{aligned} E(\overline{m}_1^2) &= \sum_{n=1}^{N_1} \int_{a_{1,n-1}}^{a_{1,n}} \frac{1}{A_1} \frac{(a_{1,n} + a_{1,n-1})^2}{4} = \frac{1}{4A_1} \sum_{n=1}^{N_1} (a_{1,n} - a_{1,n-1})(a_{1,n} + a_{1,n-1})^2 \\ &= \frac{A_1^2}{3} - \frac{A_1^2}{12N_1^2} - \frac{b_1^2x_2^2(N_1^2-1)}{3}. \end{aligned}$$

This proves part (i). To show part (ii), consider the change in $E(\overline{m}_1^2)$ when N_1 decreases to $N_1 - 1$:

$$\begin{aligned} E(\overline{m}_1^2)(N_1) - E(\overline{m}_1^2)(N_1 - 1) &= \frac{A_1^2}{12}[\frac{1}{(N_1-1)^2} - \frac{1}{N_1^2}] - \frac{b_1^2x_2^2}{3}[N_1^2 - (N_1-1)^2] \\ &\propto A_1^2 - 4b_1^2x_2^2N_1^2(N_1-1)^2 > 0 \end{aligned}$$

The last inequality follows from $a_{1,1} > 0$, which implies that $A_1 > 2b_1x_2N_1(N_1 - 1)$. Thus $E(\overline{m}_1^2)$ is strictly increasing in N_1. x_2 affects $E(\overline{m}_1^2)$ in two ways. First, a decrease in x_2 directly increases $E(\overline{m}_1^2)$. Second, by (5) a decrease in x_2 leads to a weakly larger N_1, which increases $E(\overline{m}_1^2)$ as well. Therefore, $E(\overline{m}_1^2)$ is strictly decreasing in x_2. By similar logic, $E(\overline{m}_1^2)$ is strictly decreasing in b_1. Since x_2 is decreasing in $E(\overline{m}_2^2)$, it follows that $E(\overline{m}_1^2)$ is strictly increasing in $E(\overline{m}_2^2)$.

To prove part (iii), note that $E(\overline{m}_1^2) = (E(\theta_1))^2 + var[E(\theta_1|m_1)]$. Because the conditional variance $var[E(\theta_1|m_1)] \in [0, var(\theta_1)]$, $(E(\theta_1))^2 \leq E(\overline{m}_1^2) \leq E(\theta_1^2)$, and part (iii) immediately follows. ■

Proof of Proposition 2:

Proof. By modifying the proof of Lemma 1 slightly, we can show that all PBE must be interval equilibria when the domain of state θ_i is extended to $[-B_i, A_i]$. With this new support, the difference equations that characterize the partition points, (2) and (3), remain the same. The starting boundary condition implies that $a_{i0} = -B_i$ instead of 0, but otherwise the results do not change qualitatively. If $B_2 = A_2$, then $E(\theta_2) = 0$. Now if agent 2's communication is informative ($var(\overline{m}_2) > 0$), the right hand side of (2), or the effective bias for agent 1, is 0. As a result, in the most informative equilibrium, the number of partition elements N goes to infinity and agent 1 reveals his information fully. If $B_2 \neq A_2$, then $E(\theta_2) \neq 0$ and the right hand side of (2) is not zero either. As a result, in the most informative equilibrium the number of partition elements N is still finite. ■

Proof of Proposition 3:

Proof. (i) A sufficient condition for agent 1's communication to be uninformative under both assignments is $6r|b| \geq A_1 A_2$. To see this, consider NA assignment. N_1^* is the largest N_1 such that $2r|b|x_2 N_1(N_1 - 1) < A_1$. Given $x_2 \geq \frac{3}{2A_2}$, $2r|b|x_2 N_1(N_1 - 1) \geq 3r|b|N_1(N_1 - 1)/A_2$. Now if $N_1^* \geq 2$, then $3r|b|N_1(N_1 - 1)/A_2 \geq 6r|b|/A_2$, which by the condition $6r|b| \geq A_1 A_2$ is bigger than A_1. This contradicts $2r|b|x_2 N_1^*(N_1^* - 1) < A_1$. Therefore, $N_1^* = 1$. By a similar argument, one can show that under PA assignment $N_2'^* = 1$.

Now consider NA assignment. Given $N_1^* = 1$, $E(\overline{m}_1^2) = \frac{A_1^2}{4}$ and $x_1 = \frac{2}{A_1}$. N_2^* is the largest integer such that $4|b|N_2(N_2 - 1) < A_1 A_2$. Under PA assignment, $N_2'^* = 1$, $E(\overline{m}_2'^2) = \frac{A_2^2}{4}$, $x_2' = \frac{2}{A_2}$. $N_1'^*$ is the largest integer such that $4|b|N_1'(N_1' - 1) < A_1 A_2$. It can be readily seen that $N_1'^* = N_2^* \equiv N^*$. The difference between the ex ante payoffs from NA assignment and PA assignment is

$$\begin{aligned} E(\overline{m}_1^2)E(\overline{m}_2^2) - E(\overline{m}_1'^2)E(\overline{m}_2'^2) &= \frac{A_1^2}{4}\left[\frac{A_2^2}{3} - \frac{A_2^2}{12N^{*2}} - \frac{4b^2(N^{*2}-1)}{3A_1^2}\right] \\ -\frac{A_2^2}{4}\left[\frac{A_1^2}{3} - \frac{A_1^2}{12N^{*2}} - \frac{4b^2(N^{*2}-1)}{3A_2^2}\right] &= 0. \end{aligned}$$

Therefore, both assignments lead to the same ex ante payoff.

(ii) When $b \to 0$, under NA assignment $E(\overline{m}_2^2) = \frac{A_2^2}{3}$ and $x_1 = \frac{3}{2A_2}$. Moreover, N_1^* is the largest integer such that $4r|b|N_1(N_1 - 1) < A_1 A_2$. Similarly, under PA assignment $E(\overline{m}_1'^2) = \frac{A_1^2}{3}$ and $x_2' = \frac{3}{2A_1}$. Moreover, $N_2'^*$ is the largest integer such that $4r|b|N_2'(N_2' - 1) < A_1 A_2$. It can be readily seen that $N_2'^* = N_1^* \equiv N^*$. The difference between the ex ante payoffs from NA assignment and PA assignment is

$$
\begin{aligned}
E(\overline{m}_1^2)E(\overline{m}_2^2) - E(\overline{m}_1'^2)E(\overline{m}_2'^2) &= \frac{A_2^2}{3}[\frac{A_1^2}{3} - \frac{A_1^2}{12N^{*2}} - \frac{3r^2b^2(N^{*2}-1)}{4A_2^2}] \\
-\frac{A_1^2}{3}[\frac{A_2^2}{3} - \frac{A_2^2}{12N^{*2}} - \frac{3r^2b^2(N^{*2}-1)}{4A_1^2}] &= 0.
\end{aligned}
$$

Thus both assignments lead to the same ex ante payoffs.

(iii) Define $s \equiv x_1x_2$ and $t = x_1/x_2$. Now $x_1 = \sqrt{st}$ and $x_2 = \sqrt{s/t}$. Using s and t, under NA assignment equations (8) and (9) can be rewritten as

$$
\frac{A_2}{3} - \frac{A_2}{12N_2^{*2}} - \frac{b_2^2 st(N_2^{*2}-1)}{3A_2} = \frac{A_1}{3}t - \frac{A_1}{12N_1^{*2}}t - \frac{b_1^2 s(N_1^{*2}-1)}{3A_1} = \sqrt{t/s}/2. \tag{28}
$$

From (28), we can solve for t as a function of s:

$$
t = \frac{\frac{A_2}{3} - \frac{A_2}{12N_2^{*2}} + \frac{b_1^2 s(N_1^{*2}-1)}{3A_1}}{\frac{A_1}{3} - \frac{A_1}{12N_1^{*2}} + \frac{b_2^2 s(N_2^{*2}-1)}{3A_2}}.
$$

Substituting the above expression for t into (28) and rearranging, we get an equation in s:

$$
C - D = \frac{1}{2\sqrt{s}} \times \sqrt{C + D + (\frac{1}{3} - \frac{1}{N_1^{*2}})\frac{b_1^2 s(N_1^{*2}-1)}{3} + (\frac{1}{3} - \frac{1}{N_2^{*2}})\frac{b_2^2 s(N_2^{*2}-1)}{3}}, \tag{29}
$$

where $C = A_1A_2(\frac{1}{3} - \frac{1}{N_1^{*2}})(\frac{1}{3} - \frac{1}{N_2^{*2}})$ and $D = \frac{b_1^2b_2^2s^2(N_1^{*2}-1)(N_2^{*2}-1)}{9A_1A_2}$.

Under PA assignment, define $s' \equiv x_1'x_2'$ and $t' = x_1'/x_2'$. Following the same procedure, we can get an equation in s' similar to (29). Given that $N_1^* = N_2^{*'}$ and $N_2^* = N_1^{*'}$, the equation in s' is the same as that in s with the positions of b_1 and b_2 switched and the positions of N_1^* and N_2^* switched. Inspecting (29), we see that the equation remains the same if we simply switch b_1 and b_2 and switch N_1^* and N_2^*. Therefore, both s and s' are defined by the same equation (29), and we must have $s = s'$. Thus $x_1x_2 = x_1'x_2'$, which implies $E(\overline{m}_1^2)E(\overline{m}_2^2) = E(\overline{m}_1'^2)E(\overline{m}_2'^2)$. ■

Proof of Lemma 3:

Proof. Part (i).

$$
E(\overline{m}_i^2) = \frac{1}{2}\sum_{k=1}^{N_i}\int_{a_{i,k-1}}^{a_{i,k}} (\frac{a_{i,k-1}+a_{i,k}}{2})^2 d\theta_i = \frac{1}{8}\sum_{k=1}^{N_i}(a_{i,k}-a_{i,k-1})(a_{i,k-1}+a_{i,k})^2.
$$

Using the expression of $a_{i,n}$ in (18) and then simplifying, we get the desired expression (19).

Part (ii). Inspect the expression of (19). When N_i varies in the domain between 1 and $\overline{N}_i$, $\sin(\theta_i N_i)$ is positive and increasing in N_i. Given that b_i is negative, $E(\overline{m}_i^2)$ is increasing in N_i.

Now to show that $E(\overline{m}_i^2)$ is increasing in b_i we only need to show that $E(\overline{m}_i^2)$ is increasing in b_i when N_i is fixed, because $\overline{N}_i$ is increasing in b_i and we have shown that $E(\overline{m}_i^2)$ is increasing in N_i. Specifically,

$$sgn\{\frac{\partial E(\overline{m}_i^2)}{\partial(b_i)}\} = sgn\{-(1+b_i)+\cos^2(N_i\theta_i)+(1+b_i)\frac{3+4b_i}{\sqrt{-b_i(1+b_i)}}[1+2b_iN_i\frac{\cos(N_i\theta_i)}{\sin(N_i\theta_i)}]\}.$$

Using the above expression and given that $N_i\theta_i \in [0,\pi/2]$, to show that $\frac{\partial E(\overline{m}_i^2)}{\partial(b_i)} > 0$ it is sufficient to show that $3+4b_i > \sqrt{-b_i(1+b_i)} = \sin(\theta_i)$. This indeed holds because $b_i \in [-\frac{1}{4}, 0]$. Therefore, $E(\overline{m}_i^2)$ is increasing in b_i or $E(\overline{m}_i^2)$ is decreasing in $|b_i|$.

Part (iii). By $b_i \equiv \frac{E(\theta_j)}{E(\overline{m}_j^2)} - 1$, $|b_i|$ is increasing in $E(\overline{m}_j^2)$. Thus, by part (ii), $E(\overline{m}_i^2)$ is decreasing in $E(\overline{m}_j^2)$. ■

Proof of Proposition 6:

Proof. We first show that $E(\overline{m}_2^2)$ under simultaneous communication is smaller than $E(\overline{m}_{2D}^2)$ under D1 delegation. From previous results, under simultaneous communication N_2^* is the largest integer such that $2|b_2|\frac{A_1}{2E(\overline{m}_1^2)}N_2(N_2-1) < A_2$. Under D1 delegation, N_{2D}^* is the largest integer such that $2|b_2-b_1|\frac{A_1}{2E(\theta_1^2)}N_2(N_2-1) < A_2$. Since b_1 and b_2 have the same sign and $|b_1| < |b_2|$, $|b_2-b_1| < |b_2|$. Moreover, $E(\theta_1^2) \geq E(\overline{m}_1^2)$. Therefore, $N_{2D}^* \geq N_2^*$. Comparing $E(\overline{m}_2^2)$ and $E(\overline{m}_{2D}^2)$,

$$\begin{aligned} E(\overline{m}_2^2) &= \frac{A_2^2}{3} - \frac{A_2^2}{12N_2^{*2}} - \frac{b_2^2A_1^2(N_2^{*2}-1)}{12[E(\overline{m}_1^2)]^2}, \\ E(\overline{m}_{2D}^2) &= \frac{A_2^2}{3} - \frac{A_2^2}{12N_{2D}^{*2}} - \frac{(b_2-b_1)^2A_1^2(N_{2D}^{*2}-1)}{12[E(\theta_1^2)]^2}, \\ E(\overline{m}_{2D}^2) - E(\overline{m}_2^2) &\geq \frac{b_2^2A_1^2(N_2^{*2}-1)}{12[E(\overline{m}_1^2)]^2} - \frac{(b_2-b_1)^2A_1^2(N_2^{*2}-1)}{12[E(\overline{m}_1^2)]^2} \geq 0; \end{aligned}$$

where the first inequality holds because $E(\overline{m}_{2D}^2)$ is increasing in N_{2D}^* and $N_{2D}^* \geq N_2^*$, and the second inequality follows from the fact that $|b_2-b_1| < |b_2|$ and $E(\theta_1^2) \geq E(\overline{m}_1^2)$.

The difference between the DM's ex ante payoffs can be expressed as

$$\begin{aligned} U_{D1}^P - U_{ST}^P &= E(\theta_1^2)E(\overline{m}_{2D}^2) - E(\overline{m}_1^2)E(\overline{m}_2^2) - b_1^2 \\ &\geq E(\overline{m}_2^2)[E(\theta_1^2) - \frac{b_1^2}{E(\overline{m}_2^2)} - E(\overline{m}_1^2)], \end{aligned}$$

where the inequality is due to $E(\overline{m}_{2D}^2) - E(\overline{m}_2^2) \geq 0$. Now $U_{D1}^P - U_{ST}^P > 0$ is equivalent to the term in the bracket being strictly greater than 0. More explicitly,

$$
\begin{aligned}
& E(\theta_1^2) - \frac{b_1^2}{E(\overline{m}_2^2)} - E(\overline{m}_1^2) = \frac{A_1^2}{12N_1^{*2}} + \frac{b_1^2 A_2^2 (N_1^{*2}-1)}{12[E(\overline{m}_2^2)]^2} - \frac{b_1^2}{E(\overline{m}_2^2)} \\
> \ & \frac{b_1^2 A_2^2 (N_1^{*2}-1)}{12[E(\overline{m}_2^2)]^2} - \frac{b_1^2}{E(\overline{m}_2^2)} \geq \frac{b_1^2}{E(\overline{m}_2^2)} (\frac{N_1^{*2}-1}{4} - 1),
\end{aligned}
$$

where the second inequality follows from the fact that $E(\overline{m}_2^2) \leq \frac{A_2^2}{3}$. From the above expression, it is evident that $U_{D1}^P - U_{ST}^P > 0$ if $N_1^* \geq 3$.

Now consider the case in which $N_1^* = 2$. Note that under simultaneous communication, the partition $a_{1,1} > 0$ satisfies

$$
a_{1,1} + a_{1,1} + \frac{2b_1 A_2}{E(\overline{m}_2^2)} = A_1.
$$

Given that $a_{1,1} > 0$, we have $\frac{2|b_1|A_2}{E(\overline{m}_2^2)} < A_1$ and

$$
\begin{aligned}
& E(\theta_1^2) - \frac{b_1^2}{E(\overline{m}_2^2)} - E(\overline{m}_1^2) = \frac{A_1^2}{48} + \frac{b_1^2 A_2^2}{4[E(\overline{m}_2^2)]^2} - \frac{b_1^2}{E(\overline{m}_2^2)} \\
> \ & \frac{A_2^2 b_1^2}{12E(\overline{m}_2^2)} + \frac{b_1^2 A_2^2}{4[E(\overline{m}_2^2)]^2} - \frac{b_1^2}{E(\overline{m}_2^2)} = \frac{b_1^2 A_2^2}{3[E(\overline{m}_2^2)]^2} - \frac{b_1^2}{E(\overline{m}_2^2)} \geq \frac{b_1^2}{E(\overline{m}_2^2)}(1-1) = 0,
\end{aligned}
$$

where the first inequality follows from $\frac{2|b_1|A_2}{E(\overline{m}_2^2)} < A_1$ and the second follows from the fact that $E(\overline{m}_2^2) \leq \frac{A_2^2}{3}$. Thus $U_{D1}^P - U_{ST}^P > 0$ if $N_1^* = 2$. ■

Proof of Corollary 4:

Proof. Recall that the inequalities in (11) identify the upper and lower bounds for the largest possible number of partition elements. By (11), the condition $b > A^2/36$ implies that the largest partition element $N^* = 1$ when the effective bias between the sender and the receiver is $2b$. Thus informative communication is not feasible under D1 delegation. Similarly, by (11), the condition $b < A^2/24$ implies that the largest partition element $N^* \geq 2$ when the effective bias between the sender and the receiver is b. Thus informative communication is feasible under simultaneous talk for both agents and the number of partition for each agent is at least 2. The DM's payoff difference between simultaneous talk and D1 delegation now can be expressed as

$$
U_{ST}^P - U_{D1}^P = E(\overline{m}_1^2)E(\overline{m}_2^2) + b^2 - E(\theta_1^2)[E(\theta_2)]^2 = E(\overline{m}_1^2)E(\overline{m}_2^2) + b^2 - \frac{A^4}{12}.
$$

Under simultaneous communication, for each agent i we pick an equilibrium with two partitions with the largest possible x_i, $A/2$ (recall $x_i = \frac{E(\theta_j)}{E(\overline{m}_j^2)} \leq \frac{A}{2}$). Under such an equilibrium, $E(\overline{m}_i^2) =$

$\frac{15}{48}A^2 - \frac{4b^2}{A^2}$. Note that the $E(\overline{m}_i^2)$ under the actual equilibrium with simultaneous communication is bigger than that in the equilibrium we picked. Therefore,

$$\begin{aligned} U^P_{ST} - U^P_{D1} &> [\frac{15}{48}A^2 - \frac{4b^2}{A^2}]^2 + b^2 - \frac{A^4}{12} \\ &\sim \frac{11}{768}A^4 + \frac{16}{A^4}b^4 - \frac{3}{2}b^2 \equiv f(b). \end{aligned}$$

It is easy to verify that for $b \in (\frac{A^2}{36}, \frac{A^2}{24})$, $f(b)$ is decreasing in b, and $f(b = \frac{A^2}{24}) > 0$. Therefore, $f(b) > 0$ for all $b \in (\frac{A^2}{36}, \frac{A^2}{24})$, and simultaneous talk dominates D1 delegation. ■