Targeted Search with Horizontal Differentiation in the Marriage Market

Yujing Xu*and Huanxing Yang[‡]

October 2016

Abstract

We develop a search/matching model in the marriage market with heterogeneous men (a continuum of types) and heterogeneous women (a finite number of types). The model has two distinguishing features. First, the search is targeted: each type of woman constitutes a distinctive submarket, and men are able to choose beforehand in which submarkets to participate, but the search is random within each submarket. Second, men and women are also horizontally differentiated. We show that there is always a unique equilibrium in which men are endogenously segmented into different submarkets, and that the equilibrium matching pattern is weakly positive assortative. We then explore how the equilibrium segmentation/marriage pattern changes as some exogenous shocks occur. In particular, we show that an Internet-induced increase in search efficiency would make the marriage pattern overall more assortative, while an increase in the dispersion of the horizontal match fitness could make the marriage pattern overall less assortative.

JEL Classifications: C78; D83; J12

Key Words: Targeted Search; Matching; Marriage Market; Horizontal Differentiation

1 Introduction

Search/matching is a very useful framework to study the marriage pattern in the marriage market (Burdett and Coles, 1997; and Smith, 2006). In these models, men and women are heterogeneous in their vertical types (defined by income, education level, and appearance, etc.), and they study the equilibrium matching pattern in the vertical dimension. However, one important feature has not been captured by these models: in the real world, the utility enjoyed by a married couple depends not only on their vertical types, but also on the compatibility of their personalties. That is, horizontal differentiation is also an important aspect in the marriage market. More specifically, men and women are also horizontally differentiated in terms of tastes/interests/hobbies/characters, and people also value mates with matching

*School of Economics and Finance, University of Hong Kong. Email: yujingxu@hku.hk

†Department of Economics, Ohio State University. Email: yang.1041@osu.edu

‡We would like to thank Jan Eeckhout, Jim Peck, Wing Suen and the audiences of 2nd China Workshop on Search and Matching for very helpful comments and suggestions.

tastes/interests/hobbies/characters. While it is documented that both vertical and horizontal differentiations are important in shaping the sorting pattern in the marriage market,[1] the existing literature lacks tractable theoretical models that incorporate both dimensions.

Another feature of Burdett and Coles (1997) and Smith (2006) is that they both assume random search. In the real world, however, people usually are able to narrow down their search to potential partners with a certain income or education level. That is, people can usually target their search in the dating market. This targeting ability has been greatly enhanced by the Internet. For instance, on Match.com, one can sort people according to their education levels.

To capture the above-mentioned features in the marriage market, this paper develops a search/matching model with (vertically differentiated) heterogeneous men and heterogeneous women, with the following two novel features. First, in addition to vertical differentiation, there is horizontal differentiation among men and women, which is unrelated to men's or women's vertical types. This reflects the compatibility between a woman's and a man's personalities. We model it as an i.i.d. match value between any woman-man pair. Second, search is targeted and one-sided (only men search for women). In particular, each type of woman (the number of types is finite) constitutes a distinctive submarket. Search is targeted, in the sense that men, whose types are continuous,[2] are able to choose in which submarkets to participate (or which types of women to target) beforehand. Within each submarket, however, search is random.

Within our framework, we intend to answer the following research questions. First, by making it easier to find a potential partner, how does the Internet impact the matching outcomes in the marriage market? Second, how is the marriage pattern affected when the horizontal match value becomes more dispersed, which seems to be a trend in modern and post-modern society? In particular, will those changes make the marriage pattern overall more assortative (high type men marrying high type women) or less assortative? How will these changes affect the utilities and the marrying speeds of different types of men and women? Will they increase or decrease men's or women's inequality in the marriage market? These are important questions, as they are not only important in their own right, but they also have implications for intergenerational mobility.

In our model, the man and the woman in a marriage always get the same utility, which is multiplicative in the man's type, the women's type, and their horizontal match value. Upon meeting, a man and a woman observe each other's vertical types as well as the horizontal match value. Then they simultaneously decide whether to marry. A marriage is consummated

[1] For instance, Hitsch, Hortacsu, and Ariely (2010) estimate that both horizontal and vertical components of preferences are important in driving observed sorting patterns in the marriage market.

[2] This is a simplification assumption. The model extends to settings with a finite number of men's types and the results still hold qualitatively.

if and only if both agree to marry. Once married, the couple exit the marriage market forever and are instantaneously replaced by two agents of the same vertical types. Another difference from the search/matching models of Burdett and Coles (1997) and Smith (2006) is that, while in their models the meeting rate of all types of men and all types of women are always the same, in our model the meeting rate could be different across types. In particular, we assume an urn ball search/matching technology (in continuous time). As a result, a man, who searches actively, has a constant meeting rate regardless of his type and the submarkets in which he participates; a woman, who does not search actively, has a meeting rate increasing in the men to women ratio in her submarket.

We show that there is always a unique market equilibrium. Moreover, in equilibrium men are endogenously segmented into different submarkets and the matching pattern is weakly positive assortative, with higher types of men searching for weakly higher types of women. In the horizontal dimension, every man and woman has a threshold of match value for each type of partner above which he or she is willing to accept the marriage (the acceptance cutoff). Within each submarket, higher type men, for whom women have a lower acceptance cutoff, are faster to marry (have higher probabilities of being accepted by women), since, all other things being equal, they are more desirable to women. This is also why a marginal type man can be indifferent between two adjacent submarkets. Being the highest type in one submarket and the lowest in the adjacent higher submarket, he trades off a better mate's type with a faster marrying speed. In equilibrium, all submarkets are indirectly linked through the indifference conditions of the marginal types of men. In addition, the equilibrium men to women ratio is, in general, higher in a higher submarket, which enlarges the difference between the marrying speeds of the marginal types in two adjacent submarkets and helps to sustain the indifference conditions.

The key driving force of the model is the indirect externalities men within the same submarket imposed on each other. First, more men being active in a submarket increases the men to women ratio and hence women's meeting rate in that submarket, which increases women's expected payoff. This thus makes them more choosy about men (they only accept men with a higher horizontal match fitness). This in turn reduces the attractiveness of this submarket to men. Therefore, although men's meeting rate is constant regardless of the men/women ratio, there still exists an indirect congestion effect for men. Another subtler driving force is the indirect externalities imposed by higher type men on lower type men within the same submarket. In particular, the presence of the higher type men, who are more desirable to women, boosts women's expected payoff more, relative to the lower type men of equal measure. In other words, the negative externalities imposed by higher type men on lower type men within the same submarket is stronger than the one vice versa.

We then conduct comparative statics. First, we examine how a change in women's type distribution affects equilibrium outcomes, motivated by the dramatic improvements in women's education opportunities and career prospects in China and Korea during the past few decades. When the measure of type n women increases, in all the submarkets lower than n the marginal types of men decrease, while in the higher submarkets the marginal types of men weakly increase. In all the lower submarkets, women are worse off, while men are better off. However, in the higher submarkets, either women are worse off and men are better off, or both women and men are unaffected by the change. This shows that shock in a single submarket transmits through the endogenous adjustment in segmentation to the whole market and such transmissions are asymmetric. In the upward direction, the shock transmission may stop at any submarket, while in the downward direction the shock transmission will go all the way to the lowest submarket. We also studied what happens when women's type distribution changes in several submarkets at the same time, and found similar results.

Second, we investigate the impact of the Internet. Specifically, the widespread use of the Internet in the last two decades reduces agents' search costs and makes meeting with potential spouses easier. In terms of modeling, it means that the Internet increases all men's contact rate. Under some fairly general conditions, we show that this results in an increase in the men's cutoff types, which implies a more assortative marriage pattern: fewer men are active in higher submarkets. All women are always better off. However, not all men are better off. Specifically, in each submarket the highest and the lowest types of men are in general better off, but the intermediate types might be worse off. Our simulation also indicates the following quantitative results regarding inequalities in the marriage market. First, among the higher submarkets, higher types of women benefit more than the lower types from an increase in the contact rate, while among the lower submarkets the lower types of women benefit more. Thus an Internet-induced increase in search efficiency increases women's inequality (in the marriage market) in the upper tail, but reduces women's inequality in the lower tail. Second, although within each submarket men's gains may not be monotonic in types, across the submarkets higher types of men gain more than lower types from an increase in the contact rate. Thus, an Internet-induced increase in search efficiency in general increases men's inequality in the marriage market.

Another impact of the Internet is that it makes horizontal targeting easier. That is, agents that share similar horizontal interests/hobbies/traits can now organize horizontal clubs online. We show that by reducing the possibility of getting a low horizontal match value, having horizontal clubs is essentially equivalent to an increase in the contact rate in the baseline model. Therefore, an Internet-induced horizontal targeting has similar qualitative impacts on the marriage market as an Internet-induced increase in search efficiency.

Finally, we study the impact of an increase in the dispersion of the horizontal match fitness, which can be caused by an increase in the variety of personalities or hobbies. As the dispersion of the horizontal match fitness increases, under some fairly general conditions the equilibrium cutoff types of men are very likely to decrease. This leads to overall less assortative matching, opposite to the impact of the Internet. The highest type women benefit from a more dispersed match value, but the lower types of women might be worse off. Among men within a submarket, again both the highest types and the lowest types are better off, but the middle types might be worse off. Our simulation results indicate that a more dispersed match value increases women's inequality as well as men's inequality in the marriage market. Moreover, the increase in women's inequality is more significant than the increase in men's inequality, as the endogenous adjustment in men's segmentation (more men in higher submarkets) amplifies women's inequality but dampens men's across submarkets.

Related Literature In economics, the seminal work of Becker (1973) first models the marriage market as a matching process. In a frictionless environment, he studies the marriage pattern with heterogeneous men and heterogeneous women (different vertical types). His main result is that equilibrium matching will exhibit perfect positive assortative matching if the output function of marriage is supermodular in man's and woman's types. Later works introduce search friction into Becker's matching model,[3] an approach followed by the current paper as well.

The most closely related papers to ours are Burdett and Coles (1997), Smith (2006), and Jacquet and Tan (2007), all of which are search and matching models in the marriage market with vertically differentiated men and women. A common difference between our model and theirs is that in their models there is no horizontal differentiation, which plays an important role in our model. With the presence of the horizontal match value, instead of being accepted for sure or being rejected for sure, the same type of men will be accepted by different types of women with different but positive probabilities. In some sense this acts as a shadow price, though the marriage model is the one with non-transferrable utilities.

In a completely random search model, Burdett and Coles (1997) show that equilibrium marriage exhibits block matching, which is weakly positive assortative. As mentioned earlier, a main difference is that, in our model, search is targeted. In Smith (2006), search is also completely random. His focus is on how the functional form of the output function of marriage affects the equilibrium matching pattern. In particular, he found that to ensure perfect positive assortative matching, the output function needs to be log-supermodular. Jacquet and Tan (2007) extend Burdett and Coles (1997) by allowing agents to choose with whom to meet.

[3] A non-exhaustive list includes: Burdett and Coles (1997), Burdett and Wright (1998), Eeckhout (1999), Bloch and Ryder (2000), Shimer and Smith (2000), Smith (2006), and Jacquet and Tan (2007).

That is, men and women are free to create submarkets beforehand. They show that this possibility makes the matching pattern more assortative (but not perfectly assortative). The feature that agents can choose who to meet with is related to the targeted search in our model. The difference is that in our targeted search the submarkets are exogenously fixed (defined by women types), and only men choose which type of women to meet with. Other differences between our model and its results and those of the aforementioned three papers will be elaborated on later in the text.

To the best of our knowledge, there is only one existing paper, Sundaram (2000), in the literature on search/matching in the marriage market that combines both vertically and horizontally differentiated men and women. In her model there are two types of men (on top of their vertical types), and they have different preferences for women's types/traits. Moreover, women's preference for men is homogeneous. In some sense, the horizontal differentiation in her paper, which is modeled as two different kinds of men's preferences for women's types/traits, is quite special. The horizontal differentiation in our model rather reflects personality compatibility, which is captured in an i.i.d. match value for any man-woman pair. Another important difference is that in Sundaram's model, search is completely random, but in our model, search is targeted.

The targeted search adopted in our paper is related to Moen's (1997) concept of competitive search equilibrium in the labor market context.[4] In his model, workers are endogenously segmented into different submarkets. Each submarket has the same type of firms, but across submarkets, the firm types are different. More closely related to our paper is Yang (2015). He develops a model of targeted search in a labor market setting, and shows that the Internet might have contributed to rising wage inequality as well as wage polarization. Similar to the current model, in his model, firms of different types constitute distinctive submarkets, and workers can choose beforehand in which submarket to participate, but within each submarket search is random.

The rest of the paper is organized as follows. Section 2 sets up the model. In Section 3 we characterize the market equilibrium and the endogenous segmentation of men into different submarkets. In Section 4, we study the comparative statics when the measures of women in some submarkets change. Section 5 studies the impacts of an Internet-induced increase in search efficiency. In Section 6, the impacts of an increase in the dispersion of the horizontal match value are analyzed, and Section 7 considers the possibility of horizontal targeting. Section 8 offers a conclusion and some discussion. All of the proofs are provided in the appendix.

[4] Also see the directed search models, such as Mortensen and Wright (2002), Shimer (2005), and Eeckhout and Kircher (2009).

2 Model

Consider a marriage market with heterogeneous men and heterogeneous women. Each man is characterized by his type y, which is continuously distributed on $[\underline{y}, \overline{y}]$ ($\underline{y} > 0$) with cumulative distribution function $F(y)$ and density function $f(y)$. The measure of men is normalized to 1. There are N types of women, and type n is characterized by θ_n, $\theta_1 > \theta_2 ... > \theta_N > 0$.[5] The measure of type n women is X_n, which is exogenously fixed. The total measure of women is $\sum X_n = X$, which is assumed to be close to 1. Sometimes we call type n women as nth class women. Time is continuous and all agents have the common discount rate r.

If a man i of type y and a women j of type n marry, each of them enjoys the same flow payoff $\varepsilon_{ij}\theta_n y$. The term $\theta_n y$ is the basic productivity of the marriage, which is supermodular in the man's and woman's type. The term ε_{ij} is the match fitness, which captures the horizontal aspects of match (the compatibility of temperaments, personalities, etc.). Note that ε_{ij} and the basic productivity $\theta_n y$ are supermodular as well.[6] The match fitness ε is i.i.d. across all man-woman pairs, and is independent of men's or women's types. In particular, ε is distributed on $[1-\gamma, 1+\gamma]$ with cumulative distribution function $G(\varepsilon)$ and density function $g(\varepsilon)$, assuming $\gamma \in (0, 1)$ and $E(\varepsilon) = 1$. Since $\gamma < 1$, the flow payoff of a marriage between any man and any woman is always positive. If an agent is single, his/her flow payoff is 0.

Men actively search for women[7] and the search is not random. In particular, the marriage market is segmented into N submarkets, with each type of woman constituting a distinctive submarket. The identity of each submarket (and hence women's types) is publicly observable. One can imagine that women of different classes go to different bars or attend different clubs. Each type of man can target his search by deciding in which submarket or submarkets to participate. If a man decides to participate in several submarkets, then he has to allocate his search efforts across these submarkets.

As will be shown later, generally each type of man will only search in one submarket, which uniquely gives him the highest expected discounted utility. Let Y_n be the measure of single men and $q_n = Y_n/X_n$ be the expected queue length in submarket n. We assume that the matching function is generated by an urn ball technology: at any instant the number of meetings in submarket n is αY_n. Equivalently, men's contact rate for women in any submarket is α,[8] while a type n woman's contact rate with men is αq_n, which is increasing in q_n (Mortensen and Pissarides, 1999, p. 2575-2576). One can think of α as men's common search intensity, which is

[5] In the Conclusion, we will discuss why we make women's types finite.

[6] The analysis in this paper extends qualitatively to more general situations where the basic productivity of the marriage is supermodular in y and θ_n and the flow payoff is supermodular in the basic productivity and ε_{ij}.

[7] Women do not search for men. Thus it is a one-sided search model. In the Conclusion we will discuss this feature further.

[8] If a man allocates his search efforts in several submarkets according to $\sigma = \{\sigma_n\}$, where $\sigma_n \geq 0$ and $\sum \sigma_n = 1$, then his contact rate in submarket n is $\alpha\sigma_n$.

exogenously given. The urn ball matching technology means that there is no direct externality of search congestion among men, and we will further discuss this matching technology in the Conclusion.

The search within each submarket is random. That is, when a type n woman meets a man, the man's type is a random draw from $F_n(y)$ (the density function is denoted as $f_n(y)$), which is the distribution of men's types active in submarket n. Once a man and a woman meet, they observe each other's type and the match value ε immediately. Then they simultaneously decide whether to marry. A marriage is consummated if and only if both agree to marry. Once married, they are out of the market forever, and they are replaced by clones of the corresponding types.[9] Due to the fact that all men have the same contact rate α and the clone-replacement assumption, the density function $f_n(y)$ is inherited from the the original density f, $f_n(y) = \frac{f(y)}{Y_n}$.

We label the specified search protocol as targeted search. This is because the search is partially directed, since men can choose which submarket(s) or which type(s) of women to target beforehand, and partially random, as it is within each submarket. In the existing literature of matching in the marriage market, search is either random (Burdett and Coles, 1997, Smith 2006, etc), or both men and women can endogenously form submarkets (Jacquet and Tan, 2007). Given the presence of the horizontal differentiation, men are searching for right (compatible) women of the right (vertical) type, and the same applies to women.

3 Market Equilibrium

3.1 Preliminary Analysis

A type y man's strategy consists of two parts: a participation strategy as to in which submarket(s) to participate, and a matching strategy as to which set of women (in terms of both n and the match value ε) to accept. A type n woman's (matching) strategy is a decision rule as to which set of men (in terms of both y and ε) to accept.

A participation strategy profile of all men leads to a segmentation of men into N submarkets. Denote a segmentation as $\overline{S} : [\underline{y}, \overline{y}] \rightarrow \{1, ..., N\}$. Let y_n be the set of men types participating in submarket n. Thus, $\{y_n\}_{n=1}^N$, which exhausts men's type space $[\underline{y}, \overline{y}]$, also represents a segmentation. Notice here the segmentation allows a man to target multiple submarkets.

Given the clone-replacement assumption, once a segmentation $\{y_n\}_{n=1}^N$ is determined, the measure of single men active in submarket n, Y_n, and its distribution $F_n(y)$ are both determined. The market condition in submarket n is thus summarized by $\{q_n, F_n(y)\}$. Denote

[9] This is a simplifying assumption, making the distribution of single men the same as the original distribution F. A further discussion will be offered in the Conclusion.

$U_n(y)$ as a man's expected discounted utility who is of type y and participates in submarket n, and denote V_n as a type n woman's expected discounted utility. Since the search environment is stationary, the optimal matching strategy of a type y man participating in submarket n is characterized by a reservation utility: accept a woman if and only if the overall matching utility is greater than his continuation value $U_n(y)$, or equivalently, if and only if the match value $\varepsilon \geq \min\{\max\{\frac{U_n(y)}{\theta_n y}, 1-\gamma\}, 1+\gamma\} \equiv \widehat{\varepsilon}_n^m(y)$. Similarly, a type n woman's optimal strategy is also characterized by a reservation match value $\widehat{\varepsilon}_n^w(y)$: a man of type y is accepted if and only if the match value $\varepsilon \geq \min\{\max\{\frac{V_n}{\theta_n y}, 1-\gamma\}, 1+\gamma\} \equiv \widehat{\varepsilon}_n^w(y)$. Note that $\widehat{\varepsilon}_n^w(y)$ is weakly decreasing in y, as a higher type man is more productive in marriage. Since a consummated marriage needs both parties to say yes, the acceptance cutoff (in terms of the horizontal match fitness) that ensures matching between a type y man and a type n woman is $\widehat{\varepsilon}_n(y) = \max\{\widehat{\varepsilon}_n^m(y), \widehat{\varepsilon}_n^w(y)\}$.

The value function $U_n(y)$ can be written as

$$rU_n(y) = \alpha \int_{\widehat{\varepsilon}_n^w(y)}^{1+\gamma} \max\{\varepsilon\theta_n y - U_n(y), 0\} dG(\varepsilon).$$

The constraint $\varepsilon \geq \widehat{\varepsilon}_n^w(y)$ reflects the fact that a marriage is consummated only if the woman involved agrees. Equivalently, in terms of $\widehat{\varepsilon}_n^m(y)$, it can be written as

$$U_n(y) = \max_{\widehat{\varepsilon}_n^m(y)} \frac{\alpha\theta_n y \int_{\widehat{\varepsilon}_n^m(y)}^{1+\gamma} \varepsilon dG(\varepsilon)}{r + \alpha(1 - G(\widehat{\varepsilon}_n^m(y)))} \text{ subject to } \widehat{\varepsilon}_n^m(y) \geq \widehat{\varepsilon}_n^w(y). \tag{1}$$

Assumption 1: $\gamma > \frac{r}{r+\alpha}$.

Lemma 1 *All men's optimal reservation match values are the same, regardless of their types and their chosen submarkets: $\widehat{\varepsilon}_n^m(y) = \widehat{\varepsilon}^m \in [1-\gamma, 1+\gamma)$. Moreover, if Assumption 1 is satisfied, then $\widehat{\varepsilon}^m > 1-\gamma$ and it strictly increases in α.*

Assumption 1 ensures that men care enough about the horizontal match fitness that they will not settle for women with the lowest match fitness $\varepsilon = 1-\gamma$, which is true if the lowest match fitness is bad enough (γ is large) or the cost of rejection is small enough (α is large or r is small). From now on, we will always impose Assumption 1 and hence focus on interior $\widehat{\varepsilon}^m$ only. All of the analysis and results can be extended to the case with $\widehat{\varepsilon}^m = 1-\gamma$.

The result that $\widehat{\varepsilon}^m$ is constant across all the submarkets for all men stems from the fact that in the flow payoff of marriage the horizontal and vertical components are multiplicative. That is, men's value function can be written as $z_1(\theta_n, y)z_2(\varepsilon)$ with some functions z_1 and z_2. Given this functional form, in each submarket n, maximizing the value is equivalent to maximizing $z_2(\varepsilon)$ by choosing ε, and thus all men have the same optimal $\widehat{\varepsilon}^m$.[10] The lemma also shows that

[10]If the flow payoff is additive in the horizontal and vertical components, say $\theta_n y + \varepsilon$, then the maximizer of the value $\int_{\widehat{\varepsilon}^m} (\theta_n y + \varepsilon) dG(\varepsilon)/[1 - G(\widehat{\varepsilon}^m)]$ would depend on θ_n and y. That is, different types of men in the

a higher contact rate α reduces the cost of rejection, thus men become more choosy, leading to a higher $\widehat{\varepsilon}^m$.

Denote $\overline{U}_n(y)$ as a type y man's maximum value in submarket n, which is achieved when type n women accept him whenever $\varepsilon \geq \widehat{\varepsilon}^m$. By (1), we have

$$\overline{U}_n(y) = \frac{\alpha\theta_n y \int_{\widehat{\varepsilon}^m}^{1+\gamma} \varepsilon dG(\varepsilon)}{r + \alpha(1 - G(\widehat{\varepsilon}^m))} = \frac{\alpha\theta_n y(1 - G(\widehat{\varepsilon}^m))E[\varepsilon|\varepsilon \geq \widehat{\varepsilon}^m]}{r + \alpha(1 - G(\widehat{\varepsilon}^m))}. \tag{2}$$

Given the type n women's value function V_n, we can trace the actual acceptance set of men for type n women: $\widehat{\varepsilon}_n(y) = \max\{\widehat{\varepsilon}_n^w(y), \widehat{\varepsilon}^m\}$. Note that $\widehat{\varepsilon}_n(y)$ is weakly decreasing in y. For higher types in submarket n, it might reach the lower bound $\widehat{\varepsilon}^m$. Given the set of male types active in submarket n, y_n, we can write down V_n as

$$rV_n = \alpha q_n \int_{y_n} \int_{\widehat{\varepsilon}_n(y)}^{1+\gamma} \{\varepsilon\theta_n y - V_n\} dG(\varepsilon) dF_n(y).$$

Rearranging, we get

$$V_n = \frac{\alpha \int_{y_n} \int_{\widehat{\varepsilon}_n(y)}^{1+\gamma} \theta_n y \varepsilon dG(\varepsilon) dF_n(y)}{r/q_n + \alpha \int_{y_n} [1 - G(\widehat{\varepsilon}_n(y))] dF_n(y)}. \tag{3}$$

Following the previous analysis, now we provide a definition of the equilibrium.

Definition 1 *A (marriage) market equilibrium consists of a segmentation $\overline{S}$ or $\{y_n\}_{n=1}^N$, men's matching strategy $\widehat{\varepsilon}^m$, and women's matching strategy $\{\widehat{\varepsilon}_n^w(y)\}_{n=1}^N$ such that:*

(i) Given women's optimal matching strategy $\{\widehat{\varepsilon}_n^w(y)\}_{n=1}^N$, for each type of man y the optimal matching strategy is given by $\widehat{\varepsilon}^m$.

(ii) Given $\overline{S}$ or $\{y_n\}_{n=1}^N$, and men's optimal matching strategy $\widehat{\varepsilon}^m$, women's matching strategy $\{\widehat{\varepsilon}_n^w(y)\}_{n=1}^N$ is optimal for all n.

(iii) Given $\overline{S}$ or $\{y_n\}_{n=1}^N$, and women's optimal matching strategy $\{\widehat{\varepsilon}_n^w(y)\}_{n=1}^N$, any type y man has no incentive to deviate to another submarket: for any y, $y \in y_n$, $U_n(y) \geq U_{n'}(y)$ for any $n' \neq n$.

Lemma 2 *In any market equilibrium, (i) $\frac{V_n}{\theta_n}$ is weakly decreasing in n; (ii) $\widehat{\varepsilon}_n(y)$ is weakly decreasing in n for any y.*

Lemma 2 is intuitive. For a type y man to have an incentive to participate in a lower submarket n, his probability of being accepted by all higher class women must be lower than

same submarket as well as the same type of men participating in different submarkets will choose different $\widehat{\varepsilon}^m$. But this will greatly complicate the analysis.

his probability of being accepted by class n women. This means that in equilibrium higher class women must be pickier, or $\widehat{\varepsilon}_n(y)$ is decreasing in n, which further implies that $\frac{V_n}{\theta_n}$ should be decreasing in n. This pattern also suggests that, in equilibrium, higher type men participate in weakly higher submarkets, or weakly positive assortative matching. To formally establish this single crossing property, we need an additional assumption regarding the distribution of ε.

Assumption 2: The density $g(\varepsilon)$ is logconcave and satisfies

$$\frac{g'(1+\gamma)}{g(1+\gamma)} + \frac{1}{1+\gamma} \geq 0. \tag{4}$$

The assumption that $g(\varepsilon)$ is logconcave is standard in the literature, and it is satisfied by many distributions. Since $g(\varepsilon)$ is logconcave, $g'(\varepsilon)/g(\varepsilon)$ is monotonically decreasing in ε.[11] Thus $g'(\varepsilon)/g(\varepsilon)$ reaches minimum when $\varepsilon = 1+\gamma$. Condition (4) ensures that $g'(\varepsilon)/g(\varepsilon)$ is not too negative for all ε, i.e., the density does not decrease too rapidly (relatively). This assumption guarantees that the reduction in the probability of being accepted from switching to a higher submarket is smaller for a higher type of man, as will become clear later.

Note that condition (4) is a weak condition. For uniform distribution, it is trivially satisfied. Now consider truncated normal distribution on support $[1-\gamma, 1+\gamma]$, with mean 1 and variance σ:

$$g(\varepsilon) = \frac{\phi(\frac{\varepsilon-1}{\sigma})}{\Phi(\frac{\gamma}{\sigma}) - \Phi(\frac{-\gamma}{\sigma})}\frac{1}{\sigma},$$

where ϕ and Φ are the p.d.f. and c.d.f. of a standard normal distribution, respectively. In this case, $\frac{g'(1+\gamma)}{g(1+\gamma)} = -\frac{\gamma}{\sigma^2}$. Thus condition (4) boils down to $\frac{\sigma^2}{\gamma(1+\gamma)} \geq 1$, or the variance σ^2 is big enough.

Lemma 3 *Consider two submarkets n' and n, with $n' < n$, and two types of men y and y' with $y' > y$. The following scenario cannot occur in any market equilibrium: type y men participating in submarket n', type y' men participating in submarket n, and $U_n(y) > 0$.*

Lemma 3 implies that equilibrium segmentation must exhibit weakly positive assortative matching.[12] Therefore, we have proved the following proposition.

[11] See Bagnoli and Bergstrom (2005).

[12] The lemma only applies to men whose discounted payoff is strictly positive. It is possible that there exists some y (very low types) such that $U_N(y) = 0$ (never accepted by any woman). Then $U_n(y) = 0$ for any n, and these types of men are indifferent between any submarket in equilibrium. Does that mean that an equilibrium exists that is not weakly positive assortative? Not really. This is because these types of men are always rejected in any submarket. If we only focus on the set of men that have strictly positive probabilities of being accepted by some women, then the equilibrium is still weakly positive assortative.

Proposition 1 *In any market equilibrium, a higher type man must participate in a weakly higher submarket.*

The underlying reasons for the weakly positive assortative matching (block matching) are twofold. First, given that the flow payoff in a marriage is supermodular, a higher type man gains more than a lower type man by matching with a higher class woman. Second, the reduction in the probability of being accepted by choosing a higher class woman is smaller for a higher type man. To see this, suppose $\widehat{\varepsilon}_n(y) = \frac{V_n}{\theta_n y} > \widehat{\varepsilon}^m$. Then it is immediate that

$$\widehat{\varepsilon}_{n-1}(y) - \widehat{\varepsilon}_n(y) = \frac{1}{y}\left(\frac{V_{n-1}}{\theta_{n-1}} - \frac{V_n}{\theta_n}\right)$$

is decreasing in y. Given that the distribution of ε is not too irregular (Assumption 2 ensures this), the property that $\widehat{\varepsilon}_{n-1}(y) - \widehat{\varepsilon}_n(y)$ is decreasing in y implies that switching from class n women to class $n-1$ women leads to a smaller reduction in the probability of being accepted for a higher type man. This means that, relative to a lower type man, it is less costly for a higher type man to choose a higher class woman.

Given the result of Proposition 1, an equilibrium segmentation is characterized by a nonincreasing sequence $\{\widehat{y}_n\}$, such that all men with $y \in [\widehat{y}_n, \widehat{y}_{n-1}]$ participate in submarket n. Moreover, the marginal type $\widehat{y}_n$ is indifferent between submarket n and submarket $n+1$.[13] This is possible because, being the lowest type in submarket n, he has a lower chance of being accepted, while in submarket $n+1$ he is the highest type, and thus has a higher chance of being accepted. Now we can explicitly write $F_n(y)$, the type distribution of men in submarket n. Specifically, $F_n(y) = \frac{F(y)-F(\widehat{y}_n)}{F(\widehat{y}_{n-1})-F(\widehat{y}_n)}$ for $y \in [\widehat{y}_n, \widehat{y}_{n-1}]$, and $Y_n = F(\widehat{y}_{n-1}) - F(\widehat{y}_n)$.

Remark 1 *Although this is a model of non-transferrable utilities, the presence of the horizontal match value leads to different accepting probabilities for different types of men within the same submarket, which in some sense act as shadow prices.*

Based on the analysis so far, a market equilibrium is characterized by a nonincreasing sequence of cutoff types $\{y_n^*\}$ such that:

(i) Given $\{y_n^*\}$, for any $n = 1, ..., N$: V_n satisfies equation (3), $\widehat{\varepsilon}_n^w(y) = \frac{V_n}{\theta_n y}$, and $\widehat{\varepsilon}_n(y) = \max\{\widehat{\varepsilon}^m, \widehat{\varepsilon}_n^w(y)\}$ (women's matching strategies are optimal).

(ii) Given $\widehat{\varepsilon}_n^w(y)$, the following indifference conditions are satisfied for the marginal types: for all $n = 1, ..., N-1$

$$\begin{aligned} U_n(y_n^*) &= U_{n+1}(y_n^*) \text{ if } y_n^* > \underline{y}; \text{ (interior solution)} \\ U_n(y_n^*) &\geq U_{n+1}(y_n^*) \text{ if } y_n^* = \underline{y}. \text{ (corner solution)} \end{aligned} \tag{5}$$

[13] The proposition shows that, for a generic type of man (except for the cutoff types y_n^*), there is a unique submarket which gives him the highest expected payoff, and thus he will only participate in that submarket.

More explicitly, the indifference condition (5) can be written as

$$\frac{\theta_n[1-G(\widehat{\varepsilon}_n(y_n^*))]E[\varepsilon|\varepsilon \geq (\widehat{\varepsilon}_n(y_n^*))]}{r+\alpha[1-G(\widehat{\varepsilon}_n(y_n^*))]} = \frac{\theta_{n+1}[1-G((\widehat{\varepsilon}_{n+1}(y_n^*))]E[\varepsilon|\varepsilon \geq (\widehat{\varepsilon}_{n+1}(y_n^*))]}{r+\alpha[1-G(\widehat{\varepsilon}_{n+1}(y_n^*))]}. \quad (6)$$

Since $\theta_n > \theta_{n+1}$, the indifference condition (5) means that $\widehat{\varepsilon}_{n+1}(y_n^*) < \widehat{\varepsilon}_n(y_n^*)$. That is, the marginal type y_n^* must have a strictly higher accepting probability in submarket $n+1$ than in submarket n.[14]

The following lemma is useful in later analysis.

Lemma 4 *(i) Fixing the upper bound $\widehat{y}_{n-1}$, as the lower bound $\widehat{y}_n$ decreases, V_n will weakly increase: V_n will strictly increase if $\widehat{\varepsilon}_n(\widehat{y}_n) < 1+\gamma$, and V_n will remain the same if $\widehat{\varepsilon}_n(\widehat{y}_n) = 1+\gamma$. (ii) Fixing the lower bound $\widehat{y}_n$, as the upper bound $\widehat{y}_{n-1}$ increases, V_n will strictly increase.*

Lemma 4 actually points out the driving force behind equilibrium segmentation. That is, men within the same submarket impose indirect negative externalities on each other, through the channel of changing women's payoff and their matching strategy. With more men in a particular submarket, it increases the meeting rate and the expected payoff of the women in that submarket. As a result, they become more picky about men, which means that men's expected payoff of participating in this club at least weakly decreases.[15] Another observation is that adding higher types of men to a given club has a bigger impact than adding lower types. This is because higher type men are more desirable to women, and thus will boost women's expected payoff significantly.[16]

3.2 Existence and Uniqueness of Equilibrium

The indifference condition (6) is a second order difference equation, which is highly nonlinear. Therefore, we have to establish the existence and uniqueness of equilibrium by ourselves. We proceed by induction.

Fixing the low bound $\widehat{y}_n$, let us consider partial equilibrium in submarkets $i = 1, ..., n$: a segmentation of men with types $y \in [\widehat{y}_n, \overline{y}]$ into submarkets $i = 1, ..., n$ such that the equilibrium conditions (5) are satisfied. We denote the partial equilibrium segmentation as $\{y_i^*(\widehat{y}_n)\}_{i=1}^{n-1}$. And sometimes we abuse notation and simply write it as $\{y_i^*\}_{i=1}^{n-1}$.

[14] The proof of Lemma 1 also shows that the term $\frac{[1-G(\widehat{\varepsilon})]E[\varepsilon|\varepsilon \geq (\widehat{\varepsilon})]}{r+\alpha[1-G(\widehat{\varepsilon})]}$ is strictly decreasing in $\widehat{\varepsilon}$ for $\widehat{\varepsilon} \geq \widehat{\varepsilon}^m$.

[15] Type y men's expected payoff strictly decreases if $\widehat{\varepsilon}_n(y)$ strictly increases or women's acceptance cutoff binds after the change.

[16] As the set of participating men changes in submarket n, typically the above mentioned two effects (Y_n and the average type of men both change) are triggered at the same time. But still we can conceptually distinguish these two effects. For instance, we can increase the average type of men without changing Y_n. In this case, V_n will increase.

Lemma 5 *Fix any* $\widehat{y}_2 \in [\underline{y}, \overline{y}]$*. (i) There is a unique* $y_1^* \in [\widehat{y}_2, \overline{y})$ *achieving partial equilibrium in submarkets 1 and 2. (ii)* y_1^* *is weakly increasing in* $\widehat{y}_2$*. (iii)* V_2^* *is weakly decreasing in* $\widehat{y}_2$*, and it is strictly decreasing in* $\widehat{y}_2$ *if* y_1^* *is strictly increasing in* $\widehat{y}_2$*.*

Lemma 6 *Suppose the properties in Lemma 5 hold for n. That is, given any* $\widehat{y}_n \in [\underline{y}, \overline{y}]$*, we have: (i) there is a unique* $\{y_i^*\}_{i=1}^{n-1} \in [\widehat{y}_n, \overline{y})$ *achieving partial equilibrium in submarkets* $1, ..., n-1$*; (ii)* y_{n-1}^* *is weakly increasing in* $\widehat{y}_n$*; (iii)* V_n^* *is weakly decreasing in* $\widehat{y}_n$*, and it is strictly decreasing in* $\widehat{y}_n$ *if* y_{n-1}^* *is strictly increasing in* $\widehat{y}_n$*. Then, given any* $\widehat{y}_{n+1} \in [\underline{y}, \overline{y}]$*, these properties also hold for* $n+1$*.*

Lemma 5 shows that Lemma 6 holds for $n = 2$. Applying Lemma 6 repeatedly, the results hold for any $n \leq N$. Thus, we have proved the following proposition.

Proposition 2 *There is a unique market equilibrium.*

By Lemma 5 and Lemma 6, we also have the following useful corollary.

Corollary 1 *(i) If* $\widehat{y}_n$*,* $2 \leq n \leq N-1$*, increases for exogenous reasons, then* $\{y_i^*\}_{i=1}^{n-1}$*, which ensures partial equilibrium in submarkets* $1, ..., n$*, all weakly increase, and* V_n *weakly increases as well. (ii) If* $\widehat{y}_n$*,* $1 \leq n \leq N-2$*, increases for exogenous reasons, then* $\{y_i^*\}_{i=n+1}^{N-1}$*, which ensures partial equilibrium in submarkets* $n+1, ..., N$*, all strictly increase.*

While part (i) of Corollary 1 is directly implied by Lemma 5 and Lemma 6, part (ii) can be proved in a similar way by reversing the direction. Roughly speaking, this corollary means that all equilibrium cutoffs must move in the same direction.

The existence of equilibrium is not surprising, as each man has to choose some submarkets to participate in. The uniqueness of the equilibrium is due to the indirect negative externalities that men within the same submarket impose on each other. Having more men in a particular submarket increases women's expected payoff, and they become more choosy. Thus, men's probabilities of being accepted weakly decrease (strictly, for men of low types), which reduces their expected payoff in this particular submarket. In short, more men in a submarket reduces the attractiveness of this submarket to men. On the other hand, more men in a submarket means fewer men in other submarkets. For the same reason, other submarkets become more attractive to men. This indirect negative externality means that in equilibrium the segmentation has to be right to ensure no man has an incentive to deviate to another submarket, which implies the uniqueness of equilibrium.[17]

[17] More formally, consider submarkets n and $n+1$ and the marginal type y_n^*. If the marginal type y_n^* increases, then, due to the indirect negative externality, V_n decreases and V_{n+1} increases. Thus $U_n(y_n^*)$ increases but $U_{n+1}(y_n^*)$ weakly decreases, meaning that now a type y_n^* man strictly prefers submarket n, which contradicts the fact that the marginal type increases. For a similar reason, there is no equilibrium with a marginal type lower than y_n^*.

3.3 Equilibrium features

The market equilibrium exhibits the following features. First, men are endogenously segmented into N (may be fewer) classes, with nth class men only marrying the corresponding nth class women. Note that the set of men active in the lower classes might be empty, because, all other things being equal, higher class women are more desirable to men.[18] This means that women in the lower classes might never get married, a feature similarly noted in Burdett and Coles (1997).

Second, within each class, the higher types of men get higher expected payoffs than the lower types. Moreover, the higher types are (weakly) faster to marry than the lower types. Though each type of man has the same contact rate, a higher type man has a higher chance to be accepted by women ($\widehat{\varepsilon}_n^w(y)$ is decreasing in y). This feature is different from what Burdett and Coles (1997) and Jacquet and Tan (2007) found, in which men in the same class have the same expected payoff and the same matching rate. The difference arises mainly because in our model there is horizontal match value. As a result, a higher type man has a higher chance of being accepted, which leads to a higher expected payoff.[19]

Third, although the expected payoffs of men and women in a submarket only depend on the market conditions within the same submarket (q_n and $F_n(y)$), all the submarkets are interlinked. This is because which set of men participate in a particular submarket depends on the market conditions in the adjacent submarkets. Specifically, in each submarket the two marginal types of men (the high cutoff type and the low cutoff type) have to be indifferent between adjacent submarkets. As a result, all submarkets are indirectly linked. A related feature is that men's equilibrium payoff schedule (as a function of type), $U_e(y)$, is continuous over types, as the marginal types are indifferent between adjacent submarkets.

These features mentioned in the last paragraph are different from those discovered by Burdett and Coles (1997) and Jacquet and Tan (2007). In particular, in their models, men's equilibrium payoff schedule is discontinuous across classes, with the marginal types strictly preferring the higher class to the adjacent lower class. The underlying reason is that in their models there is no horizontal match value. Actually, in both models the classes are determined from top to bottom: a lower class man will not be accepted by a higher class woman, even if he strictly prefers the higher class woman. This also means that in their models there is only one-directional linkage between classes: higher classes affect lower classes. If the market conditions in the lower classes change, they will not affect the segmentation among the higher

[18] If submarket n has a positive measure of men, then each higher submarket i, $i < n$, must be active, or have a positive measure of men. Similarly, if submarket n is inactive or has no men participating in it, then all lower submarkets ($i > n$) must be inactive as well.

[19] Another difference is that in Burdett and Coles (1997) and Jacquet and Tan (2007), an agent's flow payoff in a marriage does not depend on his/her own type, but only depends on his/her partner's type.

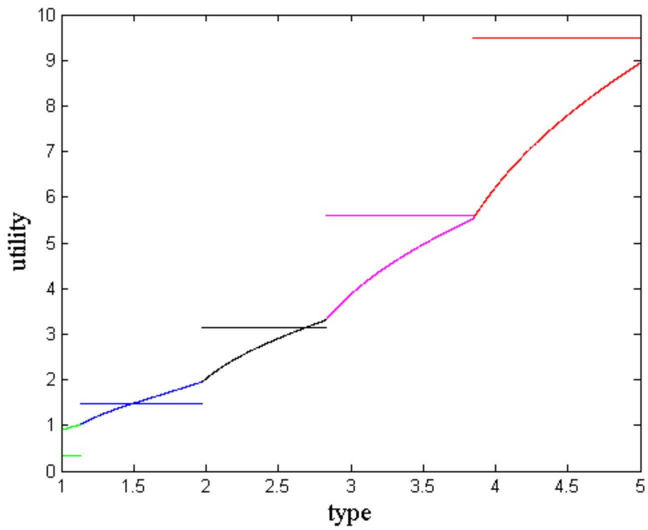

Figure 1: Men's Payoff Schedule and Women's Values

classes. In contrast, in our model, changes in the market conditions in lower classes could affect the segmentation among higher classes (if $U_{n+1}(y_n^*) < \overline{U}_{n+1}(y_n^*)$).

Is it the case that in equilibrium the expected queue length q_n^* must be higher in a higher submarket? The general pattern is yes. Recall that to make a marginal type of man indifferent between two adjacent submarkets, his probability of being accepted by the higher class women must be strictly lower than his probability of being accepted by the lower class women. A higher q_n^* in the higher submarket, by increasing V_n^*/V_{n+1}^*, would help in achieving the indifference condition.

Example 1 *(Benchmark) Suppose* $r = 0.05$*,* $\alpha = 0.25$*. Men's types follow a truncated normal distribution on* $[1, 5]$ *with* $\mu = 1.8$ *and* $var = 2$*.*[20] *Women's type distribution is given by* $[\theta_1, \theta_2, \theta_3, \theta_4, \theta_5] = [2, 1.6, 1.3, 1.1, 1]$*, and* $[X_1, X_2, X_3, X_4, X_5] = [0.05, 0.1, 0.15, 0.32, 0.3]$ *(the total measure of women* $X = 0.92$*).*[21] *The match value* ε *is uniformly distributed on* $[1-\gamma, 1+\gamma]$ *with* $\gamma = 0.5$*. The equilibrium is illustrated in the following figure.*

In Figure 1, the continuous curve indicates men's equilibrium payoff schedule (as a function of men's type), with different colors indicating men in different submarkets. The horizontal lines represent women's equilibrium values in different submarkets (the length corresponds to the type space of men participating in that submarket). The key endogenous variables in the equilibrium are listed in the following table.

[20] Both μ and var are the mean and variance of the original normal distribution before truncation. Men's type distribution here roughly resembles the income distribution in the U.S.

[21] The median type woman is a type 4 woman. The mean type lies between θ_4 and θ_3.

Table 1: Equilibrium Variables

	Submarket 1	SM 2	SM 3	SM 4	SM 5
cutoff types (y_n^*)	3.8461	2.8281	1.9723	1.1374	–
q_n^*	3.2748	2.5052	1.7988	0.8539	0.1423

One prominent feature of Table 1 is that the equilibrium expected queue length q_n^* is higher in a higher submarket. In the highest three submarkets, the q_n^* are all significantly bigger than 1; in submarket 1, q_1^* is higher than 3, meaning that the men/women ratio is bigger than 3.[22] Because q_1^* is very big, V_1^* is higher than the equilibrium payoff of the highest type men (see Figure 1). This implies that in submarket 1, women's acceptance cutoff is binding for all types of men. The same pattern also holds in submarket 2: $V_2^* > U_2(y_1^*)$. On the other hand, there are very few men participating in submarket 5, though it has many women in it: q_5^* is less than 0.15. This leads to the feature that V_5^* is lower than the lowest type men's equilibrium utility.

The above observations imply that in equilibrium men are congested in higher submarkets.[23] In other words, the difference in qs is the primary channel that makes the marginal types of men indifferent between two adjacent submarkets. Looking into the difference in the qs, we find the following pattern: the difference is largest between q_3^* and q_4^* and decreases as we move to higher or lower submarkets. The underlying reason for this pattern is as follows. The men's type (recall $\mu = 1.8$) with the highest density is in submarket 4. Thus, the density function of men's types is decreasing in submarket 1, 2, and 3, hump-shaped in submarket 4, and increasing in submarket 5. Therefore, the averages of men's types in submarket 3 and 4 are relatively close. In order to restore the indifference condition by enlarging the difference between V_3 and V_4, the difference between q_3^* and q_4^* has to be large enough. As we move to higher (or lower) submarkets, the bell-shaped density function implies that the difference in the averages of men's types in the adjacent submarkets is larger. This means that the indifference condition can hold even when the difference in qs is smaller. In addition, because q_n^* is higher in a higher submarket, men's differences in equilibrium payoffs are relatively suppressed. For instance, $U_1(\overline{y})/U_5(\underline{y}) < 10 = \frac{\theta_1 \overline{y}}{\theta_5 \underline{y}}$. For the same reason, women's differences in equilibrium values are amplified: $V_1^*/V_5^* \simeq 25 > 10 = \frac{\theta_1 \overline{y}}{\theta_5 \underline{y}}$.

Men's and women's equilibrium marrying rates are illustrated in Figure 2, with the thick horizontal lines indicating women's marrying rates.[24] From the figure, we can see that women's

[22] This means that type 1 women's contact rate is more than three times of the highest type men's contact rate.

[23] This feature has something to do with the clone-replacement assumption. A more detailed discussion is offered in the Conclusion.

[24] Specifically, a type y man's marrying rate in submarket n is $\alpha(1-G(\widehat{\varepsilon}_n(y)))$, and a type n woman's marrying rate is $\alpha q_n \int_{y_n} [1 - G(\widehat{\varepsilon}_n(y))] dF_n(y)$.

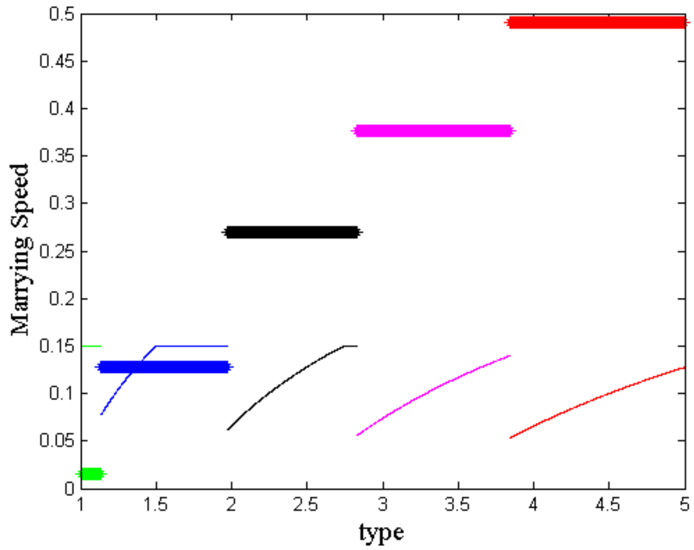

Figure 2: Men's and Women's Marrying Speed

marrying rates are higher in higher submarkets. This feature is mainly driven by the fact that q_n^* is higher in a higher submarket. For high types of men in submarkets 1 and 2, women's acceptance cutoff is always binding (since V_1^* and V_2^* are relatively big due to big q_1^* and q_2^*), and thus within each of these two submarkets men's marrying rate is monotonically increasing in men's types. In submarkets 3 and 4, for high types of men, the men's acceptance cutoff is binding. In submarket 5, men's acceptance cutoff is always binding, since V_5^* is relatively low due to a small q_5^*. Across submarkets, men's equilibrium marrying rate is not monotonic in type.

In the next three sections, we will conduct comparative statics, investigating how the equilibrium responds to shocks. To ease exposition, in the rest of the paper we assume the parameter values are such that in equilibrium each submarket of women is active or has a positive measure of men participating in it. Given that V_n is weakly decreasing in n, it is sufficient to assume that $V_N^* > 0$. Roughly speaking, it requires that the overall men/women ratio q is not too small.

4 Changes in Women's Distribution

4.1 An increase in X_n

We first study how changes in X_n, the measure of nth class women, affect equilibrium. First of all, it is easy to see that $\widehat{\varepsilon}^m$ will not change, as the equation that implicitly determines $\widehat{\varepsilon}^m$, (10), does not depend on X_n.

Proposition 3 *Suppose X_n increases to $X'_n > X_n$, while all the other parameter values of the model remain the same. Then the following results hold. (i) For $i \geq n$, y_i^* strictly decreases, V_i^* strictly decreases, and $U_i(y)$, $y \in y_i$, strictly increases. (ii) For $i \leq n-1$, y_i^* strictly increases, V_i^* strictly decreases, and $U_i(y)$, $y \in y_i$, strictly increases, if $\widehat{\varepsilon}_j(y_{j-1}^*) > \widehat{\varepsilon}^m$ for all j, $i+1 \leq j \leq n$; and y_i^*, V_i^*, and $U_i(y)$, $y \in y_i$, all remain the same if there is a j, $i+1 \leq j \leq n$, such that $\widehat{\varepsilon}_j(y_{j-1}^*) = \widehat{\varepsilon}^m$.*

The intuition for Proposition 3 is as follows. When there are more class n women, if the initial equilibrium segmentation does not change, then the meeting rate of each class n woman decreases. As a result, their expected payoff V_n decreases and they become less choosy about men. Thus, men participating in this submarket get higher utilities. This attracts the inframarginal types (close to the types participating in club n) in adjacent submarkets to switch to submarket n, which further induces adjustments in other submarkets. Although the individual shock occurs in submarket n, all men and women could be affected through the endogenous adjustment in the market segmentation.

We want to emphasize that the transmissions of a shock in submarket n are asymmetric in the upward and downward directions. When X_n increases, all lower class women (lower than n) and the corresponding men are definitely affected. However, higher class women and the corresponding men might not be affected. The underlying reason is that, for the lower marginal type men in any submarket j, in equilibrium, the women's acceptance cutoff always binds ($\widehat{\varepsilon}_j(y_j^*) > \widehat{\varepsilon}^m$), thus a change in V_j will always affect this type of man's utility in submarket j. On the other hand, for the upper marginal type men in submarket j, the men's acceptance cutoff might be binding ($\widehat{\varepsilon}_j(y_{j-1}^*) = \widehat{\varepsilon}^m$), thus a decrease in V_j might not affect this type of man's utility in submarket j. As a result, the upper bound y_{j-1}^* might not change, leaving all the higher class women and the corresponding men unaffected.

Example 2 *Suppose the benchmark example X_2 increases from* 0.1 *to* 0.13*, while all the other parameter values remain the same. The change in the equilibrium is illustrated in the following figure and table.*

Table 2: How Equilibrium Variables Change as X_2 Increases

	Submarket 1	SM 2	SM 3	SM 4	SM 5
$y_n^{*\prime} - y_n^*$	0.0015	-0.1114	-0.0955	-0.0637	–
$V_n^{*\prime} - V_n^*$	-0.0007	-0.2278	-0.1492	-0.0833	-0.1530
$q_n^{*\prime} - q_n^*$	-0.0059	-0.3230	-0.0333	-0.0272	-0.0664

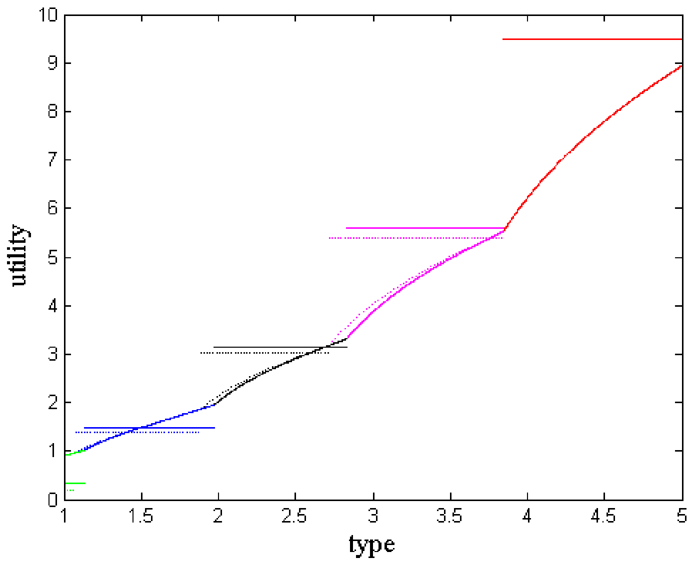

Figure 3: The Impacts of an Increase in X_2

In Figure 3, the dotted curve and the dotted V-lines are associated with the new $X_2 = 0.13$. As indicated by Figure 3 and Table 2, an increase in X_2 reduces all the cutoff male types in the lower submarkets, but y_1^* slightly increases. The equilibrium q_n^* decreases for all n. Moreover, Figure 3 and Table 2 suggest a pattern of asymmetric shock transmission. The higher submarket, submarket 1, is barely affected. However, the lower submarkets are affected relatively more significantly: for instance, a decrease in V_3^* is relatively significant.[25] Finally, in either submarket 2 or submarket 3, we see that the lower types of men in general benefit more from an increase in X_2 than higher types do. This is because for the lower types of men, women's acceptance cutoff is binding, thus they benefit more from a decrease in women's value. Moreover, among the types of men for which women's acceptance cutoff is binding, the lower types benefit more. Again, the reason is that lower types have higher acceptance cutoffs, thus the same reduction in the women's payoff results in a larger percentage increase in marrying probability for lower types of men.

4.2 An increase in X_n for any $n \leq \tilde{n}$

Consider an increase in the measures of all high classes of women (higher than class $\tilde{n}$). This could be due to more women investing in higher education. Notice that this also include, as a

[25] The reductions in V_5^* and q_5^* are relatively more significant than those in submarkets 3 and 4. The reason is that either submarket 3 or submarket 4, though they lose higher types of men to the higher submarket, gain lower types of men from the lower submarket. In contrast, submarket 5 loses higher types of men without gaining lower types, as it is the lowest submarket.

special case, the situation where the measure of all classes of women increase.

Lemma 7 *Suppose X_n increases for any $n \leq \tilde{n}$. If the initial equilibrium segmentation $\{y_n^*\}$ stays unchanged, then V_n strictly decreases for all $n \leq \tilde{n}$.*

To ease exposition, in the analytical analysis of the rest of the paper, we further impose the assumption that for any n, there exists a neighborhood of y_{n-1}^* such that $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$ for all y in the neighborhood, or $V_n < U_n(y_{n-1}^*)$.[26] This requires that q_n is not too large for any submarket n. Note that if $q_n = 1$, then the assumption is naturally satisfied. To see this, note that when $q_n = 1$, V_n can be expressed as a weighted average of $U_n(y)$: $V_n = \int_{y_n} w_n(y) U_n(y) dF_n(y)$, where the quasi-weight $w_n(y)$ is given by

$$w_n(y) = \frac{r + \alpha[1 - G(\widehat{\varepsilon}_n(y))]}{r + \alpha \int_{y_n} (1 - G(\widehat{\varepsilon}_n(y))) dF_n(y)}. \tag{7}$$

It is easy to see that $w_n(y)$ is weakly increasing in y. Combining that with the fact that $U_n(y)$ is increasing in y, we have $V_n < U_n(y_{n-1}^*)$.

Proposition 4 *Suppose X_n increases for any $n \leq \tilde{n}$, while all the other parameter values of the model remain the same. Then for any n, (i) y_n^* strictly decreases; (ii) V_n strictly decreases; and (iii) $U_n(y)$ strictly increases if in the original equilibrium $\widehat{\varepsilon}_n(y) > \widehat{\varepsilon}^m$ and stays unchanged otherwise.*

Proposition 4 shows that all women are worse off when there are more high type women. All men are weakly better off. Moreover, the set of men who are strictly better off may belong to disconnected intervals of types. In particular, it is possible that some low type men are strictly better off while some high type men's expected payoffs stay unchanged. This pattern emerges because for some types of men, their probability of being accepted by women does not depend on changes in V_n (men's acceptance cutoff binds).

As mentioned earlier, a special case of Proposition 4 is that each X_n increases by the same percentage. When each X_n decreases by the same percentage, the results of the proposition are just reversed. Note that the scenario in which each X_n decreases by the same percentage is equivalent to the scenario in which the the sex ratio Y/X increases but the distributions of men and women do not change. Thus, when that happens, all the equilibrium cutoffs y_n^* increase, all women are strictly better off, and all men are weakly worse off. Some types of men are strictly worse off, and this set of men may belong to disconnected intervals.

[26] Another case is that, for some submarkets n, $V_n \geq U_n(y_{n-1}^*)$. Then we will have too many cases to consider.

5 An Internet-Induced Increase in Search Efficiency

In this section, we study how the widespread use of the Internet affects the equilibrium in the marriage market. Essentially, the Internet reduces men's search costs for women. Instead of attending social gatherings, the Internet allows people to search and contact relevant partners at home. Given the exogenous fixed search intensity assumed in our model, the Internet, by increasing search efficiency, increases each man's contact rate. Unlike an increase in X_n, this is a universal shock applying to all men in all submarkets.

In particular, suppose α increases to $\alpha' > \alpha$ due to an Internet-induced increase in search efficiency. Note that the contact rates of all types of men increase by the same magnitude.[27] We will conduct the analysis in two steps. In the first step, we hypothetically assume that the initial equilibrium segmentation does not change, and we investigate how V_n and $U_n(y)$ will change. In the second step, we study how the endogenous segmentation will adjust.

5.1 Fixed segmentation

Suppose the initial equilibrium segmentation $\overline{S}$ does not change. In other words, y_n, q_n, and $F_n(y)$ all remain the same. The first observation is that an increase in α will cause $\widehat{\varepsilon}^m$ to increase to $\widehat{\varepsilon}^{m\prime} > \widehat{\varepsilon}^m$ (as shown in Lemma 1). Define the change in the equilibrium value of type y men in submarket n as $\Delta U_n(y) \equiv U_n'(y) - U_n(y)$. We further impose the following assumption on the distribution of ε.

Assumption 3: $\frac{g(\varepsilon)}{r+\alpha(1-G(\varepsilon))}$ is weakly increasing ε.

This assumption is a little stronger than the logconcavity of $g(\varepsilon)$, which is equivalent to $\frac{g(\varepsilon)}{1-G(\varepsilon)}$ being weakly increasing in ε. Note that it is trivially satisfied for uniform distribution. For truncated normal distribution it is satisfied as well if the variance σ is big enough. Roughly speaking, this condition says that the density $g(\varepsilon)$ cannot decrease too fast.

Lemma 8 *Suppose α increases to $\alpha' > \alpha$ while all the other parameter values of the model remain the same. In addition, suppose either $q_n \geq 1$ or q_n is close enough to 1 for each n, and the initial equilibrium segmentation $\overline{S}$ does not change. Then for any n, (i) suppose Assumption 3 holds, $\widehat{\varepsilon}^m \leq 1$, and $\frac{r+\alpha[1-G(1)]}{g(1)} - \frac{\alpha^2\gamma}{r} \geq 0$, then $V_n' > V_n$; (ii) $\Delta U_n(y)$ is increasing in y for all $y \in y_n$ and $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$; (iii) $\Delta U_{n+1}(y_n^*) > 0$; suppose Assumption 3 holds and $\frac{\partial V_n}{\partial \alpha} > \frac{\partial U_n(y_n^*)}{\partial \alpha}$, then $\Delta U_n(y_n^*) < \Delta U_{n+1}(y_n^*)$.*

The intuition for Lemma 8 is as follows. As men's contact rate α increases, women's contact rate αq_n increases as well, and this directly benefits women. The indirect impact is that an

[27] In the real world, different types of men might use the Internet (searching for partners) with different intensities. We abstract away from this heterogeneity.

increase in α makes men more choosy: men's acceptance cutoff increases. This indirect effect tends to hurt women. However, since q_n is large enough (either close to 1 or larger than 1), the direct benefit outweighs the indirect loss.[28] As a result, V_n must increase in α.

Part (iii) shows that a marginal type man y_n^* gains more from an increase in α by staying in the lower submarket $n+1$ than staying in the higher submarket n. To understand this result, we replicate the key equation in the proof:

$$\frac{\partial U_n(y_n^*)}{\partial \alpha} - \frac{\partial U_{n+1}(y_n^*)}{\partial \alpha} \propto [G(\widehat{\varepsilon}_n) - G(\widehat{\varepsilon}^m)] - [1 - \frac{U_n(y_n^*)}{V_n}]g(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n,$$

where $\widehat{\varepsilon}_n$ is a shorthand for $\widehat{\varepsilon}_n(y_n^*)$. Essentially, the result is driven by two opposite effects. The first effect favors the the higher submarket n. This is because, relative to being in the lower submarket $n+1$, in the higher submarket n a type y_n^* man's initial acceptance probability is lower and hence he can gain more in percentage terms (which is captured in the first term in the key equation: $G(\widehat{\varepsilon}_n) - G(\widehat{\varepsilon}^m) > 0$). Thus this effect implies that the marginal type man gains more by staying in the higher submarket n. The second effect, on the other hand, favors the lower submarket $n+1$. The reason is that being the highest type in submarket $n+1$, a type y_n^* man is not affected by an increase in V_{n+1}. However, he is the lowest type in submarket n, and thus an increase in V_n makes women more choosy about this type, reducing his expected payoff (this is reflected in the second term in the key equation). When V_n increases significantly in α (guaranteed by the condition $\frac{\partial V_n}{\partial \alpha} > \frac{\partial U_n(y_n^*)}{\partial \alpha}$) and the induced increase in the acceptance cutoff significantly reduces the acceptance probability of type y_n^* men in submarket n (guaranteed by the assumption on the distribution), the second effect dominates the first effect, and the marginal type men gain more from an increase in α by staying in the lower submarket.

Let us now discuss the conditions for part (i) and part (iii) of Lemma 8. Actually, the sufficient conditions specified in part (i), $\widehat{\varepsilon}^m \leq 1$ and $\frac{r+\alpha[1-G(1)]}{g(1)} - \frac{\alpha^2\gamma}{r} \geq 0$, are far from being necessary for V_n to be increasing in α. In the following lemma, we show that when the horizontal match value follows either truncated normal or uniform distribution, these two conditions are satisfied if $\frac{r}{\alpha}$ is not too small, or equivalently, if people are not too patient and the arrival rate is initially not too large.

Lemma 9 *If ε follows either truncated normal or uniform distribution, then there exists an R, such that when $\frac{r}{\alpha} \geq R$, $\widehat{\varepsilon}^m \leq 1$ and $\frac{r+\alpha[1-G(1)]}{g(1)} - \frac{\alpha^2\gamma}{r} \geq 0$ both hold.*

The condition for part (iii) to hold is $\frac{\partial V_n}{\partial \alpha} > \frac{\partial U_n(y_n^*)}{\partial \alpha}$, or women benefit more than the lowest type men in submarket n from an increase in α. This condition is not restrictive, and we expect it to hold fairly generally when $q_n \geq 1$ or q_n is close enough to 1. Our reasoning is

[28] The fact that $q_n \geq 1$ means that the increase in women's contact rate is bigger than that of men's as α increases.

as follows. As we pointed out in the previous section, V_n is a weighted average of $U_n(y)$ when $q_n = 1$, with the weight being $w_n(y) f_n(y)$. The result that $\frac{\partial V_n}{\partial \alpha} > \frac{\partial U_n(y_n^*)}{\partial \alpha}$ is due to two things. First, higher type men will generally benefit more from an increase in α: $\frac{\partial U_n(y)}{\partial \alpha}$ is increasing in y. This is because their expected payoffs per meeting are higher, which is due to their higher vertical types and higher acceptance probabilities. Moreover, their increase in the acceptance cutoffs induced by an increase in V_n are smaller: $\Delta \widehat{\varepsilon}_n^w(y) = \frac{V_n' - V_n}{\theta_n y}$ is decreasing in y.[29] Second, as α increases, the quasi-weights $w_n(y)$ increase for higher types but decrease for lower types, which tends to make V_n increase faster than $U_n(y_n^*)$ does. The following Lemma provides a set of conditions that are sufficient (far from being necessary) for $\frac{\partial V_n}{\partial \alpha} > \frac{\partial U_n(y_n^*)}{\partial \alpha}$.

Lemma 10 *Suppose ε is uniformly distributed on $[1-\gamma, 1+\gamma]$, q_n is close to 1, and $\frac{1+\gamma}{\gamma} - 4(\frac{r}{\alpha})^2 \frac{\gamma}{1+\gamma} - 4\frac{r}{\alpha} \leq 0$.[30] Then, $\frac{\partial V_n}{\partial \alpha} > \frac{\partial U_n(y_n^*)}{\partial \alpha}$.*

5.2 With endogenous adjustment in segmentation

In the previous subsection, we see that as α increases, the marginal types will no longer be indifferent between two adjacent submarkets if the segmentation remains fixed. Thus the segmentation will endogenously adjust, which is characterized in the following proposition.

Proposition 5 *Suppose α increases to α' while all the other parameter values of the model remain the same, and the conditions in Lemma 8 are satisfied. Then for each n: (i) y_n^* strictly increases; (ii) V_n strictly increases; (iii) The highest types of men (in submarket 1) are strictly better off; if the increase in α is not too large, then the highest types of men in submarket n, $1 < n \leq N$, are strictly better off, and the lowest types of men originally in submarket n, $1 \leq n < N$, are strictly better off as well.*

Proposition 5 shows that an increase in α, or an Internet-induced increase in search efficiency, makes the matching pattern more assortative in the vertical dimension. That is, there will be fewer men participating in higher submarkets and more men participating in lower submarkets. This could potentially reduce the intergenerational mobility. The underlying reason for this result is that in each submarket women value high types of men more. As the search efficiency increases, it becomes easier for women to meet with high type men, and thus they become choosier and it affects low type men more. As a result, men of low types switch to the adjacent low submarkets.

As for the expected payoffs, all women (the passive side in terms of search) are unambiguously better off. For men's side, the highest types of men (in submarket 1) are strictly better

[29] Of course, this does not translate directly to an increase in $\Delta U_n(y)$ for higher types, as it also depends on the distribution of ε.

[30] Again, this condition holds if r/α is not too small.

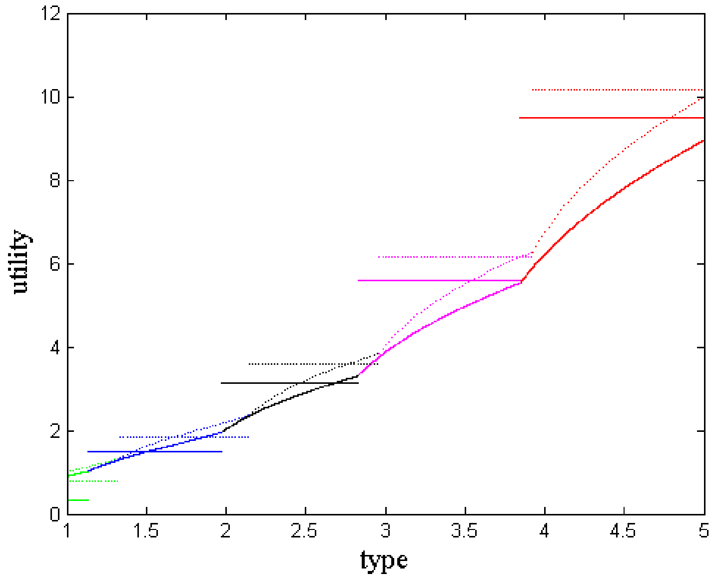

Figure 4: The Impacts of an Increase in α

off. If an increase in α is small, in each submarket all men who initially are among the highest types are better off. This is because they benefit directly from an increase in α, and an increase in women's value V_n does not affect their acceptance probabilities. At the same time, in each submarket (other than the lowest submarket N) all men who initially are among the lowest types are better off as well. The reason is that, by switching to the adjacent lower market, they become the highest types in the new equilibrium segmentation, and thus they get the direct benefit from an increase in α while avoiding being negatively affected by increases in women's values. For the middle types of men in each submarket (remaining in the same submarket), however, in general it is not analytically clear whether they are better off or worse off. This is because, although they benefit directly from an increase in α, they are negatively affected by women's becoming more choosy, and either effect can dominate.

We use the following numerical example to illustrate the above results as well as to discover more quantitative patterns.

Example 3 *Suppose in the benchmark example α increases from 0.25 to 0.4, while all the other parameter values remain the same.*[31] *The change in the equilibrium is illustrated in the following figure and table.*

[31]Note that this example does not satisfy all the assumptions of the analytical results. For instance, q_5^* is significantly below 1, $\widehat{\varepsilon}_1(\overline{y}) > \widehat{\varepsilon}^m$ and $\widehat{\varepsilon}_2(y_1^*) > \widehat{\varepsilon}^m$.

Table 3: How Equilibrium Variables Change as α Increases

	Submarket 1	Submarket 2	Submarket 3	Submarket 4	Submarket 5
$y_n^{*\prime} - y_n^*$	0.0782	0.1435	0.1740	0.2016	–
$V_n^{*\prime} - V_n^*$	0.6086	0.5291	0.4471	0.3655	0.4769
$q_n^{*\prime} - q_n^*$	−0.3018	−0.2307	−0.1276	−0.0221	0.2145

In Figure 4, the dotted curve and the dotted V-lines are associated with the new $\alpha = 0.4$. Figure 4 and Table 3 indicate the following impacts of an increase in α. First, the marginal type of all submarkets increase, with fewer men participating in higher submarkets. In other words, the matching pattern becomes more assortative. Second, all women's equilibrium values increase. Moreover, among submarkets 1, 2, 3, and 4, the increase in women's value is bigger in a higher submarket. However, the increase in V_5^* is bigger than that of V_4^* and V_3^*. This is because, submarket 5, the lowest submarket, has a net gain in the number of participating men, while both submarkets 3 and 4 have net losses (they are reflected in the changes of q_n^*). Suppose we separate women into upper tail and lower tail according to the median type (which is type 4 women). We conclude that an Internet-induced increase in search efficiency increases inequality among women in the upper tail, but reduces inequality among women in the lower tail.

All men's utilities also increase. Within each submarket (among the non-switching types), the increase in men's utility is increasing in the men's type. This is because higher type men are affected less by an increase in women's value. However, across submarkets, this monotonicity does not always hold: the switching types (that switch to a lower submarket) actually gain more than the non-switching types (which are higher than the switching types but are among the lowest types in the submarket in question in the new equilibrium). Nonetheless, the general pattern is that the utility gains of men are higher in a higher submarket. For instance, men in submarkets 4 and 5 gain very little, while men in submarkets 1 and 2 gain significantly. Thus, an Internet-induced increase in search efficiency increases the overall inequality among men. This is because the induced adjustment in the endogenous segmentation reduces the number of men participating in higher submarkets significantly but reduces only slightly or even increases the number of those participating in lower submarkets (as reflected by the change in q_n in Table 3). This effect further increases men's utilities in the highest submarket, but dampens the increases in the utilities of the men participating in the lower submarkets.

All women's marrying speeds increase. This is not always the case for men. In particular, for the low types in submarkets 3 and 4 (under the new equilibrium segmentation), their marrying speeds actually decrease as the arrival rate of meeting increases. This is because these types of men are affected significantly by increases in women's values, leading to significant increases in women's acceptance cutoff for them. This indirect negative effect dominates the direct effect of an increase in α on these men's marrying speeds, and as a result the marrying speeds of these men decrease.

6 Horizontal Match Value Becomes More Dispersed

In this section, we study the impacts of an increase in the dispersion of the horizontal match value. As the standard of living increases, usually people will have more time to enjoy leisure and develop hobbies. As a consequence, in the modern and the post-modern world people's interests become more dispersed. This trend is more pronounced in countries that experience rapid income growth in a short period of time, such as Korea and China. For simplicity, we assume that the match value ε is uniformly distributed on $[1-\gamma, 1+\gamma]$, and use an increase in γ to capture the idea that the match value ε becomes more dispersed.[32] In some sense, an increase in γ means that the horizontal match fitness becomes relatively more important. Just like an Internet-induced increase in α, an increase in γ is a universal shock applying to all men and women.

Specifically, suppose γ increases to $\gamma' > \gamma$, which applies to every man-women match pair regardless of their vertical types.[33] Again we will conduct the analysis in two steps: in the first step we hypothetically fix the initial equilibrium segmentation, and in the second step we study how the segmentation will endogenously adjust.

6.1 Fixed segmentation

Lemma 11 *Suppose γ increases to $\gamma' > \gamma$, while all the other parameter values of the model remain the same. Assume the initial equilibrium segmentation $\overline{S}$ does not change. Then, (i) $\widehat{\varepsilon}^m$ strictly increases; (ii) for any n, $U_n(y)$ strictly increases for any y with $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$; (iii) for any n, V_n strictly increases if q_n is close to 1 and $2\widehat{\varepsilon}^m < (1+\gamma) + \frac{2r\gamma}{\alpha}$; (iv) for any n, $\frac{\partial U_n(y_n^*)}{\partial \gamma} > \frac{\partial U_{n+1}(y_n^*)}{\partial \gamma}$ if and only if $\frac{\partial V_n}{\partial \gamma} < \widehat{x}\frac{\partial U_n(y_n^*)}{\partial \gamma}$, where*

$$\widehat{x} = \frac{\theta_n}{\theta_{n+1}}\{1 + (\frac{\theta_n}{\theta_{n+1}} - 1)\frac{(1+\gamma)[1 + \frac{2r}{\alpha}\frac{\gamma+\widehat{\varepsilon}^m}{1+\gamma-(1+\frac{2r}{\alpha})\widehat{\varepsilon}^m}] - \widehat{\varepsilon}_n(y_n^*)}{\frac{\theta_n}{\theta_{n+1}}\widehat{\varepsilon}_n(y_n^*) - \widehat{\varepsilon}^m}\}. \tag{8}$$

The intuition for parts (i)-(iii) of Lemma 11 is as follows. As the horizontal match value becomes more dispersed, men and women have higher probabilities of meeting someone with a higher horizontal match value. This clearly benefits men of high types (within a submarket) whose acceptance threshold is effective: their equilibrium payoffs are higher and they become

[32] Essentially, an increase in γ leads to a mean preserving spread of the distribution of ε.

[33] In the real world, high type men and high type women might have more diverse interests as they have more time and income to afford hobbies. We abstract away from this heterogeneity.

more picky. Just like an increase in α, an increase in γ has two opposite effects on women in submarket n. The direct effect is that it benefits women, as they now have higher probabilities of meeting men with a higher horizontal match value. However, the indirect effect is that now higher types of men are more picky ($\widehat{\varepsilon}^m$ increases in γ), which restricts women's choices. This effect tends to hurt women. However, when the number of women and men is similar, the direct effect dominates, and overall women benefit from an increase in γ.[34]

Part (iv) of Lemma 11 also shows that when the increase in women's payoff in submarket n is small compared to the increase in type y_n^* men's payoff in submarket n, then the marginal type men benefit more from an increase in the horizontal match value by staying in the higher submarket n. To understand the intuition behind this result, we reproduce the key inequality in the proof: $\frac{\partial U_n(y_n^*)}{\partial \gamma} \geq \frac{\partial U_{n+1}(y_n^*)}{\partial \gamma}$ if and only if

$$\begin{aligned}
&y_n^*(1+\gamma)(\theta_n - \theta_{n+1}) + \frac{G(\widehat{\varepsilon}_n^w(y_n^*)) - G(\widehat{\varepsilon}^m)}{\frac{r}{\alpha} + 1 - G(\widehat{\varepsilon}^m)}[\theta_{n+1} y_n^*(1+\gamma) - U_n(y_n^*)(1 + \frac{2r}{\alpha})] \\
&\geq \frac{\partial V_n}{\partial \gamma}[\widehat{\varepsilon}_n^w(y_n^*) - \frac{U_n(y_n^*)}{\theta_n y_n^*}].
\end{aligned}$$

When γ increases, there are three effects on the sign of $\frac{\partial U_n(y_n^*)}{\partial \gamma} - \frac{\partial U_{n+1}(y_n^*)}{\partial \gamma}$. The first effect is captured in the right-hand side of the key inequality. Since type y_n^* is the highest type in submarket $n+1$, an increase in V_{n+1} does not affect $U_{n+1}(y_n^*)$. However, an increase in V_n induced by an increase in γ will reduce type y_n^*'s probability of being accepted in submarket n, which dampens $U_n(y_n^*)$. This effect implies that the marginal type gains more by staying in submarket $n+1$. The second effect is captured by the second term in the left-hand side of the key inequality. Intuitively, an increase in γ means that the marginal type has a higher probability of finding a larger horizontal match value. In percentage terms, this effect is stronger in submarket n as this type's initial equilibrium probability of being accepted is smaller in submarket n, which means that the marginal type y_n^* gains more by staying in the higher submarket n. Notice that these above-mentioned two effects are also present when α increases. The third effect is captured by the first term in the left-hand side of the key inequality, which is absent when α increases. This effect stems from the supermodularity of men's payoff in women's type and the horizontal match value. Then, by staying in the higher submarket, the marginal type men can gain more. Overall, if $\frac{\partial V_n}{\partial \gamma}$ is relatively not too big so that the first effect is not too strong, then the second and third effects dominate. Recall that with an increase in α, we reach the opposite conclusion: the marginal type men benefit more from staying in the lower submarket. As mentioned earlier, the underlying reason is that the third effect is absent when α changes.

[34] Again, the sufficient condition $2\widehat{\varepsilon}^m < (1+\gamma) + \frac{2r\gamma}{\alpha}$ is satisfied if r/α is not too small, the same condition as in the case of an increase in α. This sufficient condition is far from being necessary.

How big is the critical value of $\widehat{x}$? From equation (8), it can be easily verified that $\widehat{x} > \frac{\theta_n}{\theta_{n+1}} > 1$, or $\widehat{x}$ is relatively big. As a result, it is relatively easy for $\frac{\partial V_n}{\partial \gamma} < \widehat{x}\frac{\partial U_n(y_n^*)}{\partial \gamma}$ to hold. This will be verified by a numerical example later.

6.2 With endogenous adjustment in segmentation

We use superscripts "a" and "$'$" to indicate variables after γ increases to γ' when the segmentation is held constant and when the segmentation adjusts endogenously, respectively. The analysis in the previous subsection shows that the comparison between $U_n^a(y_n^*)$ and $U_{n+1}^a(y_n^*)$ can go either way depending on whether $\frac{\partial V_n}{\partial \gamma}$ is larger than $\widehat{x}\frac{\partial U_n(y_n^*)}{\partial \gamma}$. It is possible that for some submarket $U_n^a(y_n^*) > U_{n+1}^a(y_n^*)$, but for another submarket the opposite holds. Moreover, the analysis in the previous subsection also suggests that $U_n^a(y_n^*) > U_{n+1}^a(y_n^*)$ is more likely. To state cleaner analytical results, we focus on the cases where $U_n^a(y_n^*)$ is always larger than $U_{n+1}^a(y_n^*)$ in any submarkets.

Proposition 6 *Suppose γ increases to γ', while all the other parameter values of the model remain the same. If $U_n^a(y_n^*) > U_{n+1}^a(y_n^*)$ for all n, then for each n: (i) y_n^* strictly decreases; (ii) $V_1' > V_1$ if q_1 is close enough to 1 and $2\widehat{\varepsilon}^m < (1+\gamma) + \frac{2r\gamma}{\alpha}$; (iii) The highest types of men (initially in submarket 1) with acceptance threshold $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$ are strictly better off; if the change in γ is not too large, then the highest types of men initially in any submarket whose acceptance threshold is still $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$ are strictly better off, and the lowest types of men who are initially in submarket n, $1 \leq n < N$, are strictly better off as well.*

Proposition 6 shows that as the horizontal match value becomes more dispersed, the matching pattern becomes less assortative in the vertical dimension. That is, the cutoff types decrease and hence more men are participating in higher submarkets. We want to point out that this result is the opposite to the change in matching pattern when α increases, under which the marriage pattern becomes more assortative. The main reason, as mentioned in the last subsection, is that an increase in γ increases the larger horizontal match value, which, due to the complementarity between the vertical types and the horizontal match value, makes the higher submarket relatively more attractive. This effect, which is absent when α increases, dominates the negative effect of moving to the higher submarket: women there become more picky since their payoff increases. As a result, men with types just below the original cutoff find it better to move to the adjacent higher submarket.

Proposition 6 also shows that the highest types of men in each submarket initially are better off, because they are not affected by the increase in women's payoff of the corresponding type. The lowest types of men in any submarket initially (except for submarket N) are also better off. To see this, note that they could move to the adjacent low submarket where men's acceptance

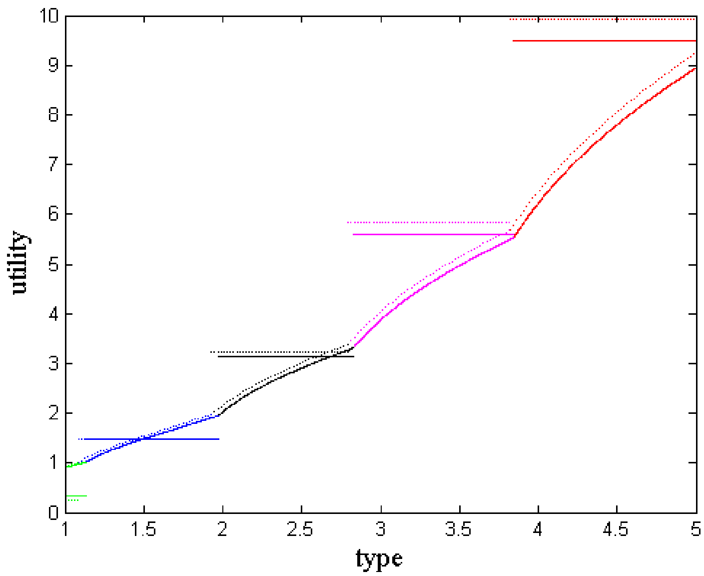

Figure 5: The Impacts of an Increase in γ

threshold is binding and their payoffs are strictly higher. The fact that they stay in the original submarket means that they must be better off as well. For the middle types of men in each submarket, analytically it is not clear whether they are better off or worse off. However, as indicated by the following numerical example, it is likely that their payoffs increase in γ too, as having types higher than y_n^*, they should be affected less by an increase in V_n.

Example 4 *Suppose in the benchmark example* γ *increases from* 0.5 *to* 0.6*, while all the other parameter values remain the same.*[35] *The change in the equilibrium is illustrated in the following figure and table.*

Table 4: How Equilibrium Variables Change as γ Increases

	Submarket 1	Submarket 2	Submarket 3	Submarket 4	Submarket 5
$y_n^{*\prime} - y_n^*$	-0.0202	-0.0339	-0.0442	-0.0505	$-$
$V_n^{*\prime} - V_n^*$	0.4866	0.2389	0.0933	0.0043	-0.1201
$q_n^{*\prime} - q_n^*$	0.0797	0.0592	0.0315	0.0037	-0.0528

In Figure 5, the dotted curve and the dotted V-lines are associated with the new $\gamma = 0.6$. Figure 5 and Table 4 indicate the following impacts of an increase in γ. First, all men's cutoff types decrease (more men participating in higher submarkets). Second, women's equilibrium

[35] Again, this example does not satisfy all the assumptions of the analytical results.

values in submarkets 1, 2, 3, and 4 all increase, while the equilibrium value of women in submarket 5 decreases. Moreover, the increase in women's value increases in women's type. Thus, an increase in the dispersion of the horizontal match value increases the inequality among women. The underlying reason for this pattern is that more men switch to higher submarkets, which benefits women in higher submarkets a lot by increasing their contact rate significantly. However, this slightly benefits (or even hurts) women in lower submarkets by barely increasing (or decreasing) their contact rate (as shown in the changes in q_n in Table 4).

Third, all men's utilities increase. Within each submarket, the increase in men's utility is increasing in men's type. This is because higher type men are affected less by an increase in women's value. The same pattern also holds across submarkets. For instance, men in submarkets 4 and 5 gain very little, while men in submarkets 1 and 2 gain significantly. Regarding the types close to the cutoff type between two submarkets, men's utilities are increasing in types for the following reason: if a switching type (switch to a higher submarket) gains less than a lower non-switching type, he will stay in the lower submarket. Therefore, we conclude that an increase in dispersion of the horizontal match value also increases the inequality among men. Finally, comparing men's and women's gains across different types, we see that the induced increase in inequality is more significant among women than among men. This is due to the endogenous adjustment in segmentation. With men switching to higher submarkets, women's gains in higher submarkets are increased further, while men's gains in higher submarkets are dampened.

7 Horizontal Clubs

The Internet not only increases the search efficiency (the contact rate α), but also enables men and women to create horizontal clubs. For instance, there is an online dating site called Farmers, which attracts men and women who are interested in becoming farmers. That is, the Internet also allows people to target specific horizontal attributes. This targeting will naturally increase the horizontal match value between men and women participating in the same horizontal club, as they share some common interests.

To study how this possibility of horizontal targeting affects the equilibrium marriage pattern, we modify our baseline model in the simplest way. Specifically, assume that each man or woman is of two possible horizontal types: L and R, which are equally likely. Moreover, each man's or woman's horizontal type is independent of his/her vertical type. If a man or woman matches with a partner of the same horizontal type, then the horizontal match value is randomly drawn from distribution G', which is a truncation of G after restricting the support to $[1, 1+\gamma]$. On the other hand, if the two partners are of different horizontal types, then the horizontal match value is randomly drawn from the truncated distribution of G after restricting the support to $[1-\gamma, 1]$. To make the comparison reasonable, we assume that the original distribution G is symmetric around 1: $g(1-x) = g(1+x)$ for any $x \in [0, \gamma]$. Assume further that there are horizontal clubs L and R in each submarket and men can also target horizontal clubs. That is, in total there are $2N$ submarkets, or each vertical club n consists of two distinctive horizontal clubs: club n_L and club n_R.

Since the measure of women in club n_L and that of club n_R are always the same, and type L men and type R men are of the same measure as well, L type men will always target L women clubs and R type men will always target R women clubs. Given this feature, we can analyze L type and R type agents separately, and the equilibrium among L type and R type will be exactly the same. For this reason, we only focus on club R in the following analysis. We compare the baseline model and the model with horizontal targeting. Specifically, in the process we fix α and γ, but only change the distribution of ε from G to G'.

In addition, we only focus on the parameter range under which men's acceptance threshold is interior with the presence of horizontal clubs. Following a similar analysis as Lemma 1, men's acceptance threshold $\widehat{\varepsilon}^{m\prime}$ under G' is implicitly determined by the following condition when it is interior:

$$-r\widehat{\varepsilon}^{m\prime} + \alpha \int_{\widehat{\varepsilon}^{m\prime}}^{1+\gamma} [\varepsilon - \widehat{\varepsilon}^{m\prime}] dG'(\varepsilon) = 0. \tag{9}$$

Thus, the following assumption guarantees that $\widehat{\varepsilon}^{m\prime}$ is strictly greater than 1.

Assumption 1A: $\int_1^{1+\gamma} \varepsilon dG'(\varepsilon) > 1 + \frac{r}{\alpha}$.

In particular, this requires that α should not be too small. It is also easy to see that $\widehat{\varepsilon}^{m\prime}$ equals $\widehat{\varepsilon}^{m}$ in the baseline model but with α changed to 2α, as equation (9) can be rewritten as

$$-r\widehat{\varepsilon}^{m} + 2\alpha \int_{\widehat{\varepsilon}^{m}}^{1+\gamma} [\varepsilon - \widehat{\varepsilon}^{m}] dG(\varepsilon) = 0,$$

since G is symmetric around 1. Then $\widehat{\varepsilon}^{m\prime} > 1$ implies that $\widehat{\varepsilon}^{m}$ in the baseline model with 2α is also greater than 1.

The following proposition shows that adding horizontal clubs is equivalent to doubling α.

Proposition 7 *Adding horizontal clubs while all the other parameter values of the model remain unchanged, is equivalent to increasing α to 2α in the baseline model.*

Proposition 7 suggests that we can use the comparative statics results regarding α to compare the equilibrium outcomes with and without horizontal clubs. In particular, adding horizontal clubs makes the marriage pattern more assortative in the vertical dimension, benefits women in all the submarkets, etc. Moreover, the two effects of the Internet on the marriage market, through an increase in α and enabling horizontal targeting, work in exactly the same direction.

8 Conclusion and Discussion

This paper, within a search/matching framework, studies the equilibrium marriage pattern with vertically differentiated men and women. Our model has two distinctive features. First, search is targeted: men can choose beforehand in which submarket to participate, but search is random within each submarket. Second, men and women are also horizontally differentiated. We show that there is a unique market equilibrium. Men (who actively search) are endogenously segmented into different submarkets, and the matching pattern is weakly positive assortative. In equilibrium, all the submarkets are indirectly linked. Within each submarket, higher types of men are faster to marry than lower types. This is because the presence of horizontal differentiation means that different types of men will be accepted by women with different probabilities, which in some sense serves a shadow price.

When the measure of a specific type of woman increases, the corresponding submarkets attracts more men. Moreover, the transmissions of the shock are asymmetric. In the downward direction (the submarkets lower than the one where the original shock occurs), all the threshold types of men decrease. However, in the upward direction, the shock transmission can stop at any submarket.

An Internet-induced increase in search efficiency causes all the threshold types of men to increase, leading to overall more assortative matching. All women are always better off, but not all men are better off. Our simulation shows that in the upper tail women's inequality (in the marriage market) increases, but in the lower tail women's inequality decreases, while men's inequality in the marriage market in general increases. We also show that an Internet-induced horizontal targeting has similar qualitative impacts on the marriage market as an Internet-induced increase in search efficiency.

Finally, as the dispersion of the horizontal match fitness increases, the equilibrium cutoff types of men are very likely to decrease, leading to overall less assortative matching. High types of women benefit from a more dispersed match value, but this is not the case for low type women. For men, not all of them benefit as the dispersion of the horizontal match fitness increases. Our simulation results indicate that a more dispersed match value increases women's inequality as well as men's inequality, with the increase in women's inequality being more significant than that of men's.

In the rest of this section, we discuss some of the simplifying assumptions we have made.

Urn ball matching technology More generally, it is reasonable to think that the contact rate of men in a submarket is also decreasing in the men/women ratio in that submarket. This direct search congestion is the key driving force in the directed search models in the labor market (such as Shimer, 2005). As mentioned earlier, in our model there are already indirect externalities imposed by men on each other within the same submarket. Introducing the direct search congestion would not qualitatively affect the main results of our model. Moreover, in the existing literature of search/matching models in the marriage market, it is commonly assumed that men and women have contact rates that are independent of the men/women ratio.

Why targeted search? As mentioned earlier, targeted search is more realistic than random search, as men can direct their search effort toward the group of women who are more suitable for them. Perhaps a little surprisingly, with both vertical and horizontal differentiation, a model with targeted search is more tractable than a model with random search. This is because with random search, one needs to trace the acceptance set (in two dimensions) of each type of man and that of each type of woman. Moreover, men's acceptance sets and women's acceptance sets interact with each other, which means that it is very hard to pin down the equilibrium acceptance strategy for each type of man and for each type of woman. Essentially, targeted search separates a matching process with two-sided (vertical) heterogeneity into a two-stage process. Specifically, in the second stage, within each submarket it becomes a matching problem with one-sided heterogeneity, as the women's side is homogeneous. In the first stage, it is again a matching problem with one-sided heterogeneity, as only men choose in which submarket to participate. This separation makes the model tractable.

The asymmetry between men and women In the current model, men and women are asymmetric in two aspects. The first asymmetry is that men actively search while women do not. We adopt this assumption for two reasons. First, it makes the model tractable. If both sides search actively, then it is hard to model targeted search. Second, it is a realistic assumption in traditional societies. In modern societies, women actively search to some extent as well. But, still it is reasonable to think that men search, on average, more actively than women do.

The second asymmetry between men and women is that while men's types are continuous, women's types are finite. Again, this is a technical assumption which makes the model tractable. If women's types are also continuous, then it is hard to define a submarket. But the current model can approximate this situation by increasing the number of women's types and letting it go to infinity. Men's types being continuous is not essential. We can work out a model in which men's types are also finite. However, men's types being continuous simplifies our analysis, as otherwise we need to worry about the mixed strategies of the marginal types of men, who are indifferent between two adjacent submarkets.[36]

The clone assumption Among the marriage search/matching models, the clone assumption is standard, with Burdett and Coles (1997) being an exception. Like Burdett and Coles (1997), we can introduce exogenous separation for existing marriages or exogenous entry of new mates and study the steady-state equilibrium. It is straightforward to see that the main equilibrium features mentioned in Section 3.3 remain valid. However, without the clone assumption, the analysis become significantly more complicated. The difficulty is that within each submarket, the higher types of men are faster to marry. Moreover, different types of women have different marrying speeds as well. Quantitatively, we conjecture that in the steady-state equilibrium the utility differences between the types who are faster to marry and those who are slower to marry will be less pronounced than in the model with clones, as the former will be fewer and the latter will be more in the steady-state equilibrium. We will leave a complete analysis of the steady-state equilibrium for future research.

Appendix

Proof of Lemma 1.

Proof. First suppose that type n women always accept a type y man (we can drop the constraint $\widehat{\varepsilon}_n^m(y) \geq \widehat{\varepsilon}_n^w(y)$) and this man adopts a reservation strategy of accepting a type n woman if and only if $\varepsilon \geq \widehat{\varepsilon}$. By (1), we can solve $U_n(y)$ as a function of $\widehat{\varepsilon}$. Now take derivative of $U_n(y)$ with respect to $\widehat{\varepsilon}$, we get

$$\frac{\partial U_n(y)}{\partial \widehat{\varepsilon}} \propto -r\widehat{\varepsilon} + \alpha \int_{\widehat{\varepsilon}}^{1+\gamma} (\varepsilon - \widehat{\varepsilon}) dG(\varepsilon) \equiv \Gamma(\widehat{\varepsilon}). \tag{10}$$

It can be shown that $\Gamma(\widehat{\varepsilon})$ is monotonically decreasing in $\widehat{\varepsilon}$:

$$\frac{\partial \Gamma(\widehat{\varepsilon})}{\partial \widehat{\varepsilon}} \propto -r - \alpha(1 - G(\widehat{\varepsilon})) < 0.$$

Moreover, $\Gamma(1+\gamma) < 0$. When Assumption 1 is satisfied, $\Gamma(1-\gamma) > 0$. Thus there is a unique $\widehat{\varepsilon}^m \in (1-\gamma, 1+\gamma)$ such that $\Gamma(\widehat{\varepsilon}^m) = 0$. Otherwise, $\Gamma(\widehat{\varepsilon}) < 0$ for any $\widehat{\varepsilon} \in (1-\gamma, 1+\gamma]$ and therefore $\widehat{\varepsilon}^m = 1-\gamma$. In both scenarios, all men participating in any submarket have the same optimal reservation match value: accept any woman with $\varepsilon \geq \widehat{\varepsilon}^m$, and reject all women with $\varepsilon \leq \widehat{\varepsilon}^m$.

Note that the cutoff that ensures matching between a type y man and a type n woman is $\widehat{\varepsilon}_n(y) = \max\{\widehat{\varepsilon}^m, \widehat{\varepsilon}_n^w(y)\}$. If $\widehat{\varepsilon}_n^w(y) \leq \widehat{\varepsilon}^m$, a type y man's optimal cutoff is still $\widehat{\varepsilon}^m$, same as

[36] In equilibrium, these types of men have to mix in the right way.

the previous analysis. If $\widehat{\varepsilon}_n^w(y) > \widehat{\varepsilon}^m$, type y men are willing to match type n women with $\varepsilon \geq \widehat{\varepsilon}_n^w(y)$, as $\Gamma(\widehat{\varepsilon}_n^w(y)) < 0$ by the previous analysis. In either case, we can write the optimal cutoff of any type of men as $\widehat{\varepsilon}^m$.

To see that $\widehat{\varepsilon}^m$ strictly increases in α when Assumption 1 holds, take derivative of Γ with respect to α and plug in the first order condition. It is easy to see that $\frac{\partial \Gamma(\widehat{\varepsilon})}{\partial \alpha} |_{\widehat{\varepsilon}=\widehat{\varepsilon}^m} > 0$. We already know that $\frac{\partial \Gamma(\widehat{\varepsilon})}{\partial \widehat{\varepsilon}} |_{\widehat{\varepsilon}=\widehat{\varepsilon}^m} < 0$. By the implicit function theorem, the claim is proved. ■

Proof of Lemma 2.

Proof. Part (i). Suppose there exists some $n > 1$ such that $\frac{V_{n-1}}{\theta_{n-1}} < \frac{V_n}{\theta_n}$. Since $\widehat{\varepsilon}_n(y) = \max\{\widehat{\varepsilon}^m, \frac{V_n}{\theta_n y}\}$, this implies $\widehat{\varepsilon}_{n-1}(y) \leq \widehat{\varepsilon}_n(y)$ for any y. In other words, women in submarket $n-1$ are less picky. As $\theta_{n-1} > \theta_n$, by (1), $\widehat{\varepsilon}_{n-1}(y) \leq \widehat{\varepsilon}_n(y)$ implies that $U_{n-1}(y) > U_n(y)$ for any y with $\widehat{\varepsilon}_{n-1}(y) < 1+\gamma$ (or $y > \frac{V_{n-1}}{\theta_{n-1}(1+\gamma)}$). Thus these men will all participate in submarket $n-1$ and will not participate in submarket n. For any y such that $\widehat{\varepsilon}_{n-1}(y) \geq 1+\gamma$ (or $y \leq \frac{V_{n-1}}{\theta_{n-1}(1+\gamma)}$), they will never marry a type n woman as they will always be rejected. As a result, $V_n = 0$. However, $V_{n-1} \geq 0$. This contradicts the initial presumption that $\frac{V_{n-1}}{\theta_{n-1}} < \frac{V_n}{\theta_n}$.

Part (ii). It directly follows part (i). ■

Proof of Lemma 3.

Proof. Suppose in a market equilibrium the scenario described in this statement occurs. It would mean that $U_{n'}(y) \geq U_n(y)$, $U_n(y') \geq U_{n'}(y')$, and $U_n(y) > 0$. Since $U_n(\cdot)$ is increasing, we have $U_{n'}(y) > 0$, $U_n(y') > 0$, and $U_{n'}(y') > 0$. These imply that all the relevant acceptance thresholds are strictly less than $1+\gamma$. Define

$$\frac{\int_{\widehat{\varepsilon}}^{1+\gamma} \varepsilon dG(\varepsilon)}{r+\alpha(1-G(\widehat{\varepsilon}))} \equiv H(\widehat{\varepsilon}).$$

Then, $U_n(y) = \alpha\theta_n y H(\widehat{\varepsilon}_n(y))$. Note that $H(\widehat{\varepsilon})$ is strictly decreasing in $\widehat{\varepsilon}$ for $\widehat{\varepsilon} \geq \widehat{\varepsilon}^m$ (proof of Lemma 1). Since $\widehat{\varepsilon}_n(y)$ is weakly decreasing in n for any y, we have the following three cases to consider.

Case (1): $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}_{n'}(y) = \widehat{\varepsilon}^m$.

Since $\widehat{\varepsilon}_n(\cdot)$ is decreasing in y, we have $\widehat{\varepsilon}_n(y') = \widehat{\varepsilon}_{n'}(y') = \widehat{\varepsilon}^m$. Now,

$$U_{n'}(y') - U_n(y') = \alpha y(\theta_{n'} - \theta_n)H(\widehat{\varepsilon}^m) > 0,$$

is a contradiction.

Case (2): $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$ and $\widehat{\varepsilon}_{n'}(y) > \widehat{\varepsilon}^m$.

Since $\widehat{\varepsilon}_n(\cdot)$ is decreasing in y, we have $\widehat{\varepsilon}_n(y') = \widehat{\varepsilon}^m$ and $\widehat{\varepsilon}_{n'}(y') < \widehat{\varepsilon}_{n'}(y)$. The condition $U_{n'}(y) \geq U_n(y)$ can be expressed as

$$U_{n'}(y) - U_n(y) = \alpha y[\theta_{n'}H(\widehat{\varepsilon}_{n'}(y)) - \theta_n H(\widehat{\varepsilon}^m)] \geq 0,$$

which implies that $\theta_{n'}H(\widehat{\varepsilon}_{n'}(y)) - \theta_n H(\widehat{\varepsilon}^m) \geq 0$. Now consider the difference between $U_n(y')$ and $U_{n'}(y')$:

$$\begin{aligned} U_{n'}(y') - U_n(y') &= \alpha y'[\theta_{n'}H(\widehat{\varepsilon}_{n'}(y')) - \theta_n H(\widehat{\varepsilon}^m)] \\ &> \alpha y'[\theta_{n'}H(\widehat{\varepsilon}_{n'}(y)) - \theta_n H(\widehat{\varepsilon}^m)] \geq 0, \end{aligned}$$

where the first inequality uses the facts that $\widehat{\varepsilon}_{n'}(y') < \widehat{\varepsilon}_{n'}(y)$ and $H(\cdot)$ is decreasing, and the second inequality follows because $\theta_{n'}H(\widehat{\varepsilon}_{n'}(y)) - \theta_n H(\widehat{\varepsilon}^m) \geq 0$. This is a contradiction.

Case (3): $\widehat{\varepsilon}_{n'}(y) > \widehat{\varepsilon}_n(y) > \widehat{\varepsilon}^m$.

Subcase (3a): $\widehat{\varepsilon}_n(y') = \widehat{\varepsilon}_{n'}(y') = \widehat{\varepsilon}^m$. By an argument similar to that in case (1), we have $U_{n'}(y') - U_n(y') > 0$. This is a contradiction.

Subcase (3b): $\widehat{\varepsilon}^m < \widehat{\varepsilon}_n(y') < \widehat{\varepsilon}_n(y)$ and $\widehat{\varepsilon}^m < \widehat{\varepsilon}_{n'}(y') < \widehat{\varepsilon}_{n'}(y)$. It is enough to show that $\frac{\partial(U_{n'}(y) - U_n(y))}{\partial y} > 0$. More explicitly,

$$\frac{\partial(U_{n'}(y) - U_n(y))}{\partial y} = \alpha[\theta_{n'}H(\widehat{\varepsilon}_{n'}(y)) - \theta_n H(\widehat{\varepsilon}_n(y))] + \alpha[\theta_{n'}y\frac{\partial H(\widehat{\varepsilon}_{n'})}{\partial \widehat{\varepsilon}_{n'}}\frac{\partial \widehat{\varepsilon}_{n'}}{\partial y} - \theta_n y\frac{\partial H(\widehat{\varepsilon}_n)}{\partial \widehat{\varepsilon}_n}\frac{\partial \widehat{\varepsilon}_n}{\partial y}].$$

In the above equation, the first term is positive, as $U_{n'}(y) - U_n(y) \geq 0$. Thus, it is sufficient that the second term

$$\theta_{n'}y\frac{\partial H(\widehat{\varepsilon}_{n'})}{\partial \widehat{\varepsilon}_{n'}}\frac{\partial \widehat{\varepsilon}_{n'}}{\partial y} - \theta_n y\frac{\partial H(\widehat{\varepsilon}_n)}{\partial \widehat{\varepsilon}_n}\frac{\partial \widehat{\varepsilon}_n}{\partial y} > 0.$$

By $\frac{\partial H(\widehat{\varepsilon})}{\partial \widehat{\varepsilon}} = \frac{g(\widehat{\varepsilon})}{[r+\alpha(1-G(\widehat{\varepsilon}))]^2}\Gamma(\widehat{\varepsilon})$ and $\frac{\partial \widehat{\varepsilon}_n}{\partial y} = -\frac{V_n}{y^2\theta_n} = -\frac{1}{y\gamma}\widehat{\varepsilon}_n(y)$, we have

$$\begin{aligned} &\theta_{n'}y\frac{\partial H(\widehat{\varepsilon}_{n'})}{\partial \widehat{\varepsilon}_{n'}}\frac{\partial \widehat{\varepsilon}_{n'}}{\partial y} - \theta_n y\frac{\partial H(\widehat{\varepsilon}_n)}{\partial \widehat{\varepsilon}_n}\frac{\partial \widehat{\varepsilon}_n}{\partial y} \\ \propto\ &\theta_n\frac{g(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n}{[r+\alpha(1-G(\widehat{\varepsilon}_n))]^2}\Gamma(\widehat{\varepsilon}_n) - \theta_{n'}\frac{g(\widehat{\varepsilon}_{n'})\widehat{\varepsilon}_{n'}}{[r+\alpha(1-G(\widehat{\varepsilon}_{n'}))]^2}\Gamma(\widehat{\varepsilon}_{n'}) \end{aligned}$$

By previous results, $\Gamma(\widehat{\varepsilon}_{n'}) < \Gamma(\widehat{\varepsilon}_n) < 0$. Therefore, it is enough to show that

$$\theta_{n'}\frac{g(\widehat{\varepsilon}_{n'})\widehat{\varepsilon}_{n'}}{[r+\alpha(1-G(\widehat{\varepsilon}_{n'}))]^2} - \theta_n\frac{g(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n}{[r+\alpha(1-G(\widehat{\varepsilon}_n))]^2} \geq 0. \tag{11}$$

The following condition is sufficient for inequality (11): $g(\varepsilon)\varepsilon$ is weakly increasing in ε for all ε. More explicitly, $\frac{g'(\varepsilon)}{g(\varepsilon)} + \frac{1}{\varepsilon} \geq 0$. The logconcavity of $g(\varepsilon)$ means that $\frac{g'(\varepsilon)}{g(\varepsilon)}$ is decreasing in ε. Thus the LHS of the above inequality is decreasing in ε, and condition (4) is sufficient.

Subcase (3c): $\widehat{\varepsilon}_n(y') = \widehat{\varepsilon}^m$, and $\widehat{\varepsilon}^m < \widehat{\varepsilon}_{n'}(y') < \widehat{\varepsilon}_{n'}(y)$.

$$U_{n'}(y) - U_n(y) = \alpha y[\theta_{n'}H(\widehat{\varepsilon}_{n'}(y)) - \theta_n H(\widehat{\varepsilon}_n(y))] \geq 0.$$

$$U_{n'}(y') - U_n(y') = \alpha y'[\theta_{n'}H(\widehat{\varepsilon}_{n'}(y')) - \theta_n H(\widehat{\varepsilon}^m)].$$

By the continuity of $\widehat{\varepsilon}_n(y)$ in y, we can find a type $y'' \in (y, y']$ such that $\frac{V_n}{\theta_n y''} = \widehat{\varepsilon}^m$. Then between types y and y'', we can apply the result of subcase (2) and get $U_{n'}(y'') - U_n(y'') > 0$.

Between types y'' and y', we can apply subcase (3a) and get $U_{n'}(y') - U_n(y') > U_{n'}(y'') - U_n(y'') > 0$. ■

Proof of Lemma 4.

Proof. Part (i). Consider the scenario in which $\widehat{y}_n$ decreases to $\widehat{y}_n^L < \widehat{y}_n$ while $\widehat{y}_{n-1}$ remains the same. Suppose after the change type n women adopt $\widehat{\varepsilon}_n^L(y)$ as the acceptance cutoff, which yields the payoff V_n^L. Notice that if type n women choose the threshold optimally, then V_n^L is the equilibrium payoff. Otherwise, V_n^L is weakly lower than the equilibrium payoff. By (3), we have

$$rX_n V_n^L = \alpha \int_{\widehat{y}_n^L}^{\widehat{y}_{n-1}} \int_{\widehat{\varepsilon}_n^L(y)}^{1+\gamma} (\theta_n y\varepsilon - V_n^L) dG(\varepsilon) dF(y).$$

Taking the difference between V_n^L and V_n and rearranging, we obtain

$$\begin{aligned} & rX_n(V_n^L - V_n) + \alpha \int_{\widehat{y}_n^L}^{\widehat{y}_{n-1}} \int_{\widehat{\varepsilon}_n^L(y)}^{1+\gamma} (V_n^L - V_n) dG(\varepsilon) dF(y) \\ =& \alpha \int_{\widehat{y}_n^L}^{\widehat{y}_n} \int_{\widehat{\varepsilon}_n(y)}^{1+\gamma} (\theta_n y\varepsilon - V_n) dG(\varepsilon) dF(y) - \alpha \int_{\widehat{y}_n^L}^{\widehat{y}_{n-1}} \int_{\widehat{\varepsilon}_n(y)}^{\widehat{\varepsilon}_n^L(y)} (\theta_n y\varepsilon - V_n) dG(\varepsilon) dF(y). \end{aligned}$$

If initially $\widehat{\varepsilon}_n(\widehat{y}_n) < 1+\gamma$, or $\widehat{y}_n > \frac{V_n}{\theta_n(1+\gamma)}$, then consider the case in which a type n woman adopts the threshold such that $\widehat{\varepsilon}_n^L(y) = \widehat{\varepsilon}_n(y)$. The above equation implies $V_n^L > V_n$. Since type n women's equilibrium payoff is weakly higher than V_n^L, we conclude that a decrease in $\widehat{y}_n$ leads to a strict increase in V_n.

If initially $\widehat{\varepsilon}_n(\widehat{y}_n) = 1+\gamma$, then V_n must remain constant. Otherwise, the two sides of the above equation would have opposite signs.

Part (ii). Consider the scenario in which $\widehat{y}_{n-1}$ increases to $\widehat{y}_{n-1}^H > \widehat{y}_{n-1}$ while $\widehat{y}_n$ remains the same. Suppose type n women adopt the same acceptance threshold as before and obtain the payoff V_n^H. Since the threshold is not necessarily optimal given $\widehat{y}_{n-1}^H$, V_n^H is weakly lower than the equilibrium payoff. Similar to part (i), we can rearrange the difference between V_n^H and V_n as follows:

$$rX_n(V_n^H - V_n) + \alpha \int_{\widehat{y}_n}^{\widehat{y}_{n-1}^H} \int_{\widehat{\varepsilon}_n(y)}^{1+\gamma} (V_n^H - V_n) dG(\varepsilon) dF(y) = \alpha \int_{\widehat{y}_{n-1}}^{\widehat{y}_{n-1}^H} \int_{\widehat{\varepsilon}_n(y)}^{1+\gamma} (\theta_n y\varepsilon - V_n) dG(\varepsilon) dF(y).$$

The right-hand side of the above equation is strictly positive. Therefore, V_n strictly increases as the upper bound $\widehat{y}_{n-1}$ increases. ■

Proof of Lemma 5.

Proof. Part (i). Denote $\overline{S}(\widehat{y}_1)$ as a segmentation with cutoff $\widehat{y}_1$. We want to trace the $U_1(\widehat{y}_1; \overline{S}(\widehat{y}_1))$ curve and $U_2(\widehat{y}_1; \overline{S}(\widehat{y}_1))$ curve as $\widehat{y}_1$ varies between $\widehat{y}_2$ and $\overline{y}$. By Lemma 4, V_1 is weakly decreasing in $\widehat{y}_1$, and thus $\widehat{\varepsilon}_1(\widehat{y}_1;\overline{S}(\widehat{y}_1))$ is decreasing in $\widehat{y}_1$. Therefore, $U_1(\widehat{y}_1;\overline{S}(\widehat{y}_1))$ is increasing in $\widehat{y}_1$. Again by Lemma 4, V_2 is increasing in $\widehat{y}_1$. Define $\Delta U(\widehat{y}_1) \equiv U_1(\widehat{y}_1;\overline{S}(\widehat{y}_1)) - U_2(\widehat{y}_1;\overline{S}(\widehat{y}_1))$. It is easy to see that $\Delta U(\widehat{y}_1)$ is continuous in $\widehat{y}_1$. In addition, $\Delta U(\overline{y}) > 0$, since $\overline{U}_1(\overline{y}) > U_2(\overline{y};\overline{S}(\overline{y}))$ (when all men are participating in club 2), which follows $\overline{U}_1(\cdot) > \overline{U}_2(\cdot)$. So we only need to consider the following two cases.

Case 1. $\Delta U(\widehat{y}_1)$ is always strictly positive for any $\widehat{y}_1 \in (\widehat{y}_2, \overline{y})$. Then a unique partial equilibrium exists with $y_1^* = \widehat{y}_2$ (a corner equilibrium with all men in the range of $[\widehat{y}_2, \overline{y}]$ participating in club 1).

Case 2. $\Delta U(\widehat{y}_1)$ is strictly negative for some $\widehat{y}_1 \in (\widehat{y}_2, \overline{y})$. Then by continuity, there must exist some $\widehat{y}_1 \in (\widehat{y}_2, \overline{y})$ such that $\Delta U(\widehat{y}_1) = 0$, and $\widehat{y}_1$ is the (interior) equilibrium cutoff. This proves the existence of an interior partial equilibrium.

Now we show the uniqueness of equilibrium in case 2. Suppose there are two interior equilibria: y_1^* and $y_1^{*\prime}$, with $y_1^{*\prime} > y_1^*$. By Lemma 4, we have $U_1(y_1^{*\prime};\overline{S}(y_1^{*\prime})) > U_1(y_1^{*\prime};\overline{S}(y_1^*))$ ($y_1^{*\prime}$ being an equilibrium marginal type means $\widehat{\varepsilon}_1(y_1^{*\prime};\overline{S}(y_1^{*\prime})) > \widehat{\varepsilon}^m$). Again by Lemma 4, $U_2(y_1^{*\prime};\overline{S}(y_1^{*\prime})) \leq U_2(y_1^{*\prime};\overline{S}(y_1^*))$ ($V_2' > V_2$, and hence $\widehat{\varepsilon}_2(y_1^{*\prime};\overline{S}(y_1^{*\prime})) \geq \widehat{\varepsilon}_2(y_1^{*\prime};\overline{S}(y_1^*))$). Combining with the equilibrium indifference condition $U_2(y_1^{*\prime};\overline{S}(y_1^{*\prime})) = U_1(y_1^{*\prime};\overline{S}(y_1^{*\prime}))$, we have $U_2(y_1^{*\prime};\overline{S}(y_1^*)) > U_1(y_1^{*\prime};\overline{S}(y_1^*))$. But $U_2(y_1^*;\overline{S}(y_1^*)) = U_1(y_1^*;\overline{S}(y_1^*))$ by the equilibrium indifference condition, which implies $U_2(y_1^{*\prime};\overline{S}(y_1^*)) < U_1(y_1^{*\prime};\overline{S}(y_1^*))$ (the single crossing property and $y_1^{*\prime} > y_1^*$). This is a contradiction. Therefore, the interior partial equilibrium must be unique.

Part (ii). Consider $\widehat{y}_2' > \widehat{y}_2$. We use the superscript $'$ to indicate endogenous variables with $\widehat{y}_2'$. Suppose $y_1^{*\prime} < y_1^*$. Then by Lemma 4, we have $U_1(y_1^*;\overline{S}(y_1^*)) \geq U_1'(y_1^*;\overline{S}(y_1^{*\prime}))$ and $U_2(y_1^*;\overline{S}(y_1^*)) \leq U_2'(y_1^*;\overline{S}(y_1^{*\prime}))$. Combining with the equilibrium condition $U_1(y_1^*;\overline{S}(y_1^*)) = U_2(y_1^*;\overline{S}(y_1^*))$, it must be the case that $U_2'(y_1^*;\overline{S}(y_1^{*\prime})) \geq U_1'(y_1^*;\overline{S}(y_1^{*\prime}))$. On the other hand, the equilibrium condition $U_1'(y_1^{*\prime};\overline{S}(y_1^{*\prime})) = U_2'(y_1^{*\prime};\overline{S}(y_1^{*\prime}))$ together with the presumption $y_1^{*\prime} < y_1^*$, due to the single crossing property, imply that $U_2'(y_1^*;\overline{S}(y_1^{*\prime})) < U_1'(y_1^*;\overline{S}(y_1^{*\prime}))$. This is a contradiction.

Part (iii). We know from part (ii) that y_1^* weakly decreases as $\widehat{y}_2$ decreases. If y_1^* remains unchanged, then V_2^* weakly increases as indicated by Lemma 4. Next consider the case that y_1^* strictly decreases as $\widehat{y}_2$ decreases. In this case V_1^* must strictly increase. This is because $\widehat{\varepsilon}_1(y_1^*)$ must be strictly less than $1+\gamma$. If $\widehat{\varepsilon}_1(y_1^*) = 1+\gamma$, then type y_1^* could have participated in submarket 2, where he will be accepted with a positive probability as he would be the highest type man there. Now suppose V_2^* weakly decreases. Since V_1^* strictly increases, type y_1^* men, who were indifferent between submarket 1 and 2 in the original equilibrium, now strictly prefer submarket 2. By the single crossing property, this means that y_1^* should strictly increase. This contradicts the fact that y_1^* strictly decreases. ■

Proof of Lemma 6.

Proof. Part (i). Fix a $\widehat{y}_{n+1}$. Denote $\overline{S}(\widehat{y}_n)$ as a segmentation with threshold $\widehat{y}_n$. Note that a given $\widehat{y}_n$, by the presumption, will induce a unique partial equilibrium segmentation in submarkets $1, ..., n$. And this has been incorporated in $\overline{S}(\widehat{y}_n)$. Again, we focus on the indifference condition between submarkets n and $n+1$. We want to trace the $U_n(\widehat{y}_n; \overline{S}(\widehat{y}_n))$ curve and the $U_{n+1}(\widehat{y}_n; \overline{S}(\widehat{y}_n))$ curve as $\widehat{y}_n$ varies. Note that $U_n(\widehat{y}_n; \overline{S}(\widehat{y}_n))$ depends on the partial equilibrium segmentation in submarkets $1, ..., n$ given $\widehat{y}_n$. Again, we consider two cases.

Case 1. Suppose $U_n(\widehat{y}_{n+1}; \overline{S}(\widehat{y}_{n+1})) \geq \overline{U}_{n+1}(\widehat{y}_{n+1})$. Then a unique partial equilibrium exists, with no men participating in club $n+1$ (a corner equilibrium).

Case 2. Suppose $U_n(\widehat{y}_{n+1}; \overline{S}(\widehat{y}_{n+1})) < \overline{U}_{n+1}(\widehat{y}_{n+1})$. But by Lemma 4, we have $\overline{U}_n(\overline{y}) > U_{n+1}(\overline{y}; \overline{S}(\overline{y}))$. By continuity, the $U_{n+1}(\widehat{y}_n; \overline{S}(\widehat{y}_n))$ curve and the $U_n(\widehat{y}_n, \overline{S}(\widehat{y}_n))$ curve must have at least one intersection within the domain $(\widehat{y}_{n+1}, \overline{y})$. Given any intersection point, $\widehat{y}_i^*$ $(i < n)$ exists and is unique according to the presumption of the previous steps. This establishes the existence of an interior partial equilibrium.

Now we establish the uniqueness of equilibrium in case 2. Suppose there are two interior equilibria: $y_n^{*\prime}$ and y_n^*, with $y_n^{*\prime} > y_n^*$. By property (iii) for n, we have $V_n^* \geq V_n^{*\prime}$. Therefore, $U_n(y_n^{*\prime}; \overline{S}(y_n^{*\prime})) \geq U_n(y_n^{*\prime}; \overline{S}(y_n^*))$. In addition, by Lemma 4 $y_n^{*\prime} > y_n^*$ implies $V_{n+1}^{*\prime} > V_{n+1}^*$. As a result, $U_{n+1}(y_n^{*\prime}; \overline{S}(y_n^{*\prime})) \leq U_{n+1}(y_n^{*\prime}; \overline{S}(y_n^*))$. The indifference condition tells us that $U_n(y_n^{*\prime}; \overline{S}(y_n^{*\prime})) = U_{n+1}(y_n^{*\prime}; \overline{S}(y_n^{*\prime}))$. Combined with the above two inequalities, we obtain $U_{n+1}(y_n^{*\prime}; \overline{S}(y_n^*)) \geq U_n(y_n^{*\prime}; \overline{S}(y_n^*))$. On the other hand, the other indifference condition $U_n(y_n^*; \overline{S}(y_n^*)) = U_{n+1}(y_n^*; \overline{S}(y_n^*))$ together with the presumption that $y_n^{*\prime} > y_n^*$, due to the single crossing property, imply $U_{n+1}(y_n^{*\prime}; \overline{S}(y_n^*)) < U_n(y_n^{*\prime}; \overline{S}(y_n^*))$. This is a contradiction.

Part (ii). Consider $\widehat{y}_{n+1}' > \widehat{y}_{n+1}$. Suppose $y_n^{*\prime} < y_n^*$. Then by Lemma 4, $V_{n+1}^{*\prime} < V_{n+1}^*$ and hence $U_{n+1}(y_n^*; \overline{S}(y_n^*)) \leq U_{n+1}(y_n^*; \overline{S}(y_n^{*\prime}))$. By property (iii) for n, $y_n^{*\prime} < y_n^*$ implies $V_n^{*\prime} > V_n^*$. Therefore, $U_n(y_n^*; \overline{S}(y_n^*)) \geq U_n'(y_n^*; \overline{S}(y_n^{*\prime}))$. By the indifference condition $U_{n+1}(y_n^*; \overline{S}(y_n^*)) = U_n(y_n^*; \overline{S}(y_n^*))$, the above two inequalities suggest $U_{n+1}'(y_n^*; \overline{S}(y_n^{*\prime})) \geq U_n'(y_n^*; \overline{S}(y_n^{*\prime}))$. On the other hand, the equilibrium condition $U_{n+1}'(y_n^{*\prime}; \overline{S}(y_n^{*\prime})) = U_n'(y_n^{*\prime}; \overline{S}(y_n^{*\prime}))$ together with the presumption $y_n^{*\prime} < y_n^*$, due to the single crossing property, imply that $U_{n+1}'(y_n^*; \overline{S}(y_n^{*\prime})) < U_n'(y_n^*; \overline{S}(y_n^{*\prime}))$. This is a contradiction.

Part (iii). First, if y_n^* remains the same as $\widehat{y}_{n+1}$ decreases, then by Lemma 4 V_{n+1}^* weakly increases. Second, if y_n^* strictly decreases as $\widehat{y}_{n+1}$ decreases, then V_n^* strictly increases by property (iii) for n. The rest of the proof is similar to that of part (iii) of Lemma 5, and thus is omitted. ■

Proof of Proposition 3.

Proof. We denote the original equilibrium segmentation as $\overline{S}$, and a new segmentation as $\overline{S}'$. We first prove the results regarding the equilibrium cutoff types $\{y_j^*\}$, and we prove the claim by ruling out the impossible cases.

Case 1: Suppose $y_n^{*\prime} \geq y_n^*$ and $y_{n-1}^{*\prime} \leq y_{n-1}^*$. By the value function (3), $V_n(X_n', \overline{S}) < V_n(X_n, \overline{S}) = V_n$. By Lemma 4, $V_n' = V_n(X_n', \overline{S}') \leq V_n(X_n', \overline{S})$. Therefore, we have $V_n' < V_n$. This implies that $U_n(y_n^*, X_n', \overline{S}') > U_n(y_n^*, X_n, \overline{S})$. Following Corollary 1 and Lemma 6, $y_n^{*\prime} \geq y_n^*$ means that $V_{n+1}' \geq V_{n+1}$. This implies that $U_{n+1}(y_n^*, X_n', \overline{S}') \leq U_{n+1}(y_n^*, X_n, \overline{S})$. Taking the two inequalities together, we have $U_n(y_n^*, X_n', \overline{S}') > U_{n+1}(y_n^*, X_n', \overline{S}')$. By the property of assortative matching, that means the indifference type $y_n^{*\prime}$ must satisfy $y_n^{*\prime} < y_n^*$. This is a contradiction.

Case 2: Suppose $y_n^{*\prime} \geq y_n^*$ and $y_{n-1}^{*\prime} > y_{n-1}^*$. By Corollary 1 and Lemma 6, $y_{n-1}^{*\prime} > y_{n-1}^*$ implies that $V_{n-1}' < V_{n-1}$. To ensure that $y_{n-1}^{*\prime} > y_{n-1}^*$ is the new marginal type between submarkets $n-1$ and n, it must be the case that $V_n' < V_n$. Following similar steps as in case 1, we get a desired contradiction.

Case 3: Suppose $y_n^{*\prime} < y_n^*$ and $y_{n-1}^{*\prime} < y_{n-1}^*$. By Corollary 1 and Lemma 6, $y_{n-1}^{*\prime} < y_{n-1}^*$ implies that $V_{n-1}' > V_{n-1}$. To ensure that $y_{n-1}^{*\prime} < y_{n-1}^*$ is the new indifference type between submarkets $n-1$ and n, it must be the case that $V_n' > V_n$. Then, using the opposite direction of the proof in case 1, again we can get a contradiction.

By ruling out the above cases, we must have $y_n^{*\prime} < y_n^*$, and $y_{n-1}^{*\prime} \geq \widehat{y}_{n-1}^*$.

Part (i). Since $y_n^{*\prime} < y_n^*$, using Corollary 1, we reach the conclusion that $y_i^{*\prime} < y_i^*$ for any $i \geq n+1$. Since $y_n^{*\prime} < y_n^*$, $V_{n+1}' < V_{n+1}$. This again implies that $V_n' < V_n$. The rest of the results follow Corollary 1, and Lemma 6.

Part (ii). Next we show that $y_{n-1}^{*\prime} = y_{n-1}^*$ if $\widehat{\varepsilon}_n(y_{n-1}^*) = \widehat{\varepsilon}^m$. Since $V_n' < V_n$, $\widehat{\varepsilon}_n(y_{n-1}^*) = \widehat{\varepsilon}^m$ implies that the new acceptance cutoff $\widehat{\varepsilon}_n'(y_{n-1}^*)$ is still $\widehat{\varepsilon}^m$. Suppose $y_{n-1}^{*\prime} > y_{n-1}^*$, then $V_{n-1}' < V_{n-1}$, and $U_{n-1}'(y_{n-1}^*) > U_{n-1}(y_{n-1}^*) = U_n(y_{n-1}^*) = U_n'(y_{n-1}^*)$, which contradicts assortative matching as $y_{n-1}^{*\prime}$ $(> y_{n-1}^*)$ is the marginal type. Therefore, we must have $y_{n-1}^{*\prime} = y_{n-1}^*$.

Now we show that $y_{n-1}^{*\prime} > y_{n-1}^*$ if $\widehat{\varepsilon}_n(y_{n-1}^*) > \widehat{\varepsilon}^m$. Since $V_n' < V_n$, $\widehat{\varepsilon}_n(y_{n-1}^*) > \widehat{\varepsilon}^m$ implies that the new acceptance cutoff $\widehat{\varepsilon}_n'(y_{n-1}^*) < \widehat{\varepsilon}_n(y_{n-1}^*)$, which means $U_n(y_{n-1}^*) < U_n'(y_{n-1}^*)$. Suppose $y_{n-1}^{*\prime} = y_{n-1}^*$, then $V_{n-1}' = V_{n-1}$, and $U_{n-1}'(y_{n-1}^*) = U_{n-1}(y_{n-1}^*) = U_n(y_{n-1}^*) < U_n'(y_{n-1}^*)$, which contradicts the presumption that y_{n-1}^* is the indifference type. Therefore, we must have $y_{n-1}^{*\prime} > y_{n-1}^*$.

As to a general $i < n$, the claim can be proved in a similar fashion. The rest of the results follow Corollary 1, and Lemma 6. ■

Proof of Lemma 7.

Proof. By the value function (3), we have

$$rV_n = \alpha \frac{Y_n}{X_n} \int_{y_n} \int_{\widehat{\varepsilon}_n(y)}^{1+\gamma} (\theta_n \varepsilon y - V_n) dG(\varepsilon) dF_n(y).$$

Suppose V_n weakly increases for some $n \leq \tilde{n}$. Then for any y with $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$, $\int_{\widehat{\varepsilon}_n(y)}^{1+\gamma} (\theta_n \varepsilon y - V_n) dG(\varepsilon)$ weakly decreases, as V_n weakly increases and $\widehat{\varepsilon}^m$ stays unchanged. For any y with

$\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}_n^w(y)$, $\int_{\widehat{\varepsilon}_n(y)}^{1+\gamma}(\theta_n \varepsilon y - V_n)dG(\varepsilon)$ also weakly decreases, as both V_n and $\widehat{\varepsilon}_n^w(y)$ weakly increases. In addition, $\frac{Y_n}{X_n}$ strictly decreases. As a result, the right-hand side of the above equation strictly decreases, while the left-hand side weakly increases. This leads to a contradiction. ■

Proof of Proposition 4.

Proof. Part (i). Suppose y_1^* weakly increases. By Lemma 4 and Lemma 7, V_1 must strictly decrease. As a result, $U_1(y_1^*)$ strictly increases. On the other hand, since $\widehat{\varepsilon}_2(y_1^*) = \widehat{\varepsilon}^m$, $U_2(y_1^*)$ remains the same. This means that a type y_1^* man now strictly prefers class 1 women, and thus y_1^* should strictly decrease. This is a contradiction. Therefore, y_1^* must strictly decrease.

Now consider submarket 2 and suppose y_2^* weakly increases. Since y_1^* strictly decreases, Lemma 4 and Lemma 7 imply that V_2 must strictly decrease. Following the same logic as in the previous step, we have a contradiction. Therefore, y_2^* must strictly decrease.

Using the induction as in the previous step, we can show that all y_n^*, $n \leq \tilde{n}$, must strictly decrease. For submarkets $n > \tilde{n}$, since $y_{\tilde{n}}^*$ strictly decreases, by Corollary (1), all y_n^*, $n > \tilde{n}$, must strictly decrease.

Part (ii). The fact that y_n^* strictly decreases directly implies that V_n strictly decreases, as $\widehat{\varepsilon}_{n+1}(y_n^*) = \widehat{\varepsilon}^m$.

Part (iii). Recall that $U_n(y)$ is affected by an increase in X_n only through a change in V_n or $\widehat{\varepsilon}_n(y)$. In addition, we have shown that V_n strictly decreases. The third part of the proposition therefore holds. ■

Proof of Lemma 8.

Proof. Part (i). By the value function of V_n, equation (3), we have

$$V_n = \int_{y_n} \frac{\alpha \theta_n y \int_{\widehat{\varepsilon}_n(y)}^{1+\gamma} \varepsilon dG(\varepsilon)}{r/q_n + \alpha \int_{y_n} (1 - G(\widehat{\varepsilon}_n(y)))dF_n(y)} dF_n(y) \equiv \int_{y_n} Z_n(y) dF_n(y)$$

Suppose α increases to α'. By previous results, men's acceptance cutoff increases to $\widehat{\varepsilon}^{m\prime} > \widehat{\varepsilon}^m$. Assume that, under α', women in submarket n adopt the same acceptance cutoff as under α. Denote $\widehat{\varepsilon}_n^a(y)$ as the resulting threshold for any $y \in \{y_n\}$, $Z_n^a(y)$ as the integrand under α' and $\widehat{\varepsilon}_n^a(y)$, and $V_n(\alpha')$ as the value of type n women under $Z_n^a(y)$ and α'. Note that the resulting $V_n(\alpha')$ is not optimal for women under α'.

First, consider the case that $\widehat{\varepsilon}_n(y) > \widehat{\varepsilon}^m$ (the lower types). Then it is straightforward that

$Z_n^a(y) \geq Z_n(y)$, as

$$Z_n^a(y) - Z_n(y) \propto \frac{\alpha' \int_{\widehat{\varepsilon}_n(y)} \varepsilon dG(\varepsilon)}{r/q_n + \alpha' \int_{y_n}[1 - G(\widehat{\varepsilon}_n^a(y))]dF_n(y)} - \frac{\alpha \int_{\widehat{\varepsilon}_n(y)} \varepsilon dG(\varepsilon)}{r/q_n + \alpha \int_{y_n}[1 - G(\widehat{\varepsilon}_n(y))]dF_n(y)}$$
$$> \frac{\alpha' \int_{\widehat{\varepsilon}_n(y)} \varepsilon dG(\varepsilon)}{r/q_n + \alpha' \int_{y_n}[1 - G(\widehat{\varepsilon}_n(y))]dF_n(y)} - \frac{\alpha \int_{\widehat{\varepsilon}_n(y)} \varepsilon dG(\varepsilon)}{r/q_n + \alpha \int_{y_n}[1 - G(\widehat{\varepsilon}_n(y))]dF_n(y)},$$

where the inequality follows because $\widehat{\varepsilon}^{m\prime} > \widehat{\varepsilon}^m$.

Next, consider y with $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$ (the higher types). Also denote the probability that, among all $\tilde{y} \in y_n$, $\widehat{\varepsilon}_n(\tilde{y}) = \widehat{\varepsilon}^m$ as $\mathbb{P}$. Then

$$\begin{aligned}
&\lim_{\alpha' \to \alpha} \frac{Z_n^a(y) - Z_n(y)}{\alpha' - \alpha} \\
&\propto \frac{r}{q_n} \int_{\widehat{\varepsilon}^m}^{1+\gamma} \varepsilon dG(\varepsilon) + \alpha g(\widehat{\varepsilon}^m)[-\frac{r}{q_n}\widehat{\varepsilon}^m - \alpha\widehat{\varepsilon}^m \int_{y_n} [1 - G(\widehat{\varepsilon}_n(y))]dF_n(y) + \alpha \int_{\widehat{\varepsilon}^m} \varepsilon dG(\varepsilon)\mathbb{P}]\frac{\partial \widehat{\varepsilon}^m}{\partial \alpha} \\
&> \frac{r}{q_n} \int_{\widehat{\varepsilon}^m}^{1+\gamma} \varepsilon dG(\varepsilon) + \alpha g(\widehat{\varepsilon}^m)[-\frac{r}{q_n}\widehat{\varepsilon}^m - \alpha\widehat{\varepsilon}^m[1 - G(\widehat{\varepsilon}^m)] + \alpha \int_{\widehat{\varepsilon}^m}^{1+\gamma} \varepsilon dG(\varepsilon)\mathbb{P}]\frac{\partial \widehat{\varepsilon}^m}{\partial \alpha} \\
&= \frac{r}{q_n} \int_{\widehat{\varepsilon}^m}^{1+\gamma} \varepsilon dG(\varepsilon) + \alpha g(\widehat{\varepsilon}^m)[r(1 - \frac{1}{q_n})\widehat{\varepsilon}^m - (1 - \mathbb{P})\alpha \int_{\widehat{\varepsilon}^m} \varepsilon dG(\varepsilon)]\frac{\partial \widehat{\varepsilon}^m}{\partial \alpha}
\end{aligned}$$

Let $q_n \geq 1$. After plugging in $\frac{\partial \widehat{\varepsilon}^m}{\partial \alpha}$, a sufficient condition for $Z_n^a(y) - Z_n(y) \geq 0$ is

$$\frac{r}{\alpha}\frac{r + \alpha[1 - G(\widehat{\varepsilon}^m)]}{g(\widehat{\varepsilon}^m)} - \alpha \int_{\widehat{\varepsilon}^m} (\varepsilon - \widehat{\varepsilon}^m) dG(\varepsilon) \geq 0. \tag{12}$$

Therefore, if (12) holds and q_n is larger than 1 or close to 1 (by continuity), then $Z_n^a(y) - Z_n(y) \geq 0$. Combined with the previous discussion, this further implies that $V_n' > V_n(\alpha') > V_n$.

The remaining task is to find sufficient conditions for (12) to hold. The second term in (12), $\alpha \int_{\widehat{\varepsilon}^m} (\varepsilon - \widehat{\varepsilon}^m) dG(\varepsilon)$, is decreasing in $\widehat{\varepsilon}^m$. Thus it is less than $\alpha\gamma$ (when $\widehat{\varepsilon}^m = 1 - \gamma$). Also, by Assumption 3 the first term in (12) is decreasing in $\widehat{\varepsilon}^m$. Therefore, with the assumption that $\widehat{\varepsilon}^m \leq 1$, the following condition is sufficient for (12)

$$\frac{r}{\alpha}\frac{r + \alpha[1 - G(1)]}{g(1)} - \alpha\gamma \geq 0.$$

Part (ii). For the high types in submarket n such that $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$, by the Envelop Theorem it can readily computed that

$$\frac{\partial U_n(y)}{\partial \alpha} = \frac{\frac{r}{\alpha} U_n(y_n)}{r + \alpha[1 - G(\widehat{\varepsilon}^m)]} > 0,$$

which is obviously increasing in y. Therefore, $\Delta U_n(y)$ increases in y whenever $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$.
Part (iii). First, we show that $\Delta U_n(y_{n-1}^*) > 0$. Following the expression of $\frac{\partial U_n(y)}{\partial \alpha}$ in part (ii), this is obvious as $\widehat{\varepsilon}_n(y_{n-1}^*) = \widehat{\varepsilon}^m$.

Next we show $\Delta U_n(y_n^*) < \Delta U_{n+1}(y_n^*)$. By previous results, we have

$$\frac{\partial U_n(y_n^*)}{\partial \alpha} = \frac{\frac{r}{\alpha}U_n(y_n^*) - \alpha[V_n - U_n(y)]g(\widehat{\varepsilon}_n)\frac{\widehat{\varepsilon}_n}{V_n}\frac{\partial V_n}{\partial \alpha}}{r + \alpha[1 - G(\widehat{\varepsilon}_n)]},$$

where $\widehat{\varepsilon}_n$ denotes $\widehat{\varepsilon}_n = \widehat{\varepsilon}_n(y_n^*) < \widehat{\varepsilon}^m$, and

$$\frac{\partial U_{n+1}(y_n^*)}{\partial \alpha} = \frac{\frac{r}{\alpha}U_{n+1}(y_n^*)}{r + \alpha[1 - G(\widehat{\varepsilon}^m)]} = \frac{\frac{r}{\alpha}U_n(y_n^*)}{r + \alpha[1 - G(\widehat{\varepsilon}^m)]} > 0.$$

If $\frac{\partial U_n(y_n^*)}{\partial \alpha} \leq 0$, then the claim is trivially satisfied. Now suppose $\frac{\partial U_n(y_n^*)}{\partial \alpha} \geq 0$. Since $\frac{\partial V_n}{\partial \alpha} > \frac{\partial U_n(y_n^*)}{\partial \alpha}$,

$$\frac{\partial U_n(y_n^*)}{\partial \alpha} < \frac{\frac{r}{\alpha}U_n(y_n^*) - \alpha[V_n - U_n(y)]g(\widehat{\varepsilon}_n)\frac{\widehat{\varepsilon}_n}{V_n}\frac{\partial U_n(y_n^*)}{\partial \alpha}}{r + \alpha[1 - G(\widehat{\varepsilon}_n)]},$$

which means

$$\frac{\partial U_n(y_n^*)}{\partial \alpha} < \frac{\frac{r}{\alpha}U_n(y_n^*)}{r + \alpha[1 - G(\widehat{\varepsilon}_n)] + \alpha[V_n - U_n(y_n^*)]g(\widehat{\varepsilon}_n)\frac{\widehat{\varepsilon}_n}{V_n}}.$$

Now, we have

$$\frac{\partial U_n(y_n^*)}{\partial \alpha} - \frac{\partial U_{n+1}(y_n^*)}{\partial \alpha} < 0 \Leftrightarrow [G(\widehat{\varepsilon}_n) - G(\widehat{\varepsilon}^m)] - [1 - \frac{U_n(y_n^*)}{V_n}]g(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n \leq 0, \tag{13}$$

which is equivalent to

$$1 - \frac{U_n(y_n^*)}{V_n} - \frac{G(\widehat{\varepsilon}_n) - G(\widehat{\varepsilon}^m)}{g(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n} \geq 0.$$

Define

$$B_n(y) \equiv 1 - \frac{U_n(y)}{V_n} - \frac{G(\widehat{\varepsilon}_n(y)) - G(\widehat{\varepsilon}^m)}{g(\widehat{\varepsilon}_n(y))\widehat{\varepsilon}_n(y)}.$$

Then (13) requires $B_n(y_n^*) \geq 0$. Observing $B_n(y)$, we first notice that $B_n(y^A) = 0$, where y^A is the type of man such that $U_n(y^A) = V_n$. Therefore, a sufficient condition for the above inequality to hold is that $B_n'(y) < 0$ whenever $B_n(y) < 0$. Using the fact that $1 - \frac{U_n}{V_n} < \frac{G(\widehat{\varepsilon}_n) - G(\widehat{\varepsilon}^m)}{g(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n}$ when $B_n(y) < 0$, we get

$$\begin{aligned} B_n'(y) &\propto -[1 - \frac{U_n}{V_n}][\frac{\alpha g(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n}{r + \alpha[1 - G(\widehat{\varepsilon}_n)]} - 1] - \frac{G(\widehat{\varepsilon}_n) - G(\widehat{\varepsilon}^m)}{g(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n}[\frac{g'(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n}{g(\widehat{\varepsilon}_n)} + 1] \\ &< -[1 - \frac{U_n}{V_n}][\frac{\alpha g(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n}{r + \alpha[1 - G(\widehat{\varepsilon}_n)]} + \frac{g'(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n}{g(\widehat{\varepsilon}_n)}] \\ &\propto -\frac{\alpha g^2(\widehat{\varepsilon}_n)}{r + \alpha[1 - G(\widehat{\varepsilon}_n)]} - g'(\widehat{\varepsilon}_n) \propto [-\frac{g(\widehat{\varepsilon}_n)}{r + \alpha[1 - G(\widehat{\varepsilon}_n)]}]', \end{aligned}$$

which is negative by Assumption 3. ■

Proof of Lemma 9.

Proof. We have already shown that $\widehat{\varepsilon}^m$ increases in α. It is easy to see that it also decreases in r. Therefore, $\widehat{\varepsilon}^m$ is decreasing in r/α. Moreover, the condition $\frac{r+\alpha[1-G(1)]}{g(1)} - \frac{\alpha^2\gamma}{r} \geq 0$ can be simplified as

$$\frac{r}{\alpha}(\frac{r}{\alpha}+\frac{1}{2})\frac{\sigma}{\phi(0)}[\Phi(\frac{\gamma}{\sigma})-\Phi(\frac{-\gamma}{\sigma})] \geq \gamma, \text{ when } \varepsilon \text{ follows truncated normal distribution,}$$

$$\frac{r}{\alpha}(\frac{r}{\alpha}+\frac{1}{2}) \geq \frac{1}{2}, \text{ when } \varepsilon \text{ follows uniform distribution.}$$

In either case, this condition is satisfied if $\frac{r}{\alpha}$ is not too small.

Actually, under uniform distribution, the conditions can be weakened. Note that a sufficient for condition (12) is that

$$\frac{r}{\alpha}\frac{r+\alpha[1-G(\widetilde{\varepsilon})]}{g(\widetilde{\varepsilon})} - \alpha\int_{\widetilde{\varepsilon}}(\varepsilon-\widetilde{\varepsilon})dG(\varepsilon) \geq 0 \text{ for any } \widetilde{\varepsilon}\in[1-\gamma,1+\gamma]. \tag{14}$$

Under uniform distribution, the derivative of the left-hand side of (14) equals to $-r+\alpha[1-G(\widetilde{\varepsilon})]$, and the second derivative is $-\frac{\alpha}{2\gamma} < 0$. This means that the left-hand side is either always decreasing in $\widetilde{\varepsilon}$ or first increasing then decreasing in $\widetilde{\varepsilon}$. Therefore, as long as the inequality holds for the two end points, (14) is satisfied. When $\widetilde{\varepsilon} = 1+\gamma$, the inequality is trivially satisfied. When $\widetilde{\varepsilon} = 1-\gamma$, the inequality is equivalent to $\frac{r}{\alpha}(\frac{r}{\alpha}+1) \geq 1/2$. Therefore, when ε is uniformly distributed, a sufficient condition is $\frac{r}{\alpha}(\frac{r}{\alpha}+1) \geq 1/2$. ■

Proof of Lemma 10.

Proof. We first show that, for $y \in y_n$ and $\widehat{\varepsilon}_n^w(y) \geq \widehat{\varepsilon}^m$, $\frac{\partial U_n(y)}{\partial\alpha}$ is increasing in y. In particular, we have

$$\frac{\partial U_n(y)}{\partial \alpha} = \frac{\frac{r}{\alpha}U_n(y) - \alpha[V_n - U_n(y)]g(\widehat{\varepsilon}_n)\frac{\widehat{\varepsilon}_n}{V_n}\frac{\partial V_n}{\partial\alpha}}{r+\alpha[1-G(\widehat{\varepsilon}_n)]},$$

where $\widehat{\varepsilon}_n$ is a shorthand for $\widehat{\varepsilon}_n(y)$. Given that ε is uniform and $\frac{\partial V_n}{\partial\alpha} > 0$, the second term is more negative for a smaller y. Thus, we only need to show that $\frac{U_n(y)}{r+\alpha[1-G(\widehat{\varepsilon}_n)]}$ is increasing in y. This is equivalent to

$$\frac{\int_{\widehat{\varepsilon}_n}^{1+\gamma}\varepsilon dG(\varepsilon)}{g(\widehat{\varepsilon}_n)\widehat{\varepsilon}_n} + \widehat{\varepsilon}_n - 2\frac{\alpha\int_{\widehat{\varepsilon}_n}^{1+\gamma}\varepsilon dG(\varepsilon)}{[r+\alpha(1-G(\widehat{\varepsilon}_n))]} \geq 0. \tag{15}$$

Using the fact that $\widehat{\varepsilon}^m = \frac{\alpha\int_{\widehat{\varepsilon}^m}^{1+\gamma}\varepsilon dG(\varepsilon)}{[r+\alpha(1-G(\widehat{\varepsilon}^m))]}$ and that $\frac{\alpha\int_{\widehat{\varepsilon}_n}^{1+\gamma}\varepsilon dG(\varepsilon)}{[r+\alpha(1-G(\widehat{\varepsilon}_n))]}$ is decreasing in $\widehat{\varepsilon}_n$ for $\widehat{\varepsilon}_n \geq \widehat{\varepsilon}^m$, the following condition is sufficient for (15):

$$\frac{\frac{r}{\alpha}+(1-G(\widehat{\varepsilon}_n))}{g(\widehat{\varepsilon}_n)} \geq (2-\frac{\widehat{\varepsilon}_n}{\widehat{\varepsilon}^m})\widehat{\varepsilon}_n.$$

Note that the LHS of the above inequality is decreasing in $\widehat{\varepsilon}_n$, and the RHS is less than $\widehat{\varepsilon}^m$. Thus the following condition is sufficient:

$$\frac{\frac{r}{\alpha}+(1-G(\widehat{\varepsilon}_n^*))}{g(\widehat{\varepsilon}_n^*)} \geq \widehat{\varepsilon}^m,$$

where $\widehat{\varepsilon}_n^*$ is the highest $\widehat{\varepsilon}_n$: $\widehat{\varepsilon}_n(y_n^*) \equiv \widehat{\varepsilon}_n^*$. Under uniform distribution, this condition becomes

$$\frac{r}{\alpha}2\gamma + [(1+\gamma) - \widehat{\varepsilon}_n^*] \geq \widehat{\varepsilon}^m.$$

The above condition holds if $\widehat{\varepsilon}^m \leq \frac{r}{\alpha}2\gamma$, which under uniform distribution is equivalent to

$$\frac{1+\gamma}{\gamma} - 4(\frac{r}{\alpha})^2\frac{\gamma}{1+\gamma} - 4\frac{r}{\alpha} \leq 0.$$

Again, this condition holds if r/α is not too small.

Combining with part (ii) of Lemma 8, we reach the conclusion that for all $y \in y_n$, $\frac{\partial U_n(y)}{\partial \alpha}$ is increasing in y.

Now we set $q_n = 1$ and show $\frac{\partial V_n}{\partial \alpha} > \frac{\partial U_n(y_n^*)}{\partial \alpha}$. Recall that when $q_n = 1$, we have $V_n = \int_{y_n} w_n(y)U_n(y)dF_n(y)$, where the quasi-weight $w_n(y)$ is defined in equation (7). Taking derivative of $w_n(y)$ with respect to α, we have

$$\begin{aligned}\frac{\partial w_n(y)}{\partial \alpha} \quad &\propto \quad [r+\alpha[1-G(\widehat{\varepsilon}_n(y))]]\{\frac{1-G(\widehat{\varepsilon}_n(y))}{r+\alpha[1-G(\widehat{\varepsilon}_n(y))]} - \frac{\alpha g(\widehat{\varepsilon}_n(y))\frac{\partial \widehat{\varepsilon}_n(y)}{\partial \alpha}}{r+\alpha[1-G(\widehat{\varepsilon}_n(y))]} \\ &\quad - \frac{\int_{y_n}[1-G(\widehat{\varepsilon}_n(\tilde{y})) - \alpha g(\widehat{\varepsilon}_n(\tilde{y}))\frac{\partial \widehat{\varepsilon}_n(\tilde{y})}{\partial \alpha}]dF_n(\tilde{y})}{r+\alpha\int_{y_n}[1-G(\widehat{\varepsilon}_n(\tilde{y}))]dF_n(\tilde{y})}\}.\end{aligned}$$

In the above expression, the third term in the bracket is independent of y. First consider the higher types of y with $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$. It is obvious that all the terms in the above expression are constant in y. Therefore, $\frac{\partial w_n(y)}{\partial \alpha}$ is constant in y. Next consider the lower types of y such that $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}_n^w(y)$. Note that $[r+\alpha[1-G(\widehat{\varepsilon}_n(y))]]$ is increasing in y and the first term in the bracket is also increasing in y. The second term in the bracket, due to uniform distribution, can be explicitly written as

$$\frac{\alpha\frac{1}{\theta_n y}\frac{\partial V_n}{\partial \alpha}}{2r\gamma + \alpha[1+\gamma-\widehat{\varepsilon}_n(y)]},$$

which is decreasing in y. Therefore, $\frac{\partial w_n(y)}{\partial \alpha}$ is increasing in y.

In addition, $\frac{\partial \widehat{\varepsilon}_n(y)}{\partial \alpha}$ is continuous in y. In particular, it is continuous at $y = y^A$ ($\widehat{\varepsilon}_n^w(y^A) = \widehat{\varepsilon}^m$). Therefore, $\frac{\partial w_n(y)}{\partial \alpha}$ is also continuous in y. Combined with the earlier discussion, this implies that $\frac{\partial w_n(y)}{\partial \alpha}$ is increasing in y for $y \leq y^A$ and constant in y when $y \geq y^A$. Moreover, since $\int_{y_n} w_n(y)dF_n(y) = 1$ for any α, thus we have $\int_{y_n}\frac{\partial w_n(y)}{\partial \alpha}dF_n(y) = 0$. As a result, $\frac{\partial w_n(y)}{\partial \alpha}$ must be negative for smaller y and then become positive for larger y.

Notice that $w_n(y)$ can also be considered as a quasi-p.d.f. of the weight distribution. Then the above discussion shows that the quasi-weight distribution with a larger α first order stochastic dominates the one with a smaller α.

Finally, we have

$$\begin{aligned}\frac{\partial V_n}{\partial \alpha} &= \int_{y_n} \frac{\partial w_n(y)}{\partial \alpha} U_n(y) dF_n(y) + \int_{y_n} w_n(y) \frac{\partial U_n(y)}{\partial \alpha} dF_n(y) \\ &\geq \int_{y_n} w_n(y) \frac{\partial U_n(y)}{\partial \alpha} dF_n(y) > \int_{y_n} w_n(y) \frac{\partial U_n(y_n^*)}{\partial \alpha} dF_n(y) \\ &= \frac{\partial U_n(y_n^*)}{\partial \alpha}.\end{aligned}$$

In the above derivation, the first inequality is due to $\int_{y_n} \frac{\partial w_n(y)}{\partial \alpha} U_n(y) dF_n(y) \geq 0$, which follows the facts that $U_n(y)$ strictly increases in y, $\frac{\partial w_n(y)}{\partial \alpha}$ is increasing in y, and $\int_{y_n} \frac{\partial w_n(y)}{\partial \alpha} dF_n(y) = 0$. The second inequality holds because $\frac{\partial U_n(y)}{\partial \alpha}$ strictly increases in y. ■

Proof of Proposition 5.

Proof. Part (i). We use notations without superscript to denote variables before the increase in α, and use superscripts "$'$" and "^{a}" to indicate variables under α' when the segmentation is endogenously adjusted and when the segmentation stays constant, respectively.

We first show that y_1^* must strictly increase. Suppose, to the contrary, $y_1^{*\prime} \leq y_1^*$. By Lemma 8, $V_1^a > V_1$ and $U_2^a(y_1^*) - U_2(y_1^*) > U_1^a(y_1^*) - U_1(y_1^*)$. Since $U_2(y_1^*) = U_1(y_1^*)$, we have $U_2^a(y_1^*) > U_1^a(y_1^*)$. Since $y_1^{*\prime} \leq y_1^*$, Lemma 4 implies that $V_1' \geq V_1^a$. Thus $U_1'(y_1^*) \leq U_1^a(y_1^*)$. On the other hand, if in the new segmentation type y_1^* men deviate to submarket 2, they will still be the highest types there, which implies $U_2'(y_1^*) = U_2^a(y_1^*)$. Taken together, we have $U_1'(y_1^*) < U_2'(y_1^*)$, which means that men of type y_1^* would strictly prefer to participate in submarket 2. This is a contradiction, and thus y_1^* must strictly increase in α.

Next we show that y_2^* must strictly increase. Suppose $y_2^{*\prime} \leq y_2^*$. Since $y_1^{*\prime} > y_1^*$ and $y_2^{*\prime} \leq y_2^*$, Lemma 4 implies that $V_2' \geq V_2^a$. Then by the same argument as in the previous step, $U_3'(y_2^*) > U_2'(y_2^*)$, i.e., men of type y_2^* strictly prefer to participate in submarket 3. This is a contradiction. This argument can be readily extended to all thresholds in the lower classes.

Part (ii). Using the implicit function theorem, we can compute the percentage change of $U_n(y_n^*)$

$$\begin{aligned}\frac{\partial U_n(y_n^*)}{\partial \alpha} \frac{\alpha}{U_n(y_n^*)} &= \frac{r - \frac{\alpha^2}{U_n(y_n^*)}[\theta_n y_n^* \widehat{\varepsilon}_n(y_n^*) - U_n(y_n^*)] g(\widehat{\varepsilon}_n(y_n^*)) \frac{\partial \widehat{\varepsilon}_n(y_n^*)}{\partial \alpha}}{\alpha[1 - G(\widehat{\varepsilon}_n(y_n^*))] + r}, \\ \frac{\partial U_{n+1}(y_n^*)}{\partial \alpha} \frac{\alpha}{U_{n+1}(y_n^*)} &= \frac{r}{\alpha[1 - G(\widehat{\varepsilon}^m)] + r}.\end{aligned}$$

From part (i), we know that y_n^* increases in α. Therefore, $\frac{\partial U_n(y_n^*)}{\partial \alpha} \frac{\alpha}{U_n(y_n^*)}$ must be smaller than $\frac{\partial U_{n+1}(y_n^*)}{\partial \alpha} \frac{\alpha}{U_{n+1}(y_n^*)}$. Since $\widehat{\varepsilon}_n(y_n^*) > \widehat{\varepsilon}^m$, it implies that $\frac{\partial \widehat{\varepsilon}_n(y_n^*)}{\partial \alpha}$ must be strictly positive. This further implies that V_n strictly increases in α.

Part (iii). According to the previous results, $U_1^a(\overline{y}) > U_1(\overline{y})$. Since $y_1^{*\prime} > y_1^*$, following Lemma 4 we have $V_1' < V_1^a$, and thus $U_1'(\overline{y}) = U_1^a(\overline{y}) > U_1(\overline{y})$. The continuity implies that the highest types of men (close to the highest type $\overline{y}$) must be better off.

Now consider the highest type men y_n^* in submarket $n > 1$. Given that an increase in α is small, for type y_n^* in submarket $n+1$, men's acceptance cutoff is still binding. Then we have $U_{n+1}'(y_n^*) = U_{n+1}^a(y_n^*) > U_{n+1}(y_n^*)$. This means that type y_n^* men are strictly better off after an increase in α. Then, by continuity, men with types slightly higher than y_n^* should also be strictly better off. This means that the lowest types of men originally in submarket n, $1 \le n < N$, are strictly better off as well. ■

Proof of Lemma 11.

Proof. Part (i). By (10), for any y with $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$, we have

$$\frac{\partial \Gamma(\widehat{\varepsilon})}{\partial \gamma}|_{\widehat{\varepsilon}=\widehat{\varepsilon}^m} \propto 1+\gamma-(1+\frac{2r}{\alpha})\widehat{\varepsilon}^m = 1+\gamma-(1+\frac{2r}{\alpha})\frac{U_n(y)}{\theta_n y} \equiv T_n(y).$$

By the definition of $T_n(y)$ and the expression of $U_n(y)$, we get

$$\begin{aligned}\theta_n y T_n(y) &= [\theta_n y(1+\gamma) - U_n(y)] - \frac{1}{\gamma}\int_{\widehat{\varepsilon}^m}^{1+\gamma}[\theta_n y\varepsilon - U_n(y)]d\varepsilon \\ &= 2[\theta_n y\frac{1+\gamma+\widehat{\varepsilon}^m}{2} - U_n] - \frac{1+\gamma-\widehat{\varepsilon}^m}{\gamma}[\theta_n y\frac{1+\gamma+\widehat{\varepsilon}^m}{2} - U_n] \\ &= [\theta_n y\frac{1+\gamma+\widehat{\varepsilon}^m}{2} - U_n]\frac{\widehat{\varepsilon}^m-(1-\gamma)}{\gamma} > 0.\end{aligned}$$

Therefore, $T_n(y) > 0$ and $\frac{\partial\Gamma(\widehat{\varepsilon})}{\partial\gamma}|_{\widehat{\varepsilon}=\widehat{\varepsilon}^m} > 0$, which implies that $\widehat{\varepsilon}^m$ is increasing in γ.

Part (ii). From the expression of $U_n(y)$ with $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$, we take derivative with respect to γ. After using the Envelope Theorem and rearranging, we get

$$\begin{aligned}2\gamma[\frac{r}{\alpha}+1-G(\widehat{\varepsilon}^m)]\frac{\partial U_n(y)}{\partial\gamma} &= [\theta_n y(1+\gamma) - U_n(y)] - \frac{1}{\gamma}\int_{\widehat{\varepsilon}^m}^{1+\gamma}[\theta_n y\varepsilon - U_n(y)]d\varepsilon \\ &= \theta_n y T_n(y) > 0.\end{aligned}$$

Part (iii). By the value function of V_n, we have

$$V_n = \int_{y_n} \frac{\alpha\theta_n y\int_{\widehat{\varepsilon}_n(y)}^{1+\gamma}\varepsilon dG(\varepsilon)}{r/q_n + \alpha\int_{y_n}(1-G(\widehat{\varepsilon}_n(y)))dF_n(y)}dF_n(y) \equiv \int_{y_n} Z_n(y)dF_n(y)$$

Let $q_n = 1$. With uniform distribution,

$$Z_n(y) = \frac{\frac{1}{2}\alpha\theta_n y[(1+\gamma)^2 - \widehat{\varepsilon}_n^2(y)]}{2r\gamma + \alpha(1+\gamma) - \alpha\int_{y_n}\widehat{\varepsilon}_n(y)dF_n(y)}.$$

Suppose γ increases to γ'. By part (i), men's acceptance threshold increases to $\widehat{\varepsilon}^{m\prime} > \widehat{\varepsilon}^m$. Assume that, under γ', women in submarket n adopt the same acceptance thresholds as under γ: $\widehat{\varepsilon}^a_n(\tilde{y}) = \widehat{\varepsilon}_n(\tilde{y})$ for any $\tilde{y} \in \{y_n\}$. Denote $Z^a_n(y)$ as the $Z_n(y)$ under γ' and when women adopt the assumed acceptance threshold. Note that the resulting payoff is not optimal for women under γ'.

First consider high types of y with $\widehat{\varepsilon}_n(y) = \widehat{\varepsilon}^m$. Among all $\tilde{y} \in y_n$, denote the probability that $\widehat{\varepsilon}_n(\tilde{y}) = \widehat{\varepsilon}^m$ as $\mathbb{P}$. Then

$$\begin{aligned}\frac{\partial Z^a_n(y)}{\partial \gamma} \quad &\propto \quad [2(1+\gamma) - 2\widehat{\varepsilon}^m \frac{\partial \widehat{\varepsilon}^m}{\partial \gamma}][2r\gamma + \alpha(1+\gamma) - \alpha \int_{y_n} \widehat{\varepsilon}_n(y) dF_n(y)] \\ &\quad -[(1+\gamma)^2 - (\widehat{\varepsilon}^m)^2][2r + \alpha - \alpha \mathbb{P} \frac{\partial \widehat{\varepsilon}^m}{\partial \gamma}].\end{aligned}$$

Plug in the optimality condition of $\widehat{\varepsilon}^m$:

$$-2\widehat{\varepsilon}^m[2r\gamma + \alpha(1+\gamma) - \alpha\widehat{\varepsilon}^m] + \alpha[(1+\gamma)^2 - (\widehat{\varepsilon}^m)^2] = 0,$$

and we get

$$\begin{aligned}\frac{\partial Z^a_n(y)}{\partial \gamma} \quad &\propto \quad [2r\gamma + \alpha(1+\gamma) - \alpha\widehat{\varepsilon}^m][(1+\gamma) - (\frac{2r}{\alpha} + 1)\widehat{\varepsilon}^m] \\ &\quad -[2r\gamma + \alpha(1+\gamma) - \alpha\widehat{\varepsilon}^m](1-\mathbb{P})\widehat{\varepsilon}^m \frac{\partial \widehat{\varepsilon}^m}{\partial \gamma} \\ &\quad +\alpha(1-\mathbb{P})\widehat{\varepsilon}^m(\bar{\widehat{\varepsilon}}_n - \widehat{\varepsilon}^m)\frac{\partial \widehat{\varepsilon}^m}{\partial \gamma},\end{aligned}$$

where $\bar{\widehat{\varepsilon}}_n$ is the average $\widehat{\varepsilon}_n(y)$ for $\widehat{\varepsilon}_n(y) > \widehat{\varepsilon}^m$. By the optimality of $\widehat{\varepsilon}^m$, we also have

$$[2r\gamma + \alpha(1+\gamma) - \alpha\widehat{\varepsilon}^m]\frac{\partial \widehat{\varepsilon}^m}{\partial \gamma} = \alpha[(1+\gamma) - (\frac{2r}{\alpha} + 1)\widehat{\varepsilon}^m].$$

After plugging in $\frac{\partial \widehat{\varepsilon}^m}{\partial \gamma}$ and observing that the last term of $\frac{\partial Z^a_n(y)}{\partial \gamma}$ is always positive, we got

$$\frac{\partial Z^a_n(y)}{\partial \gamma} > 0 \Leftarrow 2r\gamma + \alpha(1+\gamma) - 2\alpha\widehat{\varepsilon}^m \geq 0.$$

This condition is satisfied if $2\widehat{\varepsilon}^m < (1+\gamma) + \frac{2r\gamma}{\alpha}$.

Next, consider low types of y with $\widehat{\varepsilon}_n(y) > \widehat{\varepsilon}^m$. We can show that

$$\begin{aligned}\frac{\partial Z_n^a(y)}{\partial\gamma} &\propto 2(1+\gamma)[2r\gamma+\alpha(1+\gamma)-\alpha\int_{y_n}\widehat{\varepsilon}_n(y)dF_n(y)] \\ &- [(1+\gamma)^2-\widehat{\varepsilon}_n^2(y)][2r+\alpha-\alpha\mathbb{P}\frac{\partial\widehat{\varepsilon}^m}{\partial\gamma}] \\ &> [2(1+\gamma)-2\widehat{\varepsilon}^m\frac{\partial\widehat{\varepsilon}^m}{\partial\gamma}][2r\gamma+\alpha(1+\gamma)-\alpha\int_{y_n}\widehat{\varepsilon}_n(y)dF_n(y)] \\ &- [(1+\gamma)^2-(\widehat{\varepsilon}^m)^2][2r+\alpha-\alpha\mathbb{P}\frac{\partial\widehat{\varepsilon}^m}{\partial\gamma}] \\ &> 0,\end{aligned}$$

where the first inequality applies because $2r+\alpha-\alpha\frac{\partial\widehat{\varepsilon}^m}{\partial\gamma} > 0$ and the last inequality follows the result in the previous case.

Therefore, when q_n is close enough to 1 and $2\widehat{\varepsilon}^m < (1+\gamma)+\frac{2r\gamma}{\alpha}$ is satisfied, we have $\frac{\partial V_n}{\partial\gamma} = \int_{y_n}\frac{\partial Z_n(y)}{\partial\gamma}dF_n(y) > 0$.

Part (iv). Recall that $\widehat{\varepsilon}_{n+1}(y_n^*) = \widehat{\varepsilon}^m$. We can explicitly compute how $U_n(y_n^*)$ and $U_{n+1}(y_n^*)$ change as γ increases

$$\frac{\partial U_n(y_n^*)}{\partial\gamma} - \frac{\partial U_{n+1}(y_n^*)}{\partial\gamma} \propto$$

$$[\theta_n y_n^*(1+\gamma)-(1+\frac{2r}{\alpha})U_n(y_n^*)] - [\theta_{n+1}y_n^*(1+\gamma)-(1+\frac{2r}{\alpha})U_{n+1}(y_n^*)]\frac{\frac{r}{\alpha}+1-G(\widehat{\varepsilon}_n(y_n^*))}{\frac{r}{\alpha}+1-G(\widehat{\varepsilon}^m)} \tag{16}$$

$$-[\widehat{\varepsilon}_n(y_n^*)-\frac{U_n(y_n^*)}{\theta_n y_n^*}]\frac{\partial V_n}{\partial\gamma}. \tag{17}$$

Note that the term of (16) is always positive. This is because $U_n(y_n^*) = U_{n+1}(y_n^*)$, $\theta_n > \theta_{n+1}$, and $G(\widehat{\varepsilon}^m) < G(\widehat{\varepsilon}_n(y_n^*))$. On the other hand, the term of (17) is always negative, as $U_n(y_n^*) < V_n$ and $\frac{\partial V_n}{\partial\gamma} > 0$. Rearranging the above inequality, we have $\frac{\partial U_n(y_n^*)}{\partial\gamma} \geq \frac{\partial U_{n+1}(y_n^*)}{\partial\gamma}$ iff

$$\begin{aligned}& y_n^*(1+\gamma)(\theta_n-\theta_{n+1}) + \frac{G(\widehat{\varepsilon}_n(y_n^*))-G(\widehat{\varepsilon}^m)}{\frac{r}{\alpha}+1-G(\widehat{\varepsilon}^m)}[\theta_{n+1}y_n^*(1+\gamma)-U_n(y_n^*)(1+\frac{2r}{\alpha})] \\ & \geq \frac{\partial V_n}{\partial\gamma}[\widehat{\varepsilon}_n(y_n^*)-\frac{U_n(y_n^*)}{\theta_n y_n^*}].\end{aligned} \tag{18}$$

Let $\frac{\partial V_n}{\partial\gamma} = x\frac{\partial U_n(y_n^*)}{\partial\gamma}$, and define $\widehat{x}$ as the x such that $\frac{\partial U_n(y_n^*)}{\partial\gamma} = \frac{\partial U_{n+1}(y_n^*)}{\partial\gamma}$. More explicitly, using (18) and the fact that $U_{n+1}(y_n^*) = \theta_{n+1}y_n^*\widehat{\varepsilon}^m$, we can derive

$$\begin{aligned}\widehat{x} &= \frac{[2\gamma\frac{r}{\alpha}+(1+\gamma)-\widehat{\varepsilon}^m]\frac{(1+\gamma)(\frac{\theta_n}{\theta_{n+1}}-1)}{1+\gamma-(1+\frac{2r}{\alpha})\widehat{\varepsilon}^m}+\widehat{\varepsilon}_n(y_n^*)-\widehat{\varepsilon}^m}{\widehat{\varepsilon}_n(y_n^*)-\frac{\theta_{n+1}}{\theta_n}\widehat{\varepsilon}^m} \\ &= \frac{\theta_n}{\theta_{n+1}}\{1+(\frac{\theta_n}{\theta_{n+1}}-1)\frac{(1+\gamma)[1+\frac{2r}{\alpha}\frac{\gamma+\widehat{\varepsilon}^m}{1+\gamma-(1+\frac{2r}{\alpha})\widehat{\varepsilon}^m}]-\widehat{\varepsilon}_n(y_n^*)}{\frac{\theta_n}{\theta_{n+1}}\widehat{\varepsilon}_n(y_n^*)-\widehat{\varepsilon}^m}\}.\end{aligned}$$

■

Proof of Proposition 6.

Proof. Part (i). We first show that y^*_{N-1} strictly decreases. Suppose to the contrary, y^*_{N-1} weakly increases; that is $y^{*\prime}_{N-1} \geq y^*_{N-1}$. This implies that $U'_{N-1}(y^*_{N-1}) - U_{N-1}(y^*_{N-1}) \leq U'_N(y^*_{N-1}) - U_N(y^*_{N-1})$. We also know that (1) $U'_N(y^*_{N-1}) - U_N(y^*_{N-1}) = U^a_N(y^*_{N-1}) - U_N(y^*_{N-1})$ and (2) $U^a_N(y^*_{N-1}) - U_N(y^*_{N-1}) < U^a_{N-1}(y^*_{N-1}) - U_{N-1}(y^*_{N-1})$. Taken together, they imply $U'_N(y^*_{N-1}) - U_N(y^*_{N-1}) < U^a_{N-1}(y^*_{N-1}) - U_{N-1}(y^*_{N-1})$. Therefore, it must be the case that $U^a_{N-1}(y^*_{N-1}) - U_{N-1}(y^*_{N-1}) > U'_{N-1}(y^*_{N-1}) - U_{N-1}(y^*_{N-1})$. This further implies that $V^a_{N-1} < V'_{N-1}$. By Lemma 4, we must have $y^{*\prime}_{N-2} > y^*_{N-2}$. By the same argument and induction, all cut-off types strictly increase and $V^a_n < V'_n$ for all $n < N$. In particular, $y^{*\prime}_1 > y^*_1$. Recall that $U^a_1(y^*_1) > U^a_2(y^*_1)$. The fact that $y^{*\prime}_1 > y^*_1$ and the single crossing condition imply that $U^a_1(y^{*\prime}_1) > U^a_2(y^{*\prime}_1)$. By Lemma 4 and the fact $y^{*\prime}_1 > y^*_1$, we reach the conclusion that $U'_1(y^{*\prime}_1) > U'_2(y^{*\prime}_1)$, which contradicts $y^{*\prime}_1$ being the new cutoff type.

Second, we show that y^*_{N-2} strictly decreases. Suppose $y^{*\prime}_{N-2} \geq y^*_{N-2}$. Combined with $y^{*\prime}_{N-1} < y^*_{N-1}$, Lemma 4 implies that $U'_{N-2}(y^*_{N-2}) - U_{N-2}(y^*_{N-2}) \leq U'_{N-1}(y^*_{N-2}) - U_{N-1}(y^*_{N-2})$. The rest of the proof is similar to that of the previous step. Thus, $y^{*\prime}_{N-2} < y^*_{N-2}$. By induction, we can show that $y^{*\prime}_n < y^*_n$ for all n.

Part (ii). Given the conditions, part (ii) of Lemma 11 has shown that $V^a_1 > V_1$. By part (i), y^*_1 decreases, which by Lemma 4 implies that $V'_1 > V^a_1$. Therefore, $V'_1 > V_1$.

Part (iii). Consider the highest types of men in any submarket after an increase in γ. Their acceptance cutoff is $\widehat{\varepsilon}^m$ if they are in submarket 1 or if they are in submarket $n > 1$ and the change in γ is small. Then an increase in V_n does not affect their expected payoff. Therefore, their expected payoffs strictly increase as shown earlier in Lemma 11.

Next, consider y^*_n for any $n < N$. Because y^*_n strictly decreases in γ, we have $U'_n(y^*_n) > U'_{n+1}(y^*_n) = U^a_{n+1}(y^*_n) > U_{n+1}(y^*_n) = U_n(y^*_n)$. The first equality follows from the fact that for type y^*_n his acceptance cutoff in submarket $n+1$ is $\widehat{\varepsilon}^m$. This chain of inequalities implies that $U'_n(y^*_n) > U_n(y^*_n)$. According to the continuity, the expected payoffs of the lowest types of men in submarket $n < N$ also increase. ■

Proof of Proposition 7.

Proof. To highlight the importance of α, we will write α as an argument of the relevant functions whenever it is necessary. Given a segmentation $\{y_n\}$ and men's acceptance threshold

$\widehat{\varepsilon}^{m\prime}$, the value function of a type n woman can be written as

$$\begin{aligned}
rV_n'(\alpha) =&\alpha q_n \int_{y_n} \max_{\varepsilon_n^w(y)}\{\int_{\max\{\varepsilon_n^w(y),\widehat{\varepsilon}^{m\prime}\}}^{1+\gamma} [\theta_n y\varepsilon - V_n'(\alpha)]dG'(\varepsilon)\}dF_n(y) \\
=&(2\alpha) q_n \int_{y_n} \max_{\varepsilon_n^w(y)}\{\int_{\max\{\varepsilon_n^w(y),\widehat{\varepsilon}^{m\prime}\}}^{1+\gamma} [\theta_n y\varepsilon - V_n'(\alpha)]dG(\varepsilon)\}dF_n(y).
\end{aligned}$$

Similarly, given women's threshold $\widehat{\varepsilon}_n^{w\prime}(y)$, the value function of a type y man in club n is

$$\begin{aligned}
rU_n'(y;\alpha) =&\alpha \max_{\varepsilon^m}\{\int_{\max\{\widehat{\varepsilon}_n^{w\prime}(y),\varepsilon^m\}}^{1+\gamma} [\theta_n y\varepsilon - U_n'(y;\alpha)]dG'(\varepsilon)\} \\
=&(2\alpha) \max_{\varepsilon^m}\{\int_{\max\{\widehat{\varepsilon}_n^{w\prime}(y),\varepsilon^m\}}^{1+\gamma} [\theta_n y\varepsilon - U_n'(y;\alpha)]dG(\varepsilon)\}.
\end{aligned}$$

Combined with the fact that men's acceptance threshold in the baseline model with 2α is greater than 1, the second lines of the above two equations show that the maximization problems of men and women are equivalent to those in the baseline model with 2α. That is, $V_n'(\alpha) = V_n(2\alpha)$ and $U_n'(y;\alpha) = U_n(y;2\alpha)$. ■

References

[1] Bagnoli, M. and Bergstrom, T. "Log-concave Probability and Its Applications." *Economic Theory*, Vol. 26 (2005), pp. 445-469.

[2] Becker, G.S. "A Theory of Marriage: Part I." *Journal of Political Economy,* 1973, 81, 813–846.

[3] Bloch, F„ and H. Ryder. "Two-Sided Search, Marriages, and Matchmakers," *International Economic Review*, 2000, 41(1), 93–115.

[4] Burdett, K., and Coles, M. "Marriage and Class," *The Quarterly Journal of Economics*, 1997, *141-168.*

[5] Burdett, K., and R. Wright. "Two-Sided Search with Nontransferable Utility," *Review of Economic Dynamics*, 1998, 1(1), 220–245.

[6] Eeckhout, J. "Bilateral Search and Vertical Heterogeneity," *International Economic Review,* 1999, 40(4), 869–887.

[7] Eeckhout, J. and Kircher, P. "Sorting and Decentralized Competition," *Econometrica*, 2010, 78(2), 539-574.

[8] Hitsch, G., Hortacsu, A., and D. Ariely. "Matching and Sorting in Online Dating," *American Economic Review*, 2010, 100(1), 130-163.

[9] Jacquet, N., and Tan, S. "On the Segmentation of Markets," *Journal of Political Economy*, 2007, 115(4), 639-664.

[10] Moen, E. "Competitive Search Equilibrium," *Journal of Political Economy*, 1997, 105(2), 385-411.

[11] Mortensen, D. and C. Pissarides. "New Developments in Models of Search in the Labor Market," 1999, in *Handbook of Labor Economics* (O. Ashenfelter & D. Card, eds.). Amsterdam: North-Holland.

[12] Shimer, R. "The Assignment of Workers to Jobs in an Economy with Coordination Frictions," *Journal of Political Economy*, 2005, 113(5), 996-1025.

[13] Shimer, R., and Smith, L. "Assortative Matching and Search," *Econometrica*, 2000, 68(2), 343-369.

[14] Smith, L. "The Marriage Model with Search Frictions," *Journal of Political Economy*, 2006, 114(6), 1124-1144.

[15] Sundaram, R. "Multiple Traits in the Marriage Market: Does Diversity Win Sometimes," 2000, working paper, University of Pennsylvania.

[16] Yang, H. "Targeted Search, Endogenous Market Segmentation, and Wage Inequality," 2015, working paper, Ohio State University.