# Targeted Search, Endogenous Market Segmentation, and Wage Inequality

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First Version: December 2015
This Version: March 2016

**Abstract**

Is it possible that the Internet has contributed to rising wage inequality over the last two decades? To answer this question, we develop a labor search/matching model with heterogeneous workers (a continuum of types) and heterogeneous firms (a finite number of types). One novel feature of our model is that search is targeted: each type of firm constitutes a distinctive submarket, and workers are able to choose in which submarket to participate beforehand, but search is random within each submarket, and wages are determined by Nash bargaining. We show that, given the parameter values, there is always a unique equilibrium in which workers are endogenously segmented into different submarkets. The equilibrium matching pattern is weakly positively assortative, with higher ability workers matching with weakly more productive jobs. Moreover, the segmentation pattern affects the wage structure or wage distribution in the market. We then explore how the equilibrium segmentation pattern and wage inequality change as some exogenous shocks occur, which includes a skill-biased technical change in some high-productivity submarket and a decrease in the number of jobs in some low-productivity submarket. In particular, we show that an Internet-induced increase in search efficiency would make the overall matching pattern more assortative, increase wage inequality within each submarket (workers with similar jobs), and also increase overall wage inequality across submarkets.

*JEL Classifications*: C78; D31; D83; J31

*Key Words*: Wage Inequality; Targeted Search/Matching; Market Segmentation; Internet

## 1 Introduction

Since the late 1970s, wage inequality in the U.S. has been steadily rising. One pronounced feature is that the skill premia of high-skilled workers have continued to increase over the last three decades. According to Autor et al. (2006), wage inequality in the upper-tail (measured

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[†]I would like to thank David Blau, Jim Peck, Byoung Hoon Seok, Yujing Xu, and the audience of the theory seminars at HKU, HKUST, and MSU, for very helpful comments and suggestions. I also would like to thank Zhuozheng Li for excellent research assistance.

as the 90-50 percentile log-hourly wage differential) has increased steadily from 0.61 in 1973 to 0.81 in 2004. In absolute terms, Antonczyk et al. (2010) documented that only high-skilled workers (above the 80 percentile) have higher real wages in 2004 than in 1979, while the real wages of workers below the 80 percentile actually decreased. The same pattern is also found in other developed countries (Dustmann et al. (2009) for Germany, and Goos and Manning (2007) for the U.K.). The evolution of wage inequality in the lower-tail is more subtle. Autor et al. (2006) found that during the period 1973-2004, wage inequality in the lower-tail (measured as the 50-10 percentile log-hourly wage differential) increased from 0.61 to 0.69. However, after a sharp increase within 1973-1987, during 1988-2004 it actually decreased. Combined with the steady increase of inequality in the upper tail, the trend during 1988-2004 was labeled as the "polarization" of wages. Nevertheless, the evidence of wage polarization is restricted to the U.S. In other industrialized countries, wage inequality in the bottom half has been continuously increasing. Antonczyk et al. (2010) also documented the trends of wage inequality for specific skill groups in the U.S. during 1996-2004. They found that among low-skilled workers, the 80-50 difference was rather stable, but the 50-20 difference slowly declined. As to medium-skilled workers, a clear pattern of wage polarization is found: the 80-50 difference has been increasing but the 50-20 difference has decreased. Wage inequality within high-skilled workers has been continuously increasing, both for the top and bottom half of the wage distribution.

The most prominent explanation for rising wage inequality has been skill-biased technical change (Katz and Autor, 1999). In particular, a skill-biased technical change (SBTC hereafter) increases the demand for skilled workers and reduces that of unskilled ones, which tends to increase wage inequality. In response to the evidence of wage polarization, a more sophisticated version of SBTC was developed recently (Autor et al., 2004, 2006, 2008; Goos and Manning, 2007). They argue that new technologies (computerization for instance) favor highly skilled workers over less skilled routine-manual workers, and also favor less-skilled non-routine workers (cleaning jobs for instance) relative to less skilled routine-manual workers.[1]

This paper tries to provide an alternative explanation for rising wage inequality. In particular, could the widespread use of the Internet over the last two decades have contributed to rising wage inequality? For this purpose, this paper develops a labor search/matching model with heterogeneous workers and heterogeneous firms. Given that in the real world workers are of different abilities and firms have different productivities, who matches with whom would naturally affect wage inequality. One novel feature of our model is that search is targeted: workers are able to choose beforehand which types of jobs to search (which submarket to participate in), but search is random within each submarket. Our model generates endogenous segmentation of workers, with higher ability workers searching for (weakly) more productive jobs. The segmentation pattern affects the wage structure or wage distribution in the market. We then explore how the segmentation pattern and wage inequality change as some exogenous

[1] Other explanations for rising wage inequality are deunionization, falling real minimum wages, and changes in the composition of the labor force (DiNardo et al., 1996; Lemieux, 2006).

shocks occur, including SBTC and an Internet-induced increase in search efficiency. One of our main results is that an Internet-induced increase in search efficiency would make the overall matching pattern more assortative and increase wage inequality.

The main ingredients of the model are as follows. Workers have different abilities or types, and firms have different productivities. While firms' types are finite, workers' types are continuous. Each firm demands exactly one worker. When a worker matches with a firm, the output is supermodular in the worker's type and the firm's type, except for the lowest type firms, where the output does not depend on workers' types. The model is set in continuous time. Unemployed workers search for job vacancies. Search is neither completely random, nor completely directed. It lies somewhere in between (partially random and partially directed), and we label it as targeted search. In particular, each type of firm constitutes a distinctive submarket. Workers can choose which submarket to enter (or which type of firms to target). However, within each submarket, given the set of worker types participating in that submarket, search is random.

We assume an urn ball matching technology. That is, each type of worker always has the same contact rate in any submarket. However, the contact rate of unfilled job vacancies in a submarket depends on market tightness in that submarket, which in turn depends on workers' participation decisions. Once a worker meets a firm, the worker's type is immediately observed, and the firm decides whether to hire the worker. If the firm decides to hire, the wage is determined by Nash bargaining: split the surplus in the current match relative to both parties' continuation values.

The key driving force of the model is the indirect externalities that workers within the same submarket impose on each other. The first kind of externality occurs among all workers. In a particular submarket, more participating workers loosen market tightness, which increases firms' contact rate and their continuation value. Due to Nash bargaining, this reduces the wages for all types of workers participating in that submarket. The second kind of externality is the one imposed by workers of high types on low type workers. Since search is random within each submarket, Nash bargaining implies that firms' continuation value corresponds to workers' average type. The presence of high type workers in a submarket increases the average type. This increases firms' continuation value, which, again due to Nash bargaining, reduces the wages of the lower types in that submarket.

We establish that, given parameter values, there is always a unique market equilibrium. And the equilibrium features are as follows. First, workers are endogenously segmented (self-selected) into different submarkets. The endogenous segmentation exhibits weakly positively assortative matching: workers of higher types participate in weakly more productive submarkets. The marginal types are indifferent between (get the same wage in) two adjacent submarkets. This is possible mainly due to the second kind of externality mentioned earlier: being the lowest type in a higher submarket reduces the worker's share of the output, although the higher submarket is more productive. The second equilibrium feature is that all submarkets are interdependent. This is because the wages in any submarket depend on the set of participating workers or marginal types of workers, but who are the marginal types in turn depends on the market conditions in adjacent submarkets. The final equilibrium feature is that the wage schedule (as a function of worker types) is piecewise linear and continuous: it is linear and increasing in type within each submarket, but is steeper in more productive submarkets.

We then conduct comparative statics. We first consider the impacts of an SBTC. When the most productive jobs become more productive, in the most productive submarket all wages increase and the wage schedule becomes steeper. However, it also induces changes in the endogenous segmentation: all the cutoff types decrease or overall matching becomes less assortative (more workers in the higher submarkets). As a result, wages of all types of workers (in all submarkets) increase. However, the wage increases are bigger for higher types. Therefore, an SBTC in the most productive submarket not only increases overall wage inequality across submarkets, but also increases wage inequality within each submarket (among workers having similar jobs). However, our five-submarket example (high-tech, medium high-tech, medium, medium low-tech, and low-tech service jobs) shows that quantitatively the spillover to the four lower submarkets is negligible.

We then consider the case that some lower submarket loses jobs. This can be caused by a negative trade shock or computerization. Compared to the initial equilibrium, in the new equilibrium segmentation all the cutoff types in the submarkets higher than the one in question decrease, while all the cutoff types in the lower submarkets increase. Wages of all types decrease, and the wage decreases are bigger in submarkets that are closer to the submarket where the negative shock occurs. In a five-submarket example, if the negative shock occurs in the medium low-tech sector, then it will increase wage inequality in the upper tail (the three higher submarkets), but reduces wage inequality in the lower tail (the two lower submarkets), producing a pattern of wage polarization.

When a shock occurs in a middle submarket, we find that, quantitatively, the transmissions of the shock to other submarkets through the changes in the endogenous segmentation exhibit asymmetry. Specifically, the transmission of a shock to the higher submarkets is always more significant than the transmission to the lower submarkets: the wage adjustments in higher submarkets are bigger while the wage adjustments in lower submarkets are smaller. For instance, if an SBTC occurs in the medium high-tech sector, then the wage increases in the two highest submarkets are significant, while the wage increases in the three lowest submarkets are negligible.

A combination of an SBTC in the high-tech sector and a negative shock in the medium low-tech sector can generate the following pattern. Wages in the highest submarket increase, but the wages in the other four submarkets all decrease. Moreover, it generates a pattern of wage polarization: in the upper tail wage inequality increases, while in the lower tail wage inequality decreases.

Finally, we study the impacts of a decrease in search friction. The widespread use of the

Internet not only reduces workers' search costs, but also reduces the probability of a bad match in the horizontal dimension (elaborated in Section 6). In terms of modeling, both effects lead to an increase in workers' effective contact rate. Unlike an SBTC or a trade shock, this is a universal shock and is skill-neutral: the contact rate of any type of worker in any submarket increases by the same amount. To the best of our knowledge, the current paper is the first to study the impacts of the Internet on wage inequality in a search/matching framework.

As the search efficiency/contact rate increases, if the initial equilibrium segmentation remains the same, then in each submarket (except for the lowest one) the wages of higher type workers increase, while those of lower types decrease, and the wage schedule becomes steeper. This is because an increase in the contact rate directly increases each worker's continuation value. However, it also increases firms' continuation value, which is roughly parallel to the average type workers' continuation value. Due to Nash bargaining, in each submarket higher types gain and lower types lose. This pushes the lowest types in each submarket to the adjacent lower submarket.

Therefore, compared to the initial equilibrium, in the new equilibrium segmentation all the cutoff types increase. In other words, the matching pattern becomes more assortative. This is different from the impacts of an SBTC in the most productive submarket, under which more workers will participate in the higher submarkets. In the new equilibrium, the wage schedule in each submarket (except for the least productive one) becomes steeper. This means that wage inequality within each submarket increases. Moreover, the wages of the highest types improve, while the wages of the lowest types decrease. Thus, the wage inequality across submarkets increases as well. This shows that the widespread use of the Internet in the last two decades could have contributed to rising wage inequality. Moreover, our five-submarket example shows that it can generate a pattern of wage polarization: in the upper tail the wages of the higher types increase while those of the lower types decrease, and in the lower tail all wages decrease but the wage decreases of the higher types are more significant.

The rest of the paper is organized as follows. In the next subsection we discuss the related literature. Section 2 sets up the model. In Section 3 we characterize the market equilibrium. Section 4 studies the socially efficient segmentation. In Sections 5 and 6 we conduct comparative statics: how the equilibrium matching pattern and wage schedule change when some shocks occur. Section 7 offers conclusion and discussion. All the missing proofs in the text can be found in the Appendix.

## 1.1 Related literature

In explaining rising wage inequality, the models of SBTC (see the references mentioned earlier) typically treat different labor market sectors separately. As a result, they do not capture how a shock in one particular sector is transmitted to other sectors. By endogenizing the market segmentation in a search/matching framework, this paper captures the general equilibrium effects of sector-specific shocks: a shock in a particular submarket will be transmitted to other submarkets through the adjustment in the endogenous segmentation.

There have been two search protocols in the labor search/matching literature.[2] The first one is random search, pioneered by Diamond (1982), Mortensen (1982), and Pissarides (1990). The second one is directed search, initially proposed by Peters (1991) and Montgomery (1991). In these early models, at least one side of the market is assumed to be homogeneous. Later models study the situation in which both workers and firms are heterogeneous. In particular, Shimer and Smith (2000) introduce search friction into Becker's (1973) classical paper on matching/assignment. In their model, both workers and firms have a continuum of types, and search is completely random. Their focus is on the form of the match-value function which ensures positively assortative matching, which turns out has to be log-supermodular. In a directed search model, Eeckhout and Kircher (2010) show that, by allowing firms to post prices beforehand to guide search, to ensure positively assortative matching the match-value only needs to be root-supermodular (also see Shi, 2001).

More related to our paper is Shimer (2005), who develops a one-period directed search model with workers' and firms' types both being finite. In equilibrium, each worker type plays mixed strategies (applying to several types of jobs with positive probabilities). The main difference between Shimer (2005) (and the directed search literature in general) and the current paper is that in his paper the coordination/congestion friction in the application process plays a central role. In our model, there is no direct coordination/congestion friction in the search process, but workers within the same submarket impose indirect externalities on each other through Nash bargaining. In spirit, our model resembles the models of competitive search equilibrium (Shimer, 1996; Moen, 1997), in which firms of different productivities constitute distinctive submarkets. The difference is that they are models of directed search (firms post wages beforehand), and workers are homogenous.

In the labor search/matching literature, the most closely related papers to our paper are Albrecht and Vroman (2002) and Shi (2002), as they both study how SBTC affects the matching pattern and wage inequality. Both papers study a setting with two types of workers and two types of firms.[3] In contrast, in our model workers' types are continuous and firms have a finite number of types, which leads to a richer wage structure and allows us to map predictions more closely to empirical facts.[4] In Albrecht and Vroman (2002), search is completely random, and high-skill jobs can only be performed by high-skill workers. Shi (2002) is a one-period directed search model, in which firms post wages beforehand to direct workers' application process. As mentioned earlier, the search/matching process in our model is neither completely random

[2] See Rogerson et al. (2005) for a survey.

[3] In a related paper, Acemogolu (1999) shows that an increase in the number of skilled workers might induce firms to switch from offering "middling" jobs to offering specialized jobs. In his model, search is random and there are only two types of workers.

[4] In Shi (2002), high type workers only apply for high-tech jobs, while low type workers apply to both high-tech and low-tech jobs with positive probabilities. As a result, there are three different wages in total. In Albrecht and Vroman (2002), low type workers only match with low-tech jobs, while high type workers match with both types of jobs. Again in total there are only three different wages.

nor completely directed, but is rather targeted. In later sections, we will further compare the differences in predictions between our model and their models.

One may wonder why we introduce a new search protocol of targeted search. The reasons are twofold. First, compared to random search, targeted search is more realistic. In the real world, we do see workers of different skills/abilities target different sets of jobs. For instance, the jobs applied to by a PhD in economics typically do not overlap with those applied to by a high school dropout. Second, when both workers and firms are heterogeneous, targeted search turns out to be more tractable than random search and directed search. In directed search models, firms offer type-dependent wages beforehand, and workers play mixed strategies. For each worker type, one needs to figure out the set of types of firms the worker applies to and the corresponding mixing probabilities. Moreover, workers' application strategies and firms wage offers interact with each other, and workers' application strategies impose direct congestion externalities on each other. Therefore, finding the equilibrium is a complicated process, let alone doing comparative statics.[5] Compared to random search, perhaps it is surprising that targeted search is more tractable. In random search models with heterogeneous workers and heterogeneous firms, for each worker type one needs to trace his acceptance set in terms of firm types, and for each firm type one needs to trace the acceptance set in terms of worker types. Moreover, these acceptance sets interact with each other as they affect both sides' continuation values. As a result, it is very hard to fully characterize the equilibrium wage schedule and carry out comparative statics.

Mortensen and Pissarides (1999a) also study how a skill-biased shock affects unemployment and wage inequality in a search/matching framework with heterogeneous workers and heterogeneous firms. However, they assume that search is perfectly directed or matching is perfectly assortative: workers with a particular skill level only search in a submarket where firms have the corresponding skill requirement. Their main focus is on how different labor market policies across countries will lead to different responses to a skill-biased shock. In a model with ex ante homogeneous workers but heterogeneous firms, Menzio and Shi (2010a) consider perfectly directed search: on the firm side there is a continuum of submarkets, each indexed by the expected lifetime utility offered to workers. Their main focus is to develop a tractable model (they call their equilibrium block recursive equilibrium), which allows for on-the-job search, shocks to productivities, and worker-firm match-specific values, to study non-steady-state dynamics.[6]

In a search/matching model in the marriage market with non-transferable utilities and

[5] The complication is illustrated by Shimer (2005), and probably that is the reason why in his model both workers' and firms' types are finite. Moreover, the mixed strategy equilibrium in directed search models might generate some features that are not very realistic. For instance, in Shimer (2005) the lowest type workers might apply for the highest type job with a positive probability. In Shi (2002), in high-tech firms the low type workers might get a higher actual wage than the high type workers.

[6] In a related paper, Menzio and Shi (2010b) study the block recursive equilibrium in a model with ex ante heterogeneous workers but homegeneous firms.

random search, Burdett and Coles (1997) show that the equilibrium exhibits block matching (weakly positively assortative). This feature is shared by the equilibrium matching pattern in our model. Jacquet and Tan (2007) extend Burdett and Coles (1997) by allowing agents to choose with whom to meet. That is, men and women are free to create submarkets. The feature that agents can choose with whom to meet is related to the targeted search in our model. The difference is that in our targeted search the submarkets are exogenously fixed (defined by firm types), and only workers choose which type of firms to meet with.[7] In a sequel, Xu and Yang (2016) study a search/matching model in a marriage market with targeted search, in which men and women are not only vertically differentiated but also horizontally differentiated.

In a labor market setting, a recent working paper by Cheremukhin et al. (2014) also proposes a search protocol of targeted search. While the terminology is the same, the targeted search in their model is very different from the one proposed in the current paper. In particular, their model emphasizes that agents have finite information processing capacity, which limits agents' abilities to target their search to the best possible matches. The focuses of the two papers are also very different. While their paper focuses on the quality of the match, the current paper focuses on the matching pattern in the vertical dimension and wage inequality. In the context of consumer search, Yang (2013) develops a targeted search model. In particular, he models targeted search as the probability that a consumer type encounters the relevant goods in each search.[8]

## 2 Model

The model is set in continuous time, with $r$ being the common discount rate.

Consider a labor market with heterogeneous workers and heterogeneous firms. The measure of workers is normalized to 1. Workers' abilities or skill levels (types) are indexed by $x$. The support of $x$ is $[\underline{x}, \overline{x}]$, and its cumulative distribution function and density function are $F(x)$ and $f(x)$, respectively. We assume $f(x) > 0$ and it is continuous for all $x \in [\underline{x}, \overline{x}]$. There are $N + 1$ types of firms with different productivities, indexed by $\{0, 1, ..., N\}$. We assume $N$ is finite. A type $n \geq 1$ firm's productivity is $\theta_n$, with $1 \leq \theta_1 < ... < \theta_N$. The measure of type $n$ firms is $Y_n$; $\{Y_n\}$ are exogenously fixed, and $0 < Y_n < 1$ for all $n$.[9] Each firm has exactly one job. If a type $x$ worker is matched with a type $n$ ($n \geq 1$) firm, then the flow output of the match is $\theta_n x$. Note that the production function is supermodular (for $n \geq 1$), which means that from an efficiency point of view a higher type worker should be matched with a higher

[7] Another difference is that in their model in each submarket agents' contact rate do not depend on the tightness of that submarket, while in our model firms' contact rate depends on the tightness of the corresponding submarket.

[8] In a consumer search model, Lester (2011) introduces semi-directed search: the informed consumers' search is directed by the prices offered by firms, while uninformed consumers search randomly. See also Bethune et al. (2016).

[9] In Section 7 we will discuss how free entry affects the main results of the model.

type firm. If a type $x$ worker is matched with a type 0 firm, then the flow output is always 1 regardless of the worker's type. One can think of type 0 jobs as those only requiring basic skills (for instance, low-tech service jobs).

Unemployed workers actively search for unfilled job vacancies.[10] The search is not random. In particular, the labor market is segmented into $N+1$ submarkets, with each type of firms constituting a distinctive submarket. The identity of each submarket (and hence firms' types) is always publicly observable. Each worker can choose which submarket or submarkets to participate in. If a worker chooses to participate in several submarkets, then he has to allocate his search efforts across these submarkets. This setup resembles Moen's (1997) directed search model, in which firms of different productivities constitute different submarkets. In this setup, the number of submarkets is exogenously fixed, the set of firms in each submarket is also exogenously fixed, but the set of workers participating in each submarket is endogenously determined.[11][12]

Each worker will only search in the submarkets which give him the highest expected utility. As will be shown later, generically, for any type $x$ worker such a submarket is unique, and thus he will only search in one submarket. Let $G(x_n)$ and $u_n$ be the type distribution of worker types and the measure of unemployed workers, respectively, in submarket $n$. Denote $v_n$ as the measure of unfilled vacancies in submarket $n$. Let $q_n \equiv u_n/v_n$ be the expected queue length in submarket $n$, which is the inverse of market tightness. We assume that the matching function is generated by an urn ball technology. In continuous time, at any instant of time the number of meetings in submarket $n$ is $m(u_n, v_n) = \alpha u_n$, where $\alpha$ indicates the exogenous search intensity of workers (Mortensen and Pissarides, 1999b, p. 2575-2576).[13] Thus, the contact rate of a worker is always $\alpha$ (in any submarket, as it is independent of market tightness in any submarket). On the other hand, the contact rate of a type $n$ firm is $m(u_n, v_n)/v_n = \alpha q_n$. Note that an increase in $q_n$ leads to a higher contact rate for type $n$ firms. Finally, if a type $n$ firm meets a worker, the worker's type is a random draw from $G(x_n)$. This is because all type $n$ firms are symmetric and all workers in submarket $n$ have the same contact rate. In other words, search is random within each submarket.[14]

[10] Thus search is one-sided.

[11] This feature will be further discussed in Section 7.

[12] In our model there is no on-job search. Actually, in equilibrium workers have no incentive to engage in on-job search, as firms are homogeneous within the same submarket.

[13] Specifically, in per period length $dt$ the total number of vacancies who have at least one contact from workers is $v_n(1-e^{-\alpha \frac{u_n}{v_n} dt})$. Taking the limit, we get

$$m(u_n, v_n) = \lim_{dt \to 0} \frac{v_n(1-e^{-\alpha \frac{u_n}{v_n} dt})}{dt} = v_n \alpha \frac{u_n}{v_n} = \alpha u_n.$$

[14] Suppose a worker participates in several submarkets. Since each worker has a fixed search intensity, he has to spread his search effort/time across the submarkets. Let $\sigma_n$, $\sum_n \sigma_n = 1$, be the fraction of effort/time he spends in submarket $n$. Then his contact rate in submarket $n$ is $\alpha\sigma_n$. Note that $\sigma \equiv \{\sigma_n\}$ can also be interpreted as the worker's randomization probabilities.

Once an unfilled vacancy and an unemployed worker meet, the firm immediately observes the worker's type. Then the worker and the firm bargain for the wage and decide whether to match. The firm might reject the worker if the worker's type $x$ is too low. If both agree to match, then the match is consummated and they leave the market. Denote $w_n(x)$ as the wage paid to the worker in a match between a type $x$ worker and a type $n$ firm. We assume that the wage $w_n(x)$ is determined by Nash bargaining, with the worker's share of surplus being $\beta$. Specifically, denote $U_n(x)$ as the expected discounted utility of a type $x$ unemployed worker searching in submarket $n$, and $E_n(x)$ as the expected discounted utility of a type $x$ worker currently matched with a type $n$ firm. Similarly, denote $V_n$ as the expected discounted profit of an unfilled type $n$ vacancy, and $J_n(x)$ as the expected discounted profit of a type $n$ firm who is currently matched with a type $x$ worker. Nash bargaining requires that the wage $w_n(x)$ be implicitly determined by

$$E_n(x) - U_n(x) = \beta[E_n(x) + J_n(x) - U_n(x) - V_n]. \tag{1}$$

All existing matches, regardless of workers' types and firms' types, have the same exogenous separation rate $\delta$. All unemployed workers get the same flow unemployment benefit $b$. We assume that $b < 1$, which implies that it is efficient for all workers to be employed (even by the least productive jobs). Finally, each worker holds rational expectations as to whether he will be accepted by firms and the wages he will get in each submarket.

Two remarks are in order. First, the urn ball meeting technology implies that there is no direct coordination/congestion friction on worker's search: each worker's contact rate does not depend on market tightness in each submarket. This is different from the directed search literature, where the direct coordination/congestion friction plays a central role.[15] Second, search is not random as workers are able to choose in which submarket to participate beforehand. However, search is not completely directed either for two reasons. First, firms do not post wages beforehand to direct/guide search. Second, conditional on meeting a firm, within the same submarket a worker of a higher type does not have a higher probability of getting hired than a lower type worker, as long as firms in that submarket accept both workers. This is because, in continuous time, at any instant the probability that an unfilled vacancy gets two contacts from two workers, relative to the probability that an unfilled vacancy gets one contact, is negligible.[16] Loosely put, in the current model, search is directed across submarkets, but random within each submarket. To distinguish this from random search and directed search, we label our search protocol *targeted search.*

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[15] In Section 7 we offer a discussion regarding more general matching technologies.

[16] This is different from Shi's (2002) one-period directed search model. In his model, a high-tech job might get multiple applications from different types of workers, and thus the high type applicant has priority in getting employed.

# 3 Market Equilibrium

## 3.1 Preliminary analysis

A worker's strategy is to choose in which submarket to participate, which is a mapping from his type $x$ to the set of submarkets. A strategy profile of all workers is equivalent to a segmentation of workers' type space into different submarkets. We denote a segmentation as $\overline{P} : [\underline{x}, \overline{x}] \rightarrow \{0, 1, ..., N\}$. Let $x_n$ be the set of worker types who participate in submarket $n$. Thus, $\{x_n\}_{n=0}^{N}$, which exhausts the type space $[\underline{x}, \overline{x}]$, also represents a segmentation. Let $X_n$ be the measure of $x_n$. The distribution of worker types within $x_n$, $G(x_n)$, can be derived correspondingly from $F(x)$.

Given a segmentation $\overline{P}$, in each submarket $n$ the market conditions, $x_n$, $X_n$, and $G(x_n)$, are all determined. In the ensuing search/matching process, along with Nash bargaining, these market conditions determine wages in submarket $n$. Given the market conditions in submarket $n$, a type $n$ firm's strategy is a decision rule as to whether to accept a type $x$, $x \in x_n$, worker if they meet. We first investigate the determination of wages $w_n(x)$ in each individual submarket $n$, given market conditions $x_n$, $X_n$, and $G(x_n)$. In the process, we assume that type $n$ firms accept all worker types $x \in x_n$, as it will be shown to be an equilibrium feature later.[17]

Let $m_n$ be the measure of matched workers (firms as well) in submarket $n$. Then, we have the following equations

$$m_n + u_n = X_n; \quad m_n + v_n = Y_n; \quad \alpha u_n = \delta m_n.$$

In particular, the third equation is the steady state condition: the number of newly formed matches equals to that of destroyed matches. Given $X_n$, the above three equations uniquely determine the steady state $u_n$, $v_n$, and $q_n$:

$$\begin{aligned} u_n &= \frac{X_n}{\frac{\alpha}{\delta} + 1}, \quad v_n = Y_n - \frac{\frac{\alpha}{\delta} X_n}{\frac{\alpha}{\delta} + 1}; \\ q_n &= \frac{X_n}{(\frac{\alpha}{\delta} + 1) Y_n - \frac{\alpha}{\delta} X_n}. \end{aligned} \tag{2}$$

Note that since all workers active in the same submarket, regardless of their types, have the same matching rate, and all existing matches have the same break-up rate, the distribution of worker types in the unmatched pool must be the same as the distribution of worker types active in submarket $n$. That is, both are $G(x_n)$.

In submarket $n \geq 1$, a type $x$ worker's value functions are given by

$$\begin{aligned} rU_n(x) &= b + \alpha[E_n(x) - U_n(x)], \\ rE_n(x) &= w_n(x) + \delta[U_n(x) - E_n(x)]. \end{aligned}$$

[17] Once a worker has chosen a submarket, say $n$, in the search/matching stage he should accept any type $n$ firm if they meet. If this were not the case, then that worker would have chosen a different submarket in the targeting stage.

In the equation for $U_n(x)$, $b$ is the flow payoff when unemployed. With rate $\alpha$ the worker has a successful match (recall we assume type $n$ firms accept all workers active in submarket $n$), and in that case the worker enjoys an increase in value $E_n(x) - U_n(x)$. In the equation $E_n(x)$, the worker's flow payoff is $w_n(x)$. And with rate $\delta$ the current match is destroyed, and in that case the worker suffers a loss $U_n(x) - E_n(x)$.

By similar logic, type $n \geq 1$ firms' value functions are given by

$$\begin{aligned} rV_n &= \alpha q_n \{E_{x_n}[J_n(x)] - V_n\}, \\ rJ_n(x) &= (\theta_n x - w_n(x)) + \delta[V_n - J_n(x)]. \end{aligned}$$

In the equation for $V_n$, a firm's successful match rate is $\alpha q_n$. Note that type $n$ firms' optimal strategy regarding whether to accept a worker is: accept a type $x$ worker if and only if $J_n(x) \geq V_n$.

Submarket 0 is a little special in that the output does not depend on workers' types. Thus, type 0 firms always accept all types of workers. We can simplify the value functions as

$$\begin{aligned} rU_0 &= b + \alpha(E_0 - U_0), \quad rE_0 = w_0 + \delta(U_0 - E_0), \\ rV_0 &= \alpha q_0 (J_0 - V_0), \quad rJ_0 = (1 - w_0) + \delta(V_0 - J_0). \end{aligned}$$

Applying the Nash bargaining equation (1), we have $E_0 - U_0 = \beta(E_0 + J_0 - U_0 - V_0)$. From these equations, we get

$$w_0 = \frac{(1-\beta)(r + \delta + \alpha q_0)b + \beta(r + \delta + \alpha)}{r + \delta + \beta\alpha + (1-\beta)\alpha q_0}, \tag{3}$$

which is uniquely determined given $q_0$. From the expression of $w_0$ (3), it can be immediately verified that $w_0 > b$. This is because the output of a type 0 job is 1, which is bigger than $b$, and workers can always get a fraction of the surplus $1 - b$ due to Nash bargaining.

Manipulating the value functions and the Nash bargaining equation (1), we get

$$rU_n(x) = b + \frac{\alpha[w_n(x) - b]}{r + \delta + \alpha}, \tag{4}$$

$$w_n(x) = \frac{\beta(r + \delta + \alpha)(\theta_n x - rV_n) + (1-\beta)(r + \delta)b}{r + \delta + \beta\alpha}. \tag{5}$$

By equation (4), for any given type $x$ worker, the submarket $n$ with the highest $w_n(x)$ also gives him the highest expected lifetime utility. This is because the matching rate for any worker is the same across all submarkets. Therefore, a type $x$ worker will participate in the submarket that gives him the highest wage $w_n(x)$. Observing equation (5), we see that $w_n(x)$ is proportional to $\theta_n x - rV_n$, which represents the surplus created by a type $x$ worker in the current match ($\theta_n x$ is the output, and $rV_n$ is type $n$ firms' per period value or bargaining position).

Now we provide a definition of equilibrium in this economy, which we call market equilibrium.

**Definition 1** *A market equilibrium is characterized by a segmentation* $\overline{P}$ *or* $\{x_n\}$ *which satisfies the following requirements. (i) Each type* $x$ *worker,* $x \in x_n$*, should have no incentive to unilaterally deviate to participating in a submarket different from* $n$*: for any* $n$ *and any* $x \in x_n$*,* $w_n(x) \geq w_{n'}(x)$ *for any* $n' \neq n$*. (ii) For any* $n$ *and any* $x \in x_n$*, type* $n$ *firms have an incentive to accept a worker of type* $x$*:* $J_n(x) \geq V_n$*.*

Given that the number of firm types is finite and workers' types are continuous, matching must be mixed: some submarkets must have heterogeneous worker types. We show that the equilibrium matching pattern is still weakly positively assortative, meaning that higher type workers participate in weakly higher (or more productive) submarkets.

**Lemma 1** *Consider two worker types* $x'$ *and* $x''$*,* $x' < x''$*, and two submarkets* $n'$ *and* $n''$*,* $n' < n''$*. In any market equilibrium, (i) if a type* $x'$ *worker prefers submarket* $n''$ *to submarket* $n'$*, then a type* $x''$ *worker must strictly prefer submarket* $n''$ *to submarket* $n'$*; (ii) if a type* $x''$ *worker prefers submarket* $n'$ *to submarket* $n''$*, then a type* $x'$ *worker must strictly prefer submarket* $n'$ *to submarket* $n''$*.*

**Proof.** We only prove part (i), as the proof of part (ii) is similar. The fact that a type $x'$ worker prefers submarket $n''$ to submarket $n'$ means that $w_{n''}(x') \geq w_{n'}(x')$. By the wage equation (5), it implies that $\theta_{n''}x' - rV_{n''} \geq \theta_{n'}x' - rV_{n'}$, which is equivalent to $(\theta_{n''} - \theta_{n'})x' \geq rV_{n''} - rV_{n'}$. Using the fact that $x'' > x'$, we have $(\theta_{n''} - \theta_{n'})x'' > rV_{n''} - rV_{n'}$, which by the wage equation (5) implies that $w_{n''}(x'') > w_{n'}(x'')$. That is, a type $x''$ worker must strictly prefer submarket $n''$ to submarket $n'$. ■

Lemma 1 establishes a single-crossing property: in equilibrium a higher type worker must match with a weakly higher type firm. This result is formally stated in the following proposition.

**Proposition 1** *In any market equilibrium the segmentation of worker types must be an interval partition: the type space* $[\underline{x}, \overline{x}]$ *is partitioned into* $N+1$ *(maybe fewer) connected intervals, with higher type workers participating in weakly higher submarkets.*

Thus equilibrium exhibits weakly positively assortative matching. This result is quite intuitive. Given Nash bargaining, a worker always gets a certain share of the surplus created. Since the production technology is supermodular, the difference of the surplus created between any pair of types of jobs is always bigger for a higher type worker. Moreover, the bargaining position ($V_n$) of any type $n$ firms is fixed when bargaining with workers (deviation of any individual worker type, which is of measure 0, will not affect $q_n$, $E_{x_n}[x]$, or $V_n$). Therefore, the wage difference between any pair of types of jobs is always bigger for a higher type worker, which naturally leads to weakly positively assortative matching. Proposition 1 also implies that there are at most $N$ types of workers who are indifferent between participating in two adjacent submarkets. For a generic worker type $x$, there is a unique submarket that he strictly prefers.

By Proposition 1, an equilibrium segmentation is characterized by a weakly increasing sequence of cutoff types $\{\widehat{x}_n\}$ (we adopt the convention that $\underline{x} = \widehat{x}_0$ and $\overline{x} = \widehat{x}_{N+1}$), $\widehat{x}_0 \leq \widehat{x}_1 ... \leq \widehat{x}_{N+1}$, such that a worker of type $x$ chooses submarket $n$ if and only if $x \in [\widehat{x}_n, \widehat{x}_{n+1}]$. Moreover, the density function of $x_n$ is given by $g(x_n) = \frac{f(x_n)}{F(\widehat{x}_{n+1})-F(\widehat{x}_n)}$, and the measure of $x_n$ is given by $X_n = F(\widehat{x}_{n+1}) - F(\widehat{x}_n)$.

Now we explicitly solve for $w_n(x)$ and $V_n$, still assuming type $n$ firms always accept any worker type $x \in [\widehat{x}_n, \widehat{x}_{n+1}]$. From the value functions and the Nash bargaining equation (1), we have

$$(r+\delta+\alpha)w_n(x) = (1-\beta)(r+\delta)b + \beta(r+\delta+\beta\alpha)\{\theta_n x - \frac{\alpha q_n}{r+\delta+\alpha q_n} E_{x_n}[\theta_n x - w_n(x)]\}.$$

Taking the expectation of $E_{x_n}[\cdot]$ on both sides of the above equation, we get

$$E_{x_n}[w_n(x)] = \frac{(1-\beta)(r+\delta+\alpha q_n)b + \beta(r+\delta+\alpha)\theta_n E_{x_n}[x]}{r+\delta+\beta\alpha+(1-\beta)\alpha q_n}.$$

From the above equations, $w_n(x)$ can be explicitly solved as

$$w_n(x) = \frac{(1-\beta)(r+\delta+\alpha q_n)b + \beta(r+\delta+\alpha)\theta_n\{x + \frac{(1-\beta)\alpha q_n}{r+\delta+\beta\alpha}[x - E_{x_n}[x]]\}}{r+\delta+\beta\alpha+(1-\beta)\alpha q_n}. \tag{6}$$

Sometimes, it is useful to write $w_n(x)$ as

$$w_n(x) = \frac{(1-\beta)(r+\delta+\alpha q_n)b - (1-\beta)\alpha q_n \frac{\beta(r+\delta+\alpha)\theta_n E_{x_n}[x]}{r+\delta+\beta\alpha}}{r+\delta+\beta\alpha+(1-\beta)\alpha q_n} + \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha}\theta_n x \tag{7}$$

Similarly, $V_n$ can be explicitly derived as:

$$rV_n = \frac{(1-\beta)\alpha q_n[\theta_n E_{x_n}[x] - b]}{r+\delta+\beta\alpha+(1-\beta)\alpha q_n}. \tag{8}$$

Several observations are in order. First, inspecting (8) we see that $V_n$ depends on $E_{x_n}[x]$, the average ability of workers active in submarket $n$. This is intuitive, as Nash bargaining implies that a type $n$ firm gets a fraction of the expected surplus created by matching with a random worker, which is precisely the average type in expectation. Second, it can be readily verified that $\frac{\partial V_n}{\partial q_n} > 0$ and $V_n$ converges to 0 as $q_n$ approaches 0. This is also intuitive, since an increase in $q_n$ means that firms' matching rate increases, which improves firms' position. When $q_n$ approaches 0, type $n$ firms' matching rate also approaches 0, and as a result $V_n$ tends to 0. Third, by (7), we can immediately see that $w_n(x)$ is linear, and thus increasing, in $x$. Actually, the first term in (7) determines the intercept of the wage schedule $w_n(x)$, while the second term determines the slope. Specifically, the slope of the wage schedule $w_n(x)$ is

$$\frac{\partial w_n(x)}{\partial x} = \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha}\theta_n,$$

which is increasing in $\theta_n$. Thus, the wage schedule $w_n(x)$ is steeper in a higher submarket. This feature is a direct consequence of the supermodular output function and Nash bargaining. Finally, $J_n(x)$ is increasing in $x$. To see this, by (1) we can write $J_n(x)$ as

$$J_n(x) = \frac{1-\beta}{\beta}\frac{[w_n(x)-b]}{r+\delta+\alpha} + V_n. \tag{9}$$

Since $w_n(x)$ is increasing in $x$, so is $J_n(x)$. Again, this feature is mainly due to Nash bargaining.

**Lemma 2** *(i) In each submarket $n$, $w_n(x)$ is decreasing in $q_n$. (ii) In each submarket $n \geq 1$, $w_n(x)$ is decreasing in $E_{x_n}[x]$.*

**Proof.** Both results follow immediately from (5) and the facts that $\frac{\partial V_n}{\partial q_n} > 0$ and $V_n$ is increasing in $E_{x_n}[x]$. ■

The results of Lemma 2 are quite intuitive. An increase in $q_n$ or $E_{x_n}[x]$ increases firms' bargaining position $V_n$, which through Nash bargaining reduces wages. Lemma 2 implies that within the same submarket workers impose indirect negative externalities on each other by changing firms' bargaining position (recall that workers do not impose direct congestion/coordination externalities on each other, as the contact rate $\alpha$ is independent of market tightness $1/q_n$). In particular, there are two kinds of externalities. The first kind of externality occurs through $q_n$, and thus exists among all workers. More workers active in a submarket increases firms' matching rate and hence their bargaining position, which through Nash bargaining reduces the wages of all workers active in that submarket. The second kind of externality occurs through the channel of $E_{x_n}[x]$, and thus is imposed by higher type workers on lower type workers. Specifically, the presence of higher type workers increases the average ability of workers, $E_{x_n}[x]$, in submarket $n$, which improves firms' bargaining position $V_n$ and reduces the wages of the lower type workers.[18]

The negative externalities identified in Lemma 2 are the key driving forces of our model. This is because they are critical in pinning down the equilibrium segmentation. In particular, because of the second kind of externality, a worker of a relatively lower type $x'$ in submarket $n$ might be better off switching to submarket $n-1$. To see this, note that in submarket $n$ this type suffers from the negative externality by being a lower type (a higher $V_n$, or $x' - E_{x_n}[x] < 0$). However, if this type participates in submarket $n-1$, it becomes a higher type, and it can gain from a lower $V_{n-1}$, since $x' - E_{x_{n-1}}[x] > 0$.

Note that $w_n(x)$ reaches its upper bound $\overline{w}_n(x)$ when $q_n = 0$ (type $x$ is the only worker type active in submarket $n$). By equation (5), $\overline{w}_n(x)$ can be computed as:

$$\begin{aligned}
\overline{w}_n(x) &= \frac{\beta(r+\delta+\alpha)\theta_n x + (1-\beta)(r+\delta)b}{r+\delta+\beta\alpha}, \\
\overline{w}_0 &= \frac{\beta(r+\delta+\alpha) + (1-\beta)(r+\delta)b}{r+\delta+\beta\alpha}.
\end{aligned}$$

[18] Note that as the set of workers active in submarket $n$ changes, both $q_n$ and $E_{x_n}[x]$ will change. However, we can distinguish these two kinds of externalities, conceptually.

It can be readily verified that $\overline{w}_n(x)$ is increasing in $n$ and $x$.

To Summarize, a market equilibrium in this economy is characterized by the cutoff types $\{\widehat{x}_n\}_{n=1}^N$. Given the cutoffs $\{\widehat{x}_n\}_{n=1}^N$ or a segmentation, the measures and the distribution of worker types active in all submarkets, $\{X_n\}$ and $\{G(x_n)\}$, are determined. By the steady state equations, $\{u_n\}$, $\{v_n\}$, and $\{q_n\}$ are determined as well. Finally, from the value functions and the wage schedules, $\{w_n(x)\}$, $\{U_n(x)\}$, and $\{V_n\}$, are determined.

Let $\{x_n^*\}_{n=1}^N$ be the cutoff types in a market equilibrium. In light of the previous analysis, the two equilibrium conditions of market equilibrium can now be written as:

(i) The cutoff types of workers are indifferent between two adjacent submarkets: for all $n = 1, ..., N$,

$$\begin{aligned} w_n(x_n^*) &= w_{n-1}(x_n^*) \text{ if } x_n^* > \underline{x} \text{ (interior cutoff)}, \\ w_n(x_n^*) &\geq \overline{w}_{n-1}(x_n^*) \text{ if } x_n^* = \underline{x} \text{ (corner cutoff)}. \end{aligned} \tag{10}$$

(ii) In any submarket $n \geq 1$, no individual type $n$ firm can strictly increase its $V_n$ by accepting only a strict subset of worker types within $[x_n^*, x_{n+1}^*]$.

Equilibrium requirement (i) is enough for no deviation on the part of workers. This is because by Lemma 1, given that type $x_n^*$ is indifferent between submarkets $n$ and $n-1$, all workers with type $x > x_n^*$ must strictly prefer submarket $n$ to submarket $n-1$, and all workers with type $x < x_n^*$ must strictly prefer submarket $n-1$ to submarket $n$. Equilibrium requirement (ii) applies to the lower worker types in each submarket, as it may not be rewarding for firms if a worker's type is too far below the average type of the existing pool. However, the following lemma shows that equilibrium requirement (ii) is redundant if requirement (i) is satisfied.

**Lemma 3** *If equilibrium requirement (i) is satisfied, then equilibrium requirement (ii) is satisfied as well.*

The underlying reason for Lemma 3 is that workers can anticipate firms' hiring behavior (criterion). If a worker's type is too low compared to the average type of the existing pool in submarket $n$ such that type $n$ firms do not accept him, the worker will simply participate in some less productive submarkets beforehand. More precisely, a type $n$ firm is willing to accept a type $x$ worker as long as $\theta_n x \geq rV_n + b$ (the output in the current match is bigger than the sum of outside options). But when this condition is binding, the worker's wage is already pushed down to $b$, and the worker will switch to submarket 0 where he can get a wage $w_0 > b$. This exactly means that workers' equilibrium participation decisions are more stringent than firms' acceptance decisions, thus equilibrium requirement (i) implies equilibrium requirement (ii). From now on, we can safely drop equilibrium requirement (ii).

Before investigating market equilibrium more closely, we first present a useful lemma.

**Lemma 4** *(i) Consider a type $x'$ worker in submarket $n$, and two scenarios in which $V_n' \neq V_n''$. Denote $w_n'(x')$ and $w_n''(x')$ as type $x'$ worker's wage when type $n$ firms have $V_n'$ and $V_n''$,* *respectively. Then $w'_n(x') > w''_n(x')$ if and only if $V'_n < V''_n$, and vice versa. (ii) In submarket 0, $w_0$ is strictly decreasing, and $V_0$ is strictly increasing, in $\widehat{x}_1$. (iii) In submarket $N$, $V_N$ is decreasing in $\widehat{x}_N$; $V_N$ is strictly decreasing in $\widehat{x}_N$ if type $N$ firms accept type $\widehat{x}_N$. (iv) In submarket $n$, $0 < n < N$, consider two sets of participating worker types: $[\widehat{x}_n, \widehat{x}_{n+1}]$ and $[\widehat{x}'_n, \widehat{x}'_{n+1}]$. Denote type $n$ firms' equilibrium value in the first case and in the second case as $V_n$ and $V'_n$, respectively. If $[\widehat{x}_n, \widehat{x}_{n+1}] \sqsubseteq [\widehat{x}'_n, \widehat{x}'_{n+1}]$, then $V_n \leq V'_n$. If $\widehat{x}_{n+1} < \widehat{x}'_{n+1}$ and $\widehat{x}'_n \leq \widehat{x}_n$, then $V_n < V'_n$. If $\widehat{x}_{n+1} \leq \widehat{x}'_{n+1}$ and $\widehat{x}'_n < \widehat{x}_n$ and firms accept type $\widehat{x}'_n$ workers, then $V_n < V'_n$.*

The results of Lemma 4 are intuitive. In any submarket, due to Nash bargaining, an increase in firms' continuation value $V_n$ implies that any type $x$ worker's wage $w_n(x)$ decreases, and vice versa. In any submarket $n$, adding additional participating worker types will always weakly improve type $n$ firms' value $V_n$, since it potentially increases firms' matching rate, and gives them more options to choose among workers.

### 3.2 Existence and Uniqueness of Equilibrium

By equations (5) and (8), the indifference conditions of (10) can be explicitly written as: for all $n = 1, ..., N$, and $x^*_n > \underline{x}$,

$$(\theta_n - \theta_{n-1})x^*_n = \frac{(1-\beta)\alpha q_n[\theta_n E_{x_n}[x] - b]}{r + \delta + \beta\alpha + (1-\beta)\alpha q_n} - \frac{(1-\beta)\alpha q_{n-1}[\theta_n E_{x_{n-1}}[x] - b]}{r + \delta + \beta\alpha + (1-\beta)\alpha q_{n-1}}. \tag{11}$$

Note that both $q_n$ and $E_{x_n}[x]$ are functions of $x^*_n$ and $x^*_{n+1}$. Thus (11) is a second order nonlinear difference equation, with boundary conditions $x^*_0 = \underline{x}$ and $x^*_{N+1} = \overline{x}$. Since there is no general theorem regarding the existence and uniqueness of the solutions to second order nonlinear difference equations, we have to establish them by ourselves.

Our method is by induction. In particular, we consider (partial) equilibrium within a subset of submarkets. For an exogenously given $\widehat{x}_{n+1} \in (\underline{x}, \overline{x})$, we investigate the segmentation of worker types within $[\underline{x}, \widehat{x}_{n+1}]$ among submarkets $0, ..., n$. Define $x^*_{1,n+1}(\widehat{x}_{n+1}), ..., x^*_{n,n+1}(\widehat{x}_{n+1})$ as the $\{\widehat{x}_i\}_{i=1}^n$ such that all the submarkets below submarket $n$ (including submarket $n$) are in (partial) equilibrium, or the indifference conditions of (10) for all $i \leq n$ are satisfied.[19] To abuse notation, we sometimes simply denote $x^*_{i,n+1}(\widehat{x}_{n+1})$, $i = 1, ..., n$, as $x^*_i$. We start with $n = 1$.

**Partial equilibrium in submarkets 0 and 1** Consider partial equilibria in submarkets 0 and 1, given $\widehat{x}_2 \in (\underline{x}, \overline{x})$. We will show that for any $\widehat{x}_2$, $x^*_1$ exists and is unique.

Consider a segmentation with the cutoff type being $\widehat{x}_1 \in [\underline{x}, \widehat{x}_2]$, which we denote as $\overline{P}(\widehat{x}_1)$. There are three possible equilibria to consider. The first one is that $x^*_1 = \underline{x}$: all workers participate in submarket 1. In the second one, $x^*_1 \in (\underline{x}, \widehat{x}_2)$ is interior, meaning that each

[19] Similarly, for an exogenously given $\widehat{x}_n \in (\underline{x}, \overline{x})$, we can define the (partial) equilibrium segmentation of worker types within $[\widehat{x}_n, \overline{x}]$ among submarkets $n, ..., N$.

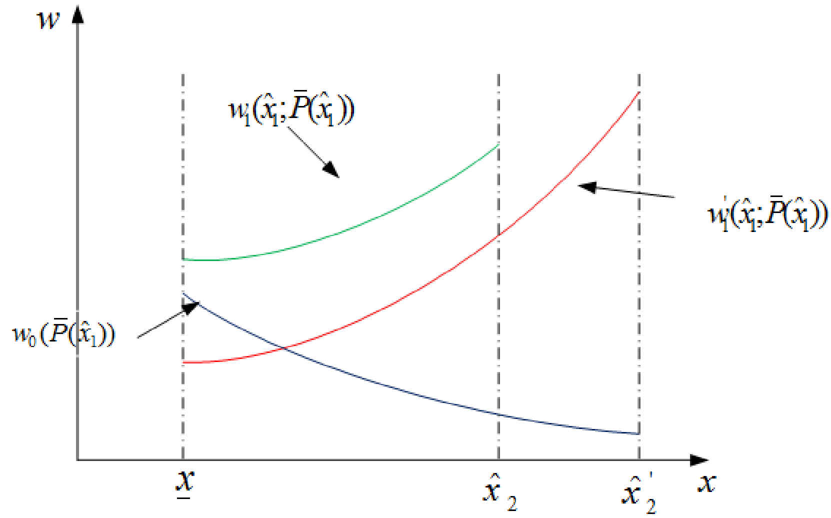


Figure 1: The Wage Schedules of the Marginal Type as Segmentation Changes

submarket gets a positive measure of workers. In the third one, $x_1^* = \widehat{x}_2$: all workers participate in submarket 0. We can immediately rule out the third equilibrium. To see this, for this to be an equilibrium, $\overline{w}_1(\widehat{x}_2) \leq w_0(\overline{P}(\widehat{x}_2))$ must hold. However, $\overline{w}_1(\widehat{x}_2) > \overline{w}_0 > w_0(\overline{P}(\widehat{x}_2))$. Thus, the third equilibrium does not exist. This result is intuitive: since submarket 1 is more productive than submarket 0, there must be a positive measure of workers participating in submarket 1.

Given a segmentation $\overline{P}(\widehat{x}_1)$, from the wage equations (3) and (6), we can compute $w_0(\overline{P}(\widehat{x}_1))$ and $w_1(\widehat{x}_1; \overline{P}(\widehat{x}_1))$, which define two wage schedules as a function of $\widehat{x}_1$. Note that both wage schedules are continuous in $\widehat{x}_1$. The equilibrium conditions are explicitly written below:

| | |
|---|---|
| Corner equilibrium: | $x_1^* = \underline{x}$ if $w_1(\underline{x}; \overline{P}(\underline{x})) \geq \overline{w}_0$, |
| Interior equilibrium: | $x_1^* = \widehat{x}_1 \in (\underline{x}, \widehat{x}_2)$ if there is an $\widehat{x}_1$ such that $w_0(\overline{P}(\widehat{x}_1)) = w_1(\widehat{x}_1; \overline{P}(\widehat{x}_1))$. |

Now we examine the properties of the wage schedules $w_0(\overline{P}(\widehat{x}_1))$ and $w_1(\widehat{x}_1; \overline{P}(\widehat{x}_1))$. As to $w_0(\cdot)$, $w_0 = \overline{w}_0$ when $\widehat{x}_1 = \underline{x}$, and it monotonically decreases as $\widehat{x}_1$ increases. This follows immediately from Lemma 4, as an increase in $\widehat{x}_1$ means more workers are active in submarket 0. Regarding $w_1(\widehat{x}_1; \overline{P}(\widehat{x}_1))$, $w_1 = w_1(\underline{x}; \overline{P}(\underline{x}))$ when $\widehat{x}_1 = \underline{x}$, it monotonically increases as $\widehat{x}_1$ increases, and it reaches $\overline{w}_1(\widehat{x}_2)$ when $\widehat{x}_1 = \widehat{x}_2$. To see that $w_1(\widehat{x}_1; \overline{P}(\widehat{x}_1))$ is monotonically increasing in $\widehat{x}_1$, consider $\widehat{x}_1' > \widehat{x}_1$. Applying part (iv) of Lemma 4, we have $w_1(\widehat{x}_1'; \overline{P}(\widehat{x}_1')) \geq w_1(\widehat{x}_1'; \overline{P}(\widehat{x}_1))$. By the monotonicity of $w_1(x; \cdot)$ in $x$, $w_1(\widehat{x}_1'; \overline{P}(\widehat{x}_1)) > w_1(\widehat{x}_1; \overline{P}(\widehat{x}_1))$. Therefore, we have $w_1(\widehat{x}_1'; \overline{P}(\widehat{x}_1')) > w_1(\widehat{x}_1; \overline{P}(\widehat{x}_1))$. The wage schedules of $w_0(\overline{P}(\widehat{x}_1))$ and $w_1(\widehat{x}_1; \overline{P}(\widehat{x}_1))$ as $\widehat{x}_1$ varies are plotted in Figure 1. We have two cases to consider.

Case 1: $w_1(\underline{x}; \overline{P}(\underline{x})) \geq \overline{w}_0$. In this case the corner equilibrium $x_1^* = \underline{x}$ exists. However, no interior equilibrium exists. This is because $w_1(\widehat{x}_1; \overline{P}(\widehat{x}_1))$ is monotonically increasing and

$w_0(\overline{P}(\widehat{x}_1))$ is monotonically decreasing, in $\widehat{x}_1$. Thus, the fact that $w_1(\underline{x};\overline{P}(\underline{x})) \geq \overline{w}_0$ implies that the two wage schedules cannot have an interior intersection.[20]

Case 2: $w_1(\underline{x};\overline{P}(\underline{x})) < \overline{w}_0$. It is immediate that the corner equilibrium $\widehat{x}_1 = \underline{x}$ does not exist. Given that $w_1(\widehat{x}_1;\overline{P}(\widehat{x}_1))$ is monotonically increasing and $w_0(\overline{P}(\widehat{x}_1))$ is monotonically decreasing, in $\widehat{x}_1$, the facts that $w_1(\underline{x};\overline{P}(\underline{x})) < \overline{w}_0$, $\overline{w}_1(\widehat{x}_2) > \overline{w}_0$, and both $w_1(\widehat{x}_1;\overline{P}(\widehat{x}_1))$ and $w_0(\overline{P}(\widehat{x}_1))$ are continuous imply that the two wage schedules must have a unique intersection, which is interior. Therefore, in this case there is a unique equilibrium $x_1^*$, which is interior: $x_1^* \in (\underline{x}, \widehat{x}_2)$.[21]

Having established the existence and uniqueness of $x_1^*$, now we investigate how $x_1^*$ changes as $\widehat{x}_2$ changes. Applying part (iv) of Lemma 4, as $\widehat{x}_2$ increases, the whole wage schedule of $w_1(\cdot;\overline{P}(\cdot))$ shifts downward (more workers active in submarket 1 pushes down $w_1(\cdot;\overline{P}(\cdot))$).[22] Moreover, as $\widehat{x}_2$ increases the wage schedule of $w_0(\overline{P}(\cdot))$ remains the same (just extending to a larger domain). This is because $w_0$ only depends on the measure of workers active in submarket 0. Therefore, from Figure 1 we reach two conclusions. First, there is a cutoff of $\widehat{x}_2$, denoted as $\widehat{x}_2^{\#}$, such that $x_{1,2}^*(\widehat{x}_2) = \underline{x}$ for all $\widehat{x}_2 \leq \widehat{x}_2^{\#}$, and $x_{1,2}^*(\widehat{x}_2)$ is interior if $\widehat{x}_2 > \widehat{x}_2^{\#}$. Second, if $\widehat{x}_2 \geq \widehat{x}_2^{\#}$, then the interior equilibrium cutoff $x_{1,2}^*(\widehat{x}_2)$ is strictly increasing in $\widehat{x}_2$; if $\widehat{x}_2 < \widehat{x}_2^{\#}$, then a marginal increase in $\widehat{x}_2$ will not change the corner equilibrium.

Finally, as $\widehat{x}_2$ increases to $\widehat{x}_2' > \widehat{x}_2$, in the partial equilibrium we must have $V_1^{*\prime} > V_1^*$. To see this, consider three scenarios. In the first scenario, $\widehat{x}_2 < \widehat{x}_2^{\#}$ and $\widehat{x}_2' < \widehat{x}_2^{\#}$ (corner equilibrium in both cases). Applying Lemma 4, it is immediate that $V_1^{*\prime} > V_1^*$. In the second scenario, $\widehat{x}_2 \leq \widehat{x}_2^{\#}$ and $\widehat{x}_2' > \widehat{x}_2^{\#}$ (corner equilibrium in the first case and interior equilibrium in the second case). Suppose $V_1^{*\prime} \leq V_1^*$. Then by Lemma 4, $w_1'(x_1^{*\prime}) \geq w_1(x_1^{*\prime})$. In the original equilibrium, we have $w_1(x_1^{*\prime}) > w_1(\underline{x}) \geq \overline{w}_0$. Therefore, we get $w_1'(x_1^{*\prime}) > \overline{w}_0$, which means that $x_1^{*\prime}$ cannot be the indifference type in the new equilibrium, a contradiction. In the third scenario, $\widehat{x}_2 > \widehat{x}_2^{\#}$ and $\widehat{x}_2' > \widehat{x}_2^{\#}$ (interior equilibrium in both cases). Again, suppose $V_1^{*\prime} \leq V_1^*$. Then by Lemma 4, $w_1'(x_1^{*\prime}) \geq w_1(x_1^{*\prime})$. Since $x_1^{*\prime} > x_1^*$, $w_1(x_1^{*\prime}) > w_1(x_1^*) = w_0$. Thus, we have $w_1'(x_1^{*\prime}) > w_0$. In submarket 0, the fact that $x_1^{*\prime} > x_1^*$ implies, by Lemma 4, that $w_0' < w_0$. Hence we have $w_1'(x_1^{*\prime}) > w_0'$, which means that $x_1^{*\prime}$ cannot be the indifference type in the new equilibrium, a contradiction.

The above results are summarized in the following lemma.

**Lemma 5** *Given any $\widehat{x}_2 \in (\underline{x}, \overline{x})$, there is a unique $x_1^*$ that achieves partial equilibrium in submarkets 0 and 1. Moreover, in the partial equilibrium $x_1^*$ is weakly increasing, and $V_1^*$ is strictly increasing, in $\widehat{x}_2$.*

The intuition for these results is as follows. Workers always go for submarket 1 first, as it is more productive. But more workers there will push down the wages in submarket 1,

[20] This case is illustrated in Figure 1 with $\widehat{x}_2$.

[21] This case is illustrated in Figure 1 with $\widehat{x}_2'$.

[22] As illustrated in Figure 1, with $\widehat{x}_2 < \widehat{x}_2'$, $w_1'(\cdot;\overline{P}(\cdot))$ lies below $w_1(\cdot;\overline{P}(\cdot))$.

leading to some lower types choosing submarket 0. When some higher types exogenously switch from submarket 2 to submarket 1 ($\widehat{x}_2$ increases), it pushes down the whole wage schedule in submarket 1. To restore the indifference condition of the marginal type, the marginal type must increase.

**Partial equilibrium in submarkets** $0, ..., n$ Now consider partial equilibria in submarkets $0, ..., n$, given $\widehat{x}_{n+1} \in (\underline{x}, \overline{x})$. Using induction, we show that, if the results of Lemma 5 hold for $n-1$, then they also hold for $n$. This is formally stated in the following lemma.

**Lemma 6** *Suppose for $n-1$, given any $\widehat{x}_n \in (\underline{x}, \overline{x})$, there is a unique equilibrium segmentation $\{x_i^*\}_{i=1}^{n-1}$ such that submarkets $0, ..., n-1$ achieve partial equilibrium. Moreover, $\{x_i^*\}_{i=1}^{n-1}$ are all weakly increasing in $\widehat{x}_{n-1}$, and $V_{n-1}^*$ is strictly increasing in $\widehat{x}_n$. Then, given any $\widehat{x}_{n+1} \in (\underline{x}, \overline{x})$, there is a unique equilibrium segmentation $\{x_i^*\}_{i=1}^{n}$ such that submarkets $0, ..., n$ achieve partial equilibrium. Moreover, in the partial equilibrium $x_n^*$ is increasing, and $V_n^*$ is strictly increasing, in $\widehat{x}_{n+1}$.*

**Equilibrium in the whole market** Given the results of Lemma 5, we can apply Lemma 6 recursively. In each step, one more higher submarket is included. This step-by-step induction will eventually reach submarket $N$, which means that a market equilibrium in the whole market exists and is unique. Thus, we have proved the following proposition.

**Proposition 2** *A market equilibrium exists. Moreover, given parameter values, the market equilibrium is unique.*

Lemma 5 and Lemma 6 also imply the following corollary, which will be useful in later analysis. Essentially, it says that if one cutoff changes for some exogenous reason, then to restore (partial) equilibrium in other submarkets, all other cutoffs must move in the same direction.

**Corollary 1** *(i) If $\widehat{x}_n$, $2 \leq n \leq N$, increases, then $\{x_i^*\}_{i=1}^{n-1}$, which ensures submarkets $0, 1, ..., n-1$ are in partial equilibrium, all weakly increase; (ii) If $\widehat{x}_n$, $1 \leq n \leq N-1$, increases, then $\{x_i^*\}_{i=n}^{N}$, which ensures submarkets $n, n+1, ..., N$ are in partial equilibrium, all weakly increase.*

**Proof.** Part (i) is directly implied by Lemma 5 and Lemma 6. Part (ii) can be proved in a similar fashion, and thus the proof is omitted. ■

The existence of market equilibrium is ensured because each worker type has to choose some submarket in which to participate. The underlying reason for the uniqueness of the equilibrium is the indirect negative externalities workers within the same submarket impose on each other, which we emphasized earlier. Specifically, more workers active in a particular submarket $n$ improve type $n$ firms' position and reduces the wages for all worker types participating in that submarket. On the other hand, more workers active in submarket $n$ means fewer workers in other submarkets, which increases the wages for workers participating in other submarkets. In short, more workers in submarket $n$ reduce the attractiveness of submarket $n$, but increases the attractiveness of other submarkets, to workers. These indirect negative externalities imply that the segmentation has to be right in equilibrium to ensure no deviation on part of the workers, which ensures the uniqueness of equilibrium.

Intuitively, one can think that an equilibrium segmentation is reached from-the-top-to-bottom. First, the most able workers participate in submarket $N$, the most productive submarket. Gradually, as the next most able workers participate in this submarket, type $N$ firms' position improves, which pushes down the whole wage schedule in submarket $N$. At some point, the next most able workers' wage in submarket $N$ becomes too low, and they start to participate in submarket $N-1$, the next most productive submarket. This same process repeats in submarket $N-1$, and so on.

### 3.3 Equilibrium properties

Several equilibrium properties are worth mentioning. First, wages in individual submarkets are semi-independent but indirectly linked. Specifically, given a segmentation, wages in each submarket are independently determined, as they only depend on the market condition in their own submarket. However, in equilibrium all the submarkets are indirectly linked through workers' participation decisions. For instance, the wages in any two adjacent submarkets are interdependent because the marginal type who is indifferent between these two submarkets depends on the wages in both submarkets. Since interdependence exists between any two adjacent submarkets, all the submarkets are indirectly linked. It means that if an exogenous shock occurs to any submarket, its impact will spread to all other submarkets, through these indirect links.

Second, in equilibrium, except for the lowest types participating in submarket 0, wages are increasing in workers' type. In particular, the equilibrium wage schedule is piecewise linear in workers' type: linear within each submarket, steeper in higher submarkets, but continuous across submarkets.[23] This further implies that the wage schedule is weakly convex in workers' type.[24]

Third, in equilibrium, $V_n^*$ must be increasing in $n$; that is, more productive job vacancies have a higher value. This is because if $V_{n+1}^* \leq V_n^*$, then all workers in submarkets $n+1$ and $n$ would have preferred submarket $n+1$ to submarket $n$.[25] As to the expected queue length $q_n^*$, the general pattern is that $q_n^*$ should be increasing in $n$, as this helps to make the marginal type $x_n^*$ indifferent between submarkets $n$ and $n-1$. However, the difference between the

[23] The continuity in the wage schedule is due to the fact that each cutoff type is indifferent between two adjacent submarkets.

[24] If the number of firm types goes to infinity, then the wage schedule will be close to being strictly convex.

[25] More explictly, the indifference condition (11) can be written compactly as $(\theta_n - \theta_{n-1})x_n^* = rV_n^* - rV_{n-1}^*$.

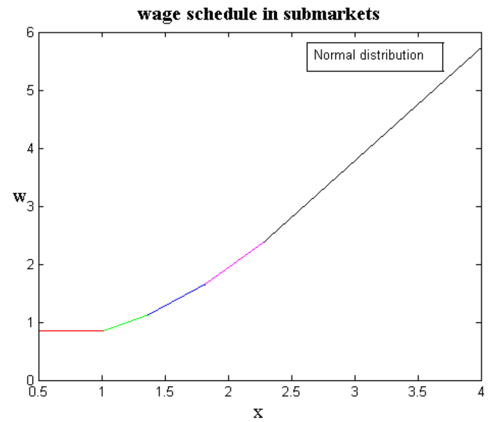


Figure 2: The Equilibrium Wage Schedule

average types $E_{x_n}[x]$ and $E_{x_{n-1}}[x]$ can also help to make the marginal type $x_n^*$ indifferent. This implies that $q_n^*$ is not necessarily monotonic. Instead, whether $q_n^*$ is monotonic depends on the distribution of worker types and the distribution of job types.

**Example 1** *(The benchmark case) Workers' type distribution is truncated normal on* $[0.5, 4]$, *with mean* 1.5 *and variance* 0.7.[26] *There are five types of firms. Firms' productivities are* $(\theta_1, \theta_2, \theta_3, \theta_4) = (1, 1.5, 2, 2.5)$, *and the measures of firms are* $(Y_0, Y_1, Y_2, Y_3, Y_4) = (0.25, 0.25, 0.2, 0.15, 0.1)$, *with the total measure of firms being* 0.95. *These five submarkets correspond to (from high to low) high-tech, medium high-tech, medium (manufacturing), medium low-tech (low-skill manufacturing), and low-tech service jobs, respectively.*[27] *The other parameter values are:* $b = 0.5$, $r = 0.05$, $\delta = 0.06$, $\beta = 0.65$, *and* $\alpha = 0.1$. *The equilibrium wage schedule is illustrated in Figure 2 (each submarket is of a different color). Workers' type distribution and the equilibrium wage distribution are illustrated in Figure 3, and the key endogenous variables are recorded in Table 1.*

**Table 1: Key Endogenous Variables**

| | SM0 | SM1 | SM2 | SM3 | SM4 |
|---|---|---|---|---|---|
| $x_n^*$ | - | 1.0230 | 1.3678 | 1.8186 | 2.2894 |
| $E_{x_n}[x]$ | 0.7949 | 1.2015 | 1.5900 | 2.0355 | 2.6378 |
| $X_n$ | 0.1855 | 0.1920 | 0.2712 | 0.2110 | 0.1403 |
| $q_n^*$ | 0.5187 | 0.5541 | 3.3325 | 4.3679 | 4.2718 |

[26] We choose the worker type distribution to be truncated normal because empirical evidence (see Herrnstein and Murray, 1994) indicates that it is the case.

[27] Cleaning jobs are typical low-tech service jobs, which have a lower wage rate than that of low-skill manufacturing jobs. The measures of these jobs are chosen to match the distribution of job types in the real world.

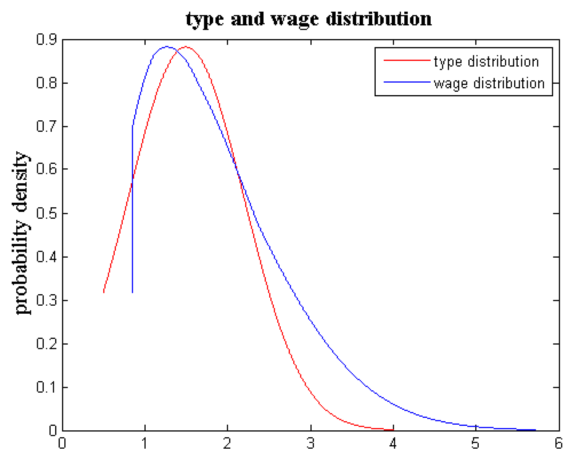


Figure 3: Type Distribution and The Equilibrium Wage Distribution

From Table 1, we observe that there are more workers than jobs in submarkets 2, 3, and 4, but the reverse is true in submarkets 0 and 1. As to $q_n^*$, we see that $q_0^* < q_1^* < q_2^* < q_3^*$, but $q_3^* > q_4^*$. In particular, $q_0^*$ and $q_1^*$ are low and close to each other, but there is a big upward jump from $q_1^*$ to $q_2^*$. The reason for $q_0^*$ and $q_1^*$ being close is that the marginal type $x_1^*$ is close to 1, which means that for this type the output difference between a type 0 job and a type 1 job is very small. Similarly, the underlying reason for the big upward jump from $q_1^*$ to $q_2^*$ is that the marginal type $x_2^* \simeq 1.37$, which means that for this type the output difference between a type 1 job and a type 2 job is relatively large.[28] The underlying reason behind $q_3^* > q_4^*$ is the second kind of externality mentioned before. Due to the decreasing density for $x \geq 1.5$, the difference between the average type in submarket 3 and that of submarket 4 is relatively large (roughly 0.6). This makes submarket 3 relatively more attractive and leads to $q_3^* > q_4^*$.

In Figure 3, we can see that, compared to the type distribution, the wage distribution is more condensed for the low types (lower than the mean 1.5), but is stretched out for the high types (higher than the mean).[29] Actually, the wage distribution is close to a truncated log normal distribution, which is consistent with empirical evidence (Battistin et al., 2009).

Conceptually, we can distinguish two kinds of wage inequality. The first kind is wage inequality within the same submarket (for jobs with similar productivities). This inequality is solely due to the fact that workers have different abilities. As $\theta_n$ increases, wage inequality within the same submarket increases (the wage schedule becomes steeper). If the type distribution of workers participating in submarket $n$ becomes more dispersed, the wage inequality within this submarket also increases. The second kind is wage inequality across different sub-

[28] More generally, since the marginal type is higher in a higher submarket, the output difference for the marginal type across two adjacent submarkets is also larger in higher submarkets. This implies that, other things equal, $q_n^*$ should increase in an increasing rate.

[29] Actually, there is a mass point on the lowest wage $w_0^*$, which is not shown in Figure 3.

markets (across jobs with different productivities). This inequality results from workers having different abilities as well as different jobs having different productivities. In other words, it depends on the endogenous segmentation or matching pattern. Note that the classification of wage inequality in our model is different from those in the two-type models of Albrecht and Vroman (2002) and Shi (2002). In particular, they focus on the skill-premium (wage inequality between skilled and unskilled workers) and within-group wage inequality (workers of the same type get different wages when matched with different types of firms).[30] In our model, there is only the skill-premium, though it also depends on the matching pattern between workers and jobs.

# 4 Socially Efficient Segmentation

In this section we study the socially efficient segmentation which maximizes total social welfare. In particular, a social planner can determine the set of worker types participating in each submarket (the segmentation). However, the social planner is not able to affect the search friction in each submarket (the search/matching technology is still the same). We assume that the social planner is utilitarian, which means that she only cares about the total social welfare and has no preference over inequality. Given that the output function is supermodular, a social planner will naturally assign higher types of workers to more productive jobs. That is, the optimal segmentation must exhibit weakly positively assortative matching. Thus, a segmentation can be represented as $\{\widehat{x}_n\}_{n=1}^N$ such that workers with type $x \in [\widehat{x}_n, \widehat{x}_{n+1}]$ participate in submarket $n$.

Given a segmentation $\{\widehat{x}_n\}_{n=1}^N$, in the steady state, at any instant of time the total social welfare in submarket $n$ is given by $m_n\theta_n E[x_n] + u_n b$. Specifically, the measure of matches is $m_n$, and each match on average produces an output of $\theta_n E_{x_n}[x]$. The measure of unemployed workers is $u_n$, and each of them gets a flow payoff $b$. Let $\frac{\alpha}{\alpha+\delta} \equiv \pi \in (0,1)$. From the steady state conditions, we have $m_n = \min\{\pi X_n, Y_n\}$, and $u_n = \max\{(1-\pi)X_n, X_n - Y_n\}$. This is because we have the restriction that $m_n \leq Y_n$. When $\pi X_n \geq Y_n$, we have $m_n = Y_n$, $q_n$ approaches infinity, and all unfilled vacancies are filled instantaneously.

Now the total social welfare can be expressed as $\sum_{n=0}^N \{m_n\theta_n E[x_n] + u_n b\}$. Since $m_n \leq Y_n$ and each match is more productive than $b$, the social planner would never choose a segmentation such that $\pi X_n > Y_n$ (additional workers in submarket $n$ do not affect $m_n$). Thus, the social planner's problem can be formulated as

$$\max_{\{\widehat{x}_n\}_{n=1}^N} \sum_{n=0}^N X_n\{\pi\theta_n E[x_n] + (1-\pi)b\} \quad \Leftrightarrow \quad \begin{aligned} &\max_{\{\widehat{x}_n\}_{n=1}^N} \sum_{n=0}^N \theta_n \int_{\widehat{x}_n}^{\widehat{x}_{n+1}} x f(x)dx \\ &\text{s.t. } \pi X_n \leq Y_n \text{ for all } n \end{aligned}$$

[30] In Albrecht and Vroman (2002), the within-group wage inequality only exists among skilled workers, while in Shi (2002) it only exists among unskilled workers.

Denote a socially optimal segmentation (a solution to the above programming problem) as $\{x_n^o\}_{n=1}^N$, and let $X_n^o = F(x_{n+1}^o) - F(x_n^o)$.

**Proposition 3** *There is a unique socially optimal segmentation $\{x_n^o\}_{n=1}^N$, which has the following properties. (i) $X_N^o = \min\{Y_N/\pi, 1\}$. (ii) For $n < N$, $X_n^o = Y_n/\pi$ if $\sum_{i=n}^N [Y_i/\pi] \leq 1$, $X_n^o = 1 - \sum_{i=n+1}^N [Y_i/\pi]$ if $\sum_{i=n+1}^N [Y_i/\pi] < 1$ but $\sum_{i=n}^N [Y_i/\pi] \geq 1$, and $X_n^o = 0$ if $\sum_{i=n+1}^N [Y_i/\pi] \geq 1$.*

**Proof.** Taking the derivative of the objective function in the programming problem with respect to $\widehat{x}_n$, we get

$$-\theta_n \widehat{x}_n f(\widehat{x}_n) + \theta_{n-1} \widehat{x}_n f(\widehat{x}_n) = -(\theta_n - \theta_{n-1}) \widehat{x}_n f(\widehat{x}_n) < 0.$$

Thus $\widehat{x}_n$ should be as low as possible to maximize the objective function. This implies that the solution to the problem is a corner one, or the constraint binds: $\pi X_n = Y_n$. From this observation, the results immediately follow. ■

Roughly speaking, the socially efficient segmentation maximizes the number of steady-state matches in more productive submarkets. First, enough workers of the highest types should be assigned to the most productive submarket $N$ such that $m_N = Y_N$. If there are still workers left ($Y_N/\pi < 1$), then among the remaining workers the highest types should be assigned to the second most productive submarket $N-1$ such that $m_{N-1} = Y_{N-1}$, and so on. The intuition for the efficient assignment being "lexicographical" is straightforward. Due to the urn ball matching technology, assigning any group of workers to two different submarkets (suppose both have unfilled vacancies in the steady state) will lead to the same number of additional matches. Thus, assigning these workers to the more productive submarket will result in a bigger increase in the total output. Therefore, in the efficient assignment more productive submarkets should have no unfilled vacancy before assigning any worker to less productive submarkets.

**Proposition 4** *Comparing the equilibrium segmentation $\{x_n^*\}_{n=1}^N$ and the socially optimal segmentation $\{x_n^o\}_{n=1}^N$, we have, for all $n$, $x_n^* \geq x_n^o$; moreover, $x_n^* > x_n^o$ if $x_n^* > \underline{x}$.*

**Proof.** First consider $x_N^*$ and $x_N^o$. If $x_N^o = \underline{x}$, then $x_N^* \geq x_N^o$ is trivially satisfied. Now suppose $x_N^o > \underline{x}$. By Proposition 3, $X_N^o = Y_N/\pi$, $v_N^o = 0$ and $q_N^o \to \infty$. With $x_N^o$ being the marginal type, by (8), in submarket $N$ firms' equilibrium value can be computed as

$$rV_N = \lim_{q_N \to \infty} \frac{(1-\beta)\alpha q_N [\theta_n E_{x_N}[x] - b]}{r + \delta + \beta\alpha + (1-\beta)\alpha q_N} = \theta_n E_{x_N}[x] - b.$$

And from (5), we get

$$w_N(x_N^o) = b + \frac{\beta[r + \delta + \alpha]}{r + \delta + \beta\alpha} \theta_N [x_N^o - E_{x_N}[x]] < b,$$

where the inequality follows because $x_N^o < E_{x_N}[x]$. The fact that $w_N(x_N^o) < b$ implies that the equilibrium $x_N^*$ must be strictly greater than $x_N^o$, since in equilibrium we have $w_N(x_N^*) > b$, and $w_N(\widehat{x}_N; \overline{P}(\widehat{x}_N))$ is increasing in $\widehat{x}_N$.

Now suppose the results hold for $n+1$. We want to show that the results also hold for $n$. If $x_n^o = \underline{x}$, then the statement is trivially satisfied. Now suppose $x_n^o > \underline{x}$. By a similar logic as in the previous step, we can show that, with worker types within $[x_n^o, x_{n+1}^o]$ participating in submarket $n$, $w_n(x_n^o; \overline{P}(x_{n+1}^o)) < b$. Thus, $x_n^*(\overline{P}(x_{n+1}^o)) > x_n^o(\overline{P}(x_{n+1}^o)) = x_n^o$. By Corollary 1, $x_n^* = x_n^*(\overline{P}(x_{n+1}^*)) > x_n^*(\overline{P}(x_{n+1}^o))$, since $x_{n+1}^* > x_{n+1}^o$ by the presumption. Therefore, $x_n^* > x_n^o$. ■

Proposition 4 illustrates that, compared to the socially optimal segmentation, in equilibrium too few workers are participating in more productive submarkets. Or loosely speaking, the equilibrium segmentation exhibits more assortative matching relative to the optimal segmentation. Intuitively, the equilibrium segmentation is determined by wage equalization for marginal types, while the efficient segmentation is driven by output equalization for marginal types. Since output cannot be equalized for marginal types, efficient segmentation leads to a corner solution. That is, the most productive submarkets are extremely loose (the queue length goes to infinity) under the efficient segmentation. This means that firms in these submarkets have a higher value (their vacancies are filled instantaneously), which drives down the wages of the marginal types below the unemployment benefit $b$. This implies that in equilibrium there must be fewer workers participating in more productive submarkets than under the efficient segmentation.

## 5 Shocks to Individual Submarkets

In this section and the next section we investigate how the equilibrium, including the segmentation pattern and the wage schedule, will change as some of the exogenous parameters vary. We focus on shocks to individual submarkets in this section. In particular, we conduct two comparative statics analyses: an SBTC in a more productive submarket, and a reduction in the number of jobs in some less productive submarket. To make the analysis clean, in the rest of the paper we focus on interior equilibria: $x_1^* > \underline{x}$. That is, each submarket has a positive measure of participating workers. We also denote $w_e(x)$ as the equilibrium wage schedule. Note that $w_e(x)$ depends on the equilibrium segmentation.

### 5.1 SBTC in submarket $N$

Consider an SBTC in submarket $N$. That is, the most-productive jobs become more productive. In particular, suppose $\theta_N$ increases to $\theta_N' > \theta_N$, while all the other parameters of the model remain the same. We use superscript "′" to denote the endogenous variables under $\theta_N'$.

To proceed, we first fix the original equilibrium segmentation pattern $\overline{P}$, and investigate how an increase in $\theta_N$ upsets the original equilibrium. Since all other parameter values remain the same and $\overline{P}$ remains the same, in all submarkets other than submarket $N$ the wage schedules and firms' values do not change either. Now consider submarket $N$. For any $x \geq x_N^*$, by (6),

$$\frac{\partial w_N(x;\overline{P})}{\partial \theta_N} \propto x + \frac{(1-\beta)\alpha q_N}{r+\delta+\beta\alpha}[x - E_{x_n}[x]] > 0.$$

The inequality holds because in equilibrium $w_N(x) \geq b$, which by (6) implies that the above expression is bigger than $b$. Thus, an increase in $\theta_N$ shifts the whole wage schedule up in submarket $N$. This is intuitive, as by Nash bargaining the increases in outputs will be shared by workers and firms. In particular, the wage of the original marginal type increases as well: $w'_N(x_N^*;\overline{P}) > w_N(x_N^*;\overline{P})$. But this means that a type $x_N^*$ worker is no longer indifferent between submarkets $N-1$ and $N$. Therefore, the original equilibrium segmentation will change in order to restore equilibrium.

**Proposition 5** *Suppose $\theta_N$ increases to $\theta'_N > \theta_N$, while all the other parameters of the model remain the same. Then, the following results hold. (i) $x_n^{*\prime} < x_n^*$ for all $n = 1, ..., N$; (ii) $w'_e(x) > w_e(x)$ for any $x$ (the whole wage schedule shifts up); (iii) $V_n^{*\prime} < V_n^*$ for $n \leq N-1$, and $V_N^{*\prime} > V_N$; (iv) $u_1^{*\prime} < u_1^*$ and $q_1^{*\prime} < q_1^*$, and $u_N^{*\prime} > u_N^*$ and $q_N^{*\prime} > q_N^*$.*

The results of Proposition 5 are intuitive. As type $N$ firms become more productive, due to Nash bargaining both the wages and firms' value in submarket $N$ increase. This attracts some high types in submarket $N-1$ (just below the initial cutoff $x_N^*$) to switch to submarket $N$. This increases market tightness in submarket $N-1$, which improves wages but makes type $N-1$ firms worse off. But this further attracts some high types in submarket $N-2$ (just below the cutoff $x_{N-1}^*$) to switch to submarket $N-1$. A similar adjustment process continues in lower submarkets, leading to a decrease in the marginal type, a reduction in firms' value, and improvements in workers' wages, in each lower submarket.

In submarket $N$, workers' wages improve because they benefit directly from the SBTC (having more workers in submarket $N$ only dampens the gain but does not overturn it). In all lower submarkets, workers' wages improve because of the indirect trickling down effect through the changes in the endogenous segmentation. With workers switching to higher submarkets, firms in lower submarkets are worse off, which improve workers' wages in these submarkets. However, firms in submarket $N$ benefit not only directly from the SBTC, but also indirectly from more participating workers.

As to unemployment and market tightness, in submarket $N$ unemployment increases and market tightness decreases, as now it attracts more workers. In submarket 0 the unemployment decreases and the market tightness increases, as it loses workers to submarket 1. However, for an intermediate submarket $n$ it is not clear whether it becomes tighter or not.[31] This depends on the parameters of the model.

[31] Given the urn ball matching technology, aggregate unemployment in this economy is fixed, as long as the total number of jobs and the total number of workers do not change.

We are also interested in the magnitudes of wage increases across different submarkets. Let $\Delta V_n^* = V_n^* - V_n^{*\prime}$ be the change in type $n$ firms' equilibrium value, and $\Delta w_e(x) = w_e'(x) - w_e(x)$ be the wage increase of a type $x$ worker, moving from the original equilibrium to the new equilibrium. Note that $\Delta w_e(x) > 0$ for all $x$ and $\Delta V_n^* > 0$ for all $n \leq N-1$.

**Proposition 6** *For all $x$, the wage increase $\Delta w_e(x)$ is weakly increasing in $x$. In particular, (i) for all $n \leq N-1$, the decrease in firms' equilibrium value $\Delta V_n^*$ is increasing in $n$; (ii) for $x \in [x_n^*, x_{n+1}^{*\prime}]$, $n \leq N-1$, $\Delta w_e(x)$ is constant; (iii) for $x \in [x_{n+1}^{*\prime}, x_{n+1}^*]$, $n \leq N-1$, $\Delta w_e(x)$ is strictly increasing in $x$; (iv) for $x \geq x_N^{*\prime}$, $\Delta w_e(x)$ is strictly increasing in $x$.*

Proposition 6 implies that an SBTC in the most productive submarket, though causing all wages to increase, increases wage inequality. In particular, it increases wage inequality within each submarket (except for submarket 0). Within the most productive submarket $N$, wage becomes more unequal because an increase in $\theta_N$, through Nash bargaining, makes the wage schedule steeper. Within submarket $n$, $1 \leq n \leq N$, the wage becomes more unequal because the wage increases of the low switching types ($x \in [x_n^{*\prime}, x_n^*]$) are smaller than the wage increase of the high types ($x \in [x_n^*, x_{n+1}^{*\prime}]$), who do not switch submarkets. The underlying reason for this pattern is that the low switching types suffer from an increase in firms' value as they switch to a higher submarket ($V_n^* > V_{n-1}^*$). Since the amount of the wage increases becomes smaller in lower submarkets, the SBTC also worsens wage inequality across submarkets. Using an analogy, since the initial positive shock happens in submarket $N$, the trickling down effect through the changes in endogenous segmentation tapers off in lower submarkets, which are further away from the source of the shock.

**Example 2** *In the benchmark case, suppose $\theta_4$ increases from 2.5 to 3.5. The changes in the equilibrium wage schedule are illustrated in Figure 4. Quantitatively, we can see that the change in the endogenous segmentation is small (for instance, $x_4^*$ changes from 2.2894 to 2.2683), which leads to negligible increases in wages for workers in submarkets 0, 1, 2, and 3. On the other hand, the wage increases among the workers in submarket 4 are significant. In short, the positive shock significantly benefits workers in submarket 4, but has little impact on workers in other lower submarkets.*

The underlying reason for this pattern is twofold. First, fixing the initial equilibrium segmentation, the initial marginal type $x_4^*$ does not benefit much from an increase in $\theta_4$. This is because it also increases type 4 firms' value at a rate of $E_{x_4}[x]$, which is bigger than $x_4^*$. Second, both $q_3^*$ and $q_4^*$ are relatively big. This means that firms' values $V_3^*$ and $V_4^*$, and hence the wages in submarkets 3 and 4, are sensitive to changes in the marginal type $x_4^*$. Taken together, the adjustment of $x_4^*$ is small, which means that the positive shock has very little spillover to other lower submarkets.

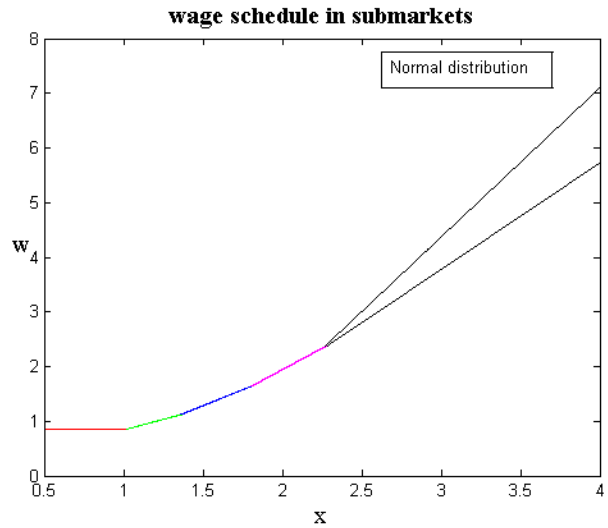


Figure 4: The Impact of an SBTC in Submarket $N$

### 5.2 Fewer low-productivity jobs

Suppose the number of jobs in submarket 0 decreases. That is, $Y_0$ decreases to $Y_0' < Y_0$, while the other parameter values remain unchanged. As mentioned earlier, in the real world type 0 jobs correspond to low-tech service jobs. Thus computerization can cause a reduction in the number of these type of jobs. Again, we use superscript "$'$" to denote the endogenous variables under $Y_0'$.

To study its impacts on the equilibrium, again we first assume that the initial equilibrium segmentation pattern $\overline{P}$ does not change. Consider submarket 0. Since $Y_0' < Y_0$, the steady state equations imply $q_0' > q_0^*$. By Lemma 4, we have $w_0'(\overline{P}) < w_0(\overline{P})$. This change would initiate an adjustment process in other submarkets.

**Proposition 7** *Suppose the measure of type 0 jobs, $Y_0$, decreases to $Y_0' < Y_0$, while all the other parameters of the model remain the same. Then, the following results hold. (i) $x_n^{*\prime} < x_n^*$ for all $n$; (ii) $w_e'(x) < w_e(x)$ for any $x$ (the whole wage schedule shifts down); $V_n^{*\prime} > V_n^*$ for all $n$, and $u_0^{*\prime} < u_0^*$, $u_N^{*\prime} > u_N^*$, $q_N^{*\prime} > q_N^*$, but $q_0'^* > q_0^*$; (iii) for all $x$, the wage decrease $\Delta w_e(x) \equiv w_e(x) - w_e'(x)$ is weakly decreasing in $x$. In particular, for $x \in [x_n^*, x_{n+1}^{*\prime}]$, $1 \le n \le N-1$, and for $x \le x_1^{*\prime}$, $\Delta w_e(x)$ is constant; for $x \in [x_n^{*\prime}, x_n^*]$, $1 \le n \le N$, $\Delta w_e(x)$ is strictly decreasing in $x$.*

One can see that the changes induced by a reduction in the number of jobs in submarket 0 are parallel to those induced by an SBTC in submarket $N$. As the measure of type 0 firms decreases, in submarket 0 the market loosens and the wage decreases. Thus some higher type workers in submarket 0 (just below the initial cutoff $x_1^*$) will switch to submarket 1.

This in turn loosens submarket 1 and dampens wages there, which induces some higher types in submarket 1 to switch to submarket 2. This adjustment process will continue in all higher submarkets in a similar fashion. As a result, all workers get lower wages and all firms get higher values. In submarket 0, workers are worse off due to the initial negative shock.[32] In all other higher submarkets, workers suffer because of the trickling up effect through the adjustments in endogenous segmentation.

Part (iii) of Proposition 7 implies that the negative shock, though causing all wages to decrease, increases wage inequality within all submarkets (except for submarket 0), as well as across submarkets. In particular, within submarket $n \geq 1$, the wage decreases of the low switching types ($x \in [x_n^{*\prime}, x_n^*]$) are bigger than the wage decrease of the high types ($x \in [x_n^*, x_{n+1}^{*\prime}]$), who do not switch submarkets. This is because the low switching types suffer from an increase in firm value ($V_{n+1}^* > V_n^*$). Across submarkets, the wage decreases become smaller in higher submarkets because they are located further away from submarket 0, where the initial negative shock occurs. As a result, the trickling up effect through the changes in endogenous segmentation gradually tapers off in higher submarkets.

**Example 3** *In the benchmark case, suppose $Y_0$ decreases from 0.25 to 0.15. The changes in the equilibrium wage schedule are illustrated in Figure 5. Quantitatively, we can see that the adjustment of $x_1^*$ is significant (from 1.0230 to 0.9782 ). However, the adjustments in the segmentation among the other 4 submarkets are very small (for instance, $x_2^*$ changes from 1.3678 to 1.3655), which leads to negligible decreases in wages for workers in those submarkets. On the other hand, the wage decrease among the workers in submarket 0 is intermediate (from 0.8533 to 0.8097). In short, the negative shock hurts workers in submarket 0, but has little impact on workers in other higher submarkets.*

The underlying reason for this pattern is as follows. The adjustment of $x_1^*$ is significant because both $q_0^*$ and $q_1^*$ are small, which means that the wages in both markets are not sensitive to changes in the marginal type $x_1^*$. For the same reason that $q_1^*$ is small, although the adjustment of $x_1^*$ is significant, it has little impact on the wages in submarket 1, which means that the negative shock has very little spillover to other higher submarkets.

### 5.3 Shocks to middle submarkets

Now we investigate the impacts of a shock to some middle submarket $j$, $1 \leq j < N$. We start with a decrease in the number of jobs in submarket $j$, $Y_j$.

**Proposition 8** *Suppose $Y_j$, $1 \leq j < N$, decreases to $Y_j' < Y_j$, and all other parameter values remain the same. Then, (i) $x_n^{*\prime} > x_n^*$ for all $n \leq j$, and $x_n^{*\prime} < x_n^*$ for all $n > j$; (ii) $w_e'(x) < w_e(x)$ for any $x$ (the whole wage schedule shifts down); (iii) for $x \leq x_{j+1}^*$, the wage*

[32] Although submarket 0 will have fewer workers, the endogenous adjustment in segmentation is not big enough to overturn the impact of the initial decrease in the number of type 0 jobs on market tightness: $q_0$ increases.

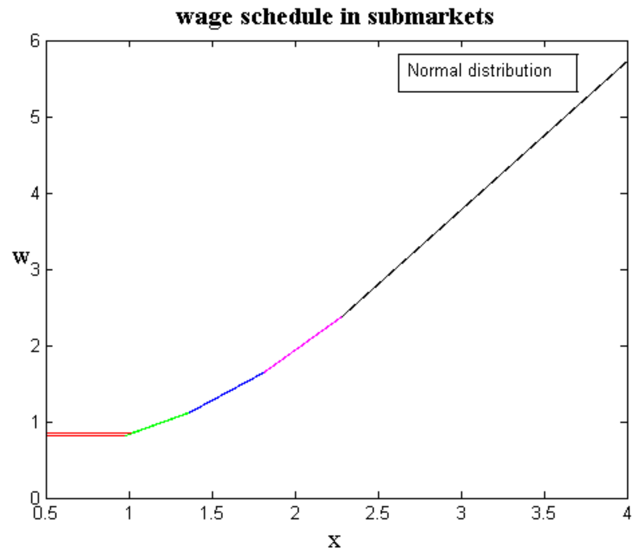


Figure 5: The Impact of Losing Jobs in Submarket 0

*decreases* $\Delta w_e(x) \equiv w_e(x) - w'_e(x)$ *are weakly increasing in* $x$*; for* $x > x^*_{j+1}$*,* $\Delta w_e(x)$ *is weakly decreasing in* $x$*.*

Proposition 8 indicates that a decrease in $Y_j$ induces some high types in submarket $j$ to switch to submarket $j+1$, and some low types to switch to submarket $j-1$. As a result, the cutoff types in the higher submarkets and those in the lower submarkets adjust in opposite directions. However, wages in all submarkets adjust in the same direction: all wages decrease. Moreover, the wage decreases become smaller in submarkets that are further away from submarket $j$. In particular, in submarkets lower than submarket $j$, the wage decreases are smaller in lower submarkets. Using an analogy, submarket $j$ is the epicenter of an earthquake, with shocks tapering off in more distant submarkets.

As to the impacts on wage inequality within submarkets, a decrease in $Y_j$ does not affect the wage inequality within submarket $j$, as all workers remain in it experience the same amount of wage decrease. However, it increases wage inequality within submarkets higher than submarket $j$, but decreases wage inequality within submarkets lower than submarket $j$ (except for submarket 0). This is because in a submarket higher than $j$, the switching types in the new equilibrium segmentation are relatively low types, and their wage decreases are bigger than that of the non-switching types. On the other hand, in a submarket lower than $j$, the switching types in the new equilibrium segmentation are relatively high types, and their wage decreases are bigger than that of the non-switching types. Regarding wage inequality across submarkets, among submarkets higher than submarket $j$ a decrease in $Y_j$ increases wage inequality. However, it decreases wage inequality among submarkets lower than submarket $j$, as the wage decreases are smaller in lower submarkets. This shows that a reduction in the

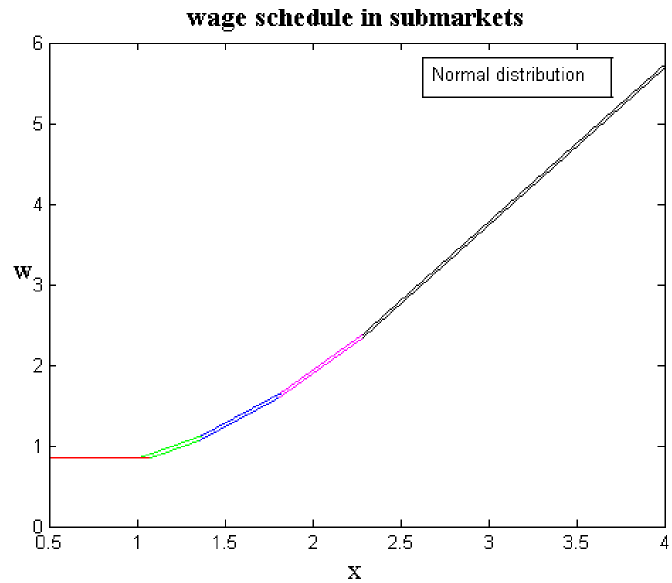


Figure 6: The Impact of Losing Jobs in Submarket 1

number of jobs in a middle submarket will lead to wage polarization.

**Example 4** *In the benchmark case, suppose $Y_1$ decreases from 0.25 to 0.15. The changes in the equilibrium wage schedule are illustrated in Figure 6. Quantitatively, we can see that the adjustment of $x_1^*$ and $x_2^*$ are significant, but the adjustments in the other cutoff types are negligible. The wage decrease in submarket 0 is insignificant (from 0.8533 to 0.8436), while the wage decreases in all the other submarkets are intermediate and almost the same (for type $x_2^{*\prime} = 1.355$, its wage decreases from 1.1115 to 1.0605; for the highest type $\overline{x} = 4$, its wage decreases from 5.7188 to 5.6756). This pattern suggests that, in terms of the impacts on wages, quantitatively the* ***shock transmission is asymmetric*** *in the two directions: the upward transmission (to higher submarkets) is significant while the downward transmission (in lower submarkets) is negligible.*

The underlying reason for the quantitatively asymmetric shock transmission is the following. In submarkets 0 and 1, in the initial equilibrium $q_0^*$ and $q_1^*$ are both low are similar (slightly above 0.5), while in the other three submarkets $q_n^*$ are high (all above 3). This means that the wage in submarket 0 is insensitive to the adjustment of $x_1^*$, while the wages in submarkets 2, 3, and 4 are sensitive to the adjustments in $x_2^*$, $x_3^*$, and $x_4^*$. As a result, the decrease in $w_0$ is insignificant, while the decrease in the wages in submarkets 2, 3, and 4 are of similar magnitudes to that in submarket 1.

As mentioned earlier, type 1 jobs correspond to low-tech manufacturing jobs. Both negative trade shocks (competition from cheap labor in China) and computerization can lead to a decrease in $Y_1$. This example shows that a negative shock in the low-tech manufacturing sector alone could lead to a pattern of wage polarization.[33]

Next we consider a SBTC in submarket $j$, $1 \leq j < N$.

**Proposition 9** *Suppose $\theta_j$, $1 \leq j < N$, increases to $\theta_j' > \theta_j$, and all other parameter values remain the same. Then, (i) $x_n^{*\prime} > x_n^*$ for all $n > j$; (ii) $w_e'(x) > w_e(x)$ for any $x \geq x_{n+1}^*$; (iii) for $x \geq x_{n+1}^*$, the wage increases $\Delta w_e(x) \equiv w_e'(x) - w_e(x)$ are weakly decreasing in $x$.*

In many aspects, an SBTC in submarket $j$ has similar impacts as an increase in $Y_j$. In particular, as $\theta_j$ increases, we can unambiguously state its impacts on submarkets higher than $j$. In those submarkets, some low types switch to lower submarkets. All wages increase, but the wage increases are weakly decreasing in types. Wage inequality within any submarket higher than $j$ decreases, and wage inequality across submarkets higher than $j$ decreases as well.

However, the impacts of an increase in $\theta_j$ on submarkets lower than $j$ are ambiguous, as we cannot pin down in which direction $x_j^*$ will adjust. It is possible that $x_j^*$ will increase, reducing all wages in all the submarkets lower than $j$. This suggests that the shock transmission might be asymmetric qualitatively: while an increase in $\theta_j$ always increases all wages in all higher submarkets, it might reduce all wages in lower submarkets. This is different from a change in $Y_j$, under which the shock transmission is qualitatively symmetric in higher and lower submarkets: wages in all submarkets change in the same direction.

Why might an increase in $\theta_j$ trigger asymmetric transmission? This is because an increase in $\theta_j$ reduces the difference between submarkets $j$ and $j+1$, but widens the difference between submarkets $j$ and $j-1$. Moreover, other things equal, the higher types in submarket $j$ benefit more than the lower types as the wage schedule becomes steeper. As a result, submarket $j$ might attract many low inframarginal types from submarket $j+1$ (depending on the distribution of worker types), which might push down the wage of the low types in submarket $j$ (overcoming the direct effect of an increase in $\theta_j$). In contrast, a change in $Y_j$ does not affect the difference between submarkets $j$ and $j+1$ or the difference between submarkets $j$ and $j-1$. Moreover, the initial shock affects the wages in submarket $j$ through the channel of changing market tightness $1/q_j$, thus the amount of the wage change is the same for all types in submarket $j$. These imply that the adjustment or transmission must be qualitatively symmetric in higher submarkets and in lower submarkets.

**Example 5** *In the benchmark case, suppose $\theta_3$ increases from 2 to 2.2. The changes in the equilibrium wage schedule are illustrated in Figure 7. We can see that the adjustments of all cutoffs are negligible. The cutoffs of $x_1^*$, $x_2^*$, and $x_3^*$ all decrease slightly, and wages in submarkets 0, 1, and 2 all increase. However, again we observe the pattern of asymmetric shock transmission in quantitative terms: the wage increases in the lower submarkets (0, 1, and 2) are negligible, while the wage increases in the higher submarket (4) are significant.*

[33] However, in the upper tail all wages decrease and the increase in wage inequality is quantatively negligible.

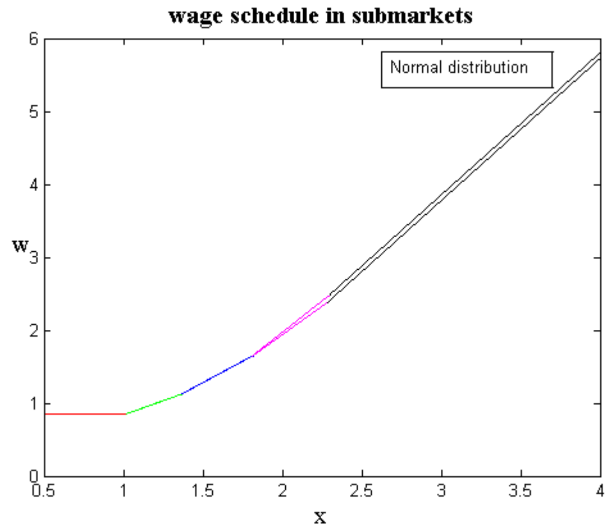


Figure 7: The Impact of an Increase in $\theta_3$

*Actually, the wage increases for all types in submarket 4 are almost of the same magnitudes as that of the highest type in submarket 3, and higher than those of most types in submarket 3. It definitely increases overall wage inequality, as the higher types benefit significantly from the shock, while the lower types almost do not benefit.*

The underlying reason for the quantitatively asymmetric shock transmission is the following. Fixing the initial equilibrium segmentation, the initial low marginal type in submarket 3, $x_3^*$, does not benefit much from an increase in $\theta_3$, while the initial high marginal type, $x_4^*$, benefits significantly (recall that the wage schedule in submarket 3 becomes steeper). This translates into a big increase in wages in submarket 4, but very little increase in wages in submarket 2.[34]

As mentioned earlier, type 3 jobs correspond to medium-high tech jobs in the real world. This example shows that, due to the asymmetric shock transmission, an SBTC shock in the medium-high tech sector alone can significantly increase overall wage inequality: the workers in the high-tech sector benefit from this shock almost by the same amount as the highest types in the medium-high tech sector, while the low types of workers in the medium-high tech sector and those in the other three sectors benefit very little from this shock.

## 5.4 Combination of shocks

In the previous subsections, we see that an SBTC in submarket $N$ causes all wages to increase, while a negative shock in submarket 1 causes all wages to decrease. The following example

[34] Note that both $q_2^*$ and $q_4^*$ are big (bigger than 3). Therefore, the asymmetric shock transmission cannot be attributed to the difference in $q$'s.

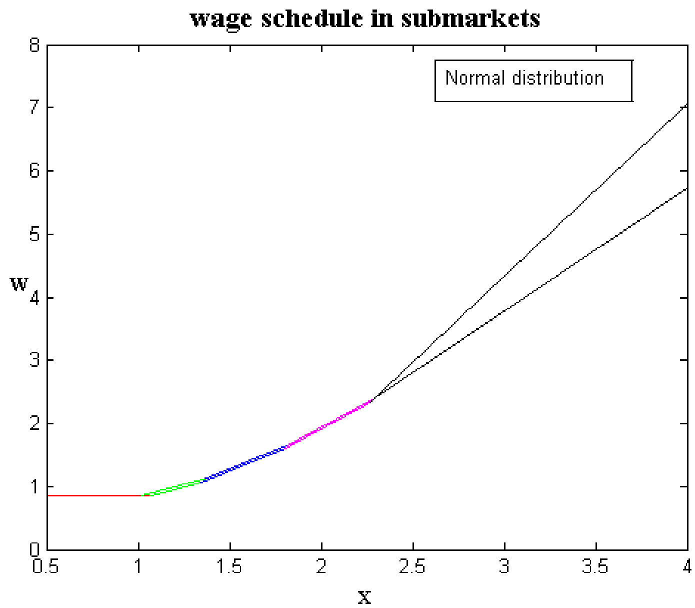


Figure 8: The Impact of a Combination of Shocks

combines these two individual shocks.

**Example 6** *In the benchmark case, suppose* $Y_1$ *decreases from* 0.25 *to* 0.15*, and* $\theta_4$ *increases from* 2.5 *to* 3.5*. The changes in the equilibrium wage schedule are illustrated in Figure 8. Quantitatively, we can see that the adjustment of* $x_1^*$ *is significant (from* 1.0230 *to* 1.0695*), while the adjustments of other cutoffs are rather small. The wages in submarket 4 increase significantly, and the wage in submarket 0 decreases slightly (change from* 0.8533 *to* 0.8448*). The wages in submarkets 1, 2, and 3 all decrease: among these submarkets, though the amount of the wage decrease is decreasing in type, all the workers experience almost the same immediate amount of wage decrease. In particular, the wage of type* $x = 1.25$ *(submarket 1) decreases from* 1.0296 *to* 0.9845*, that of* $x = 1.75$ *(submarket 2) decreases from* 1.5681 *to* 1.5311*, and that of* $x = 2.25$ *(submarket 3) decreases from* $x = 2.3205$ *to* 2.2895*. Moreover, the lowest types in submarket 4 also experience wage decreases. For instance, the wage of* $x = 2.29$ *decreases from* 2.3829 *to* 2.3749*.*

This example shows that the combination of shocks generates a stronger version of wage polarization. In the upper tail, the wages of the high types increase while those of the low types decrease, and wage inequality worsens. In the lower tail, the wages of all types decrease but the lowest types' wage decreases are smaller, hence wages become less unequal. The fact that the wage in submarket 0 decreases only slightly while the wages in submarket 1, 2, and 3 decrease by almost the same immediate amount is due to the asymmetric transmission of the negative shock in submarket 1 as well as of the SBTC in submarket 4. That the lowest types in submarket 4 also experience wage decreases, despite a big increase in $\theta_4$, is because they would have directly benefited only a little from an increase in $\theta_4$, while the spillover from the negative shock in submarket 1 on them is relatively large due to the asymmetric shock transmission.

# 6 Internet-Induced Increase in Search Efficiency

In this section, we study the impacts of the Internet on the equilibrium segmentation pattern and wage inequality. In particular, the Internet has two effects on workers' search for jobs. First, the Internet reduces workers' search costs (or time). Instead of spending time going to job agencies or buying and reading newspaper job ads, the Internet enables workers to stay at home and search jobs online. Since we implicitly assume that workers have a fixed search intensity (or time), it means that a reduction in search costs increases $\alpha$, the workers' contact rate.

Second, the Internet also allows firms to post more detailed job descriptions online, thus making workers' pre-search screening more efficient. In the real world, jobs are differentiated not only in the vertical dimension (in terms of productivity), but also on the horizontal dimension (location, the nature of job tasks, firm culture, etc.), and different workers value these "horizontal" features differently. Thus more realistically, workers are also searching for suitable jobs in the horizontal dimension. In this aspect, the Internet allows workers to more efficiently screen suitable jobs before sending applications (or contact), thus increasing the conversion rate from contacts to a successful match. Although the interpretation is different, in terms of modelling the second effect is equivalent to an increase in $\alpha$, the same as the first effect. In the appendix, we formalize this point by explicitly modeling the horizontal aspect of search.

In summary, the Internet increases workers' search efficiency, and in our setting its impact is modeled as an increase in $\alpha$. Unlike an SBTC shock to an individual submarket, it is a universal shock affecting all workers and all submarkets. Moreover, it is skill-neutral, as all workers' contact rate increases by the same amount.

## 6.1 The impacts on wages with fixed segmentation

To study the impacts of an increase in $\alpha$, we first hypothetically assume that the initial equilibrium segmentation pattern does not change. Now consider the change of $w_n(x)$, $n > 1$, caused by an increase in $\alpha$. In particular,

$$\frac{dw_n(x)}{d\alpha} = \frac{\partial w_n(x)}{\partial \alpha} + \frac{\partial w_n(x)}{\partial q_n}\frac{\partial q_n}{\partial \alpha}. \tag{12}$$

In the above equation (12), the first term captures the direct impact: an increase in $\alpha$ directly changes the worker's and firms' bargaining positions and hence $w_n(x)$. The second term reflects the indirect impact: an increase in $\alpha$ will affect $q_n$ in the steady state, which induces a change in $w_n(x)$.

First consider the indirect impact. By the steady state equation, it can be computed that $\frac{\partial q_n}{\partial \alpha} \propto (X_n - Y_n)$. Thus $q_n$ is increasing in $\alpha$ if and only if the initial $q_n > 1$ or $X_n > Y_n$. Intuitively, an increase in $\alpha$ increases the number of matches, thus making the tightness of the market more extreme: if initially the market is loose ($X_n > Y_n$ or $q_n > 1$) then it becomes looser ($q_n$ increases), and if initially the market is tight ($X_n < Y_n$ or $q_n < 1$) then it becomes tighter ($q_n$ decreases). Recall that $\frac{\partial w_n(x_n)}{\partial q_n} < 0$. Therefore, the indirect impact is negative if and only if the initial $q_n > 1$.

Next we study the direct impact. First, we consider the average type $\overline{x}_n \equiv E_{x_n}[x]$. By the wage equation (6), it can be calculated that

$$\frac{\partial w_n(\overline{x}_n)}{\partial \alpha} \propto \beta(1-\beta)(\theta_n \overline{x}_n - b)(1-q_n).$$

By the above equation, the direct impact $\frac{\partial w_n(\overline{x}_n)}{\partial \alpha}$ is positive if and only if $q_n < 1$. This is intuitive, as an increase in $\alpha$ by $\Delta\alpha$ increases the worker's contact rate by $\Delta\alpha$, but it improves firms' contact rate by $q_n \Delta\alpha$. Therefore, if $q_n > 1$, then firms' position improves more than the worker's, leading to a decrease in the wage.

Taking the two effects together, we conclude that $\frac{dw_n(\overline{x})}{d\alpha} > 0$ if $q_n < 1$, $\frac{dw_n(\overline{x})}{d\alpha} < 0$ if $q_n > 1$, and $\frac{dw_n(\overline{x})}{d\alpha} = 0$ if $q_n = 1$. Since in submarket 0 both the output and the wage do not depend on workers' type, the same result applies to $w_0$: $\frac{dw_0}{d\alpha} > 0$ if $q_0 < 1$, $\frac{dw_0}{d\alpha} < 0$ if $q_0 > 1$, and $\frac{dw_0}{d\alpha} = 0$ if $q_0 = 1$.

Next consider an arbitrary type $x \in x_n$, $n \geq 1$. By equations (5) and (8), we have

$$\frac{dw_n(x)}{d\alpha} = \frac{(1-\beta)(r+\delta)\beta(\theta_n x - rV_n - b) - (r+\delta+\beta\alpha)\beta(r+\delta+\alpha)r\frac{dV_n}{d\alpha}}{(r+\delta+\beta\alpha)^2}.$$

From the above expression, we can clearly see that $\frac{dw_n(x)}{d\alpha}$ is increasing in $x$. That is, a higher type worker benefits more from an increase in $\alpha$ than a lower type. This is because, as the contact rate increases, firms' improvement in position is proportional to $\overline{x}_n$, while a type $x$ worker's improvement in position is proportional to $x$. This implies the following general pattern as $\alpha$ increases if $q_n$ is close to 1: in submarket $n$ the wages of higher types of workers increase, while those of lower types decrease. Intuitively, the negative externality imposed by higher types on lower types is magnified when $\alpha$ increases.

A related observation is that in each submarket $n$ ($n \geq 1$), the wage schedule $w_n(x)$ becomes steeper as $\alpha$ increases. To see this, from the wage equation (7),

$$\frac{\partial w_n(x)}{\partial x} = \frac{\beta[r+\delta+\alpha]}{r+\delta+\beta\alpha}\theta_n \text{ and } \frac{\partial^2 w_n(x)}{\partial x \partial \alpha} \propto (1-\beta)(r+\delta)\beta\theta_n > 0.$$

This is because an increase in $\alpha$ means that a worker of type $x$ gets a bigger share of $\theta_n x$, while an increase in $\alpha$ means type $n$ firms get a bigger share of $\theta_n \overline{x}_n$, which is independent of $x$

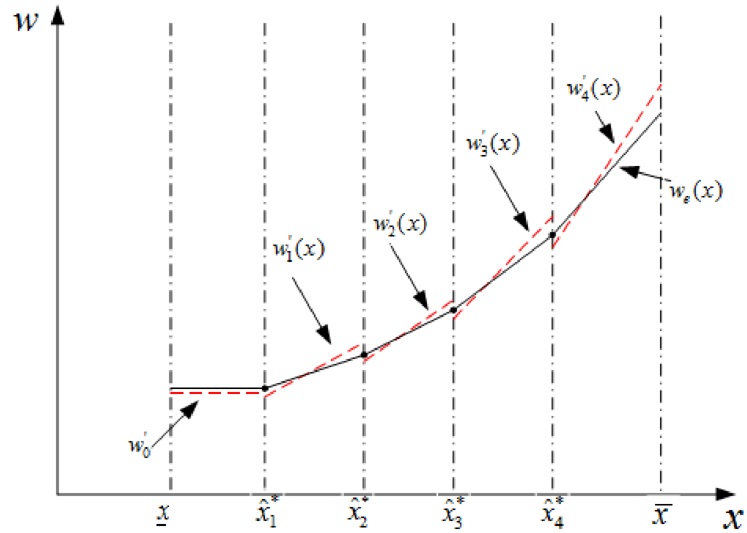


Figure 9: The Impacts of an Increase in $\alpha$ with Fixed Segmentation

given a fixed segmentation. In addition, the intercept of the wage schedule $w_n(x)$ also changes due to the change in type $n$ firms' position. Previous results indicate that, as long as $q_n$ is not too small relative to 1, firms' position will improve and the intercept of the wage schedule decreases.

The impacts of an increase in $\alpha$ on the wage schedules, fixing the initial equilibrium segmentation, are illustrated in Figure 9, where the solid lines indicate the initial equilibrium wage schedules, while the dashed lines represent the new ones.

### 6.2 Overall Impacts

As indicated in Figure 9, after an increase in $\alpha$, under the initial equilibrium segmentation the wage schedule is no longer continuous. Thus, the segmentation must adjust to restore equilibrium. But how the segmentation will adjust depends on the initial equilibrium segmentation, which in turn depends on the primitives of the model: the distribution of worker types and the distribution of firm types. In the ensuing analysis, we will mainly focus on the following scenario (illustrated in Figure 9): the primitives of the model are such that in the initial equilibrium in each submarket $q_n^* > 1$ but not far away from 1. Roughly speaking, the following conditions are largely adequate to generate such an initial equilibrium segmentation pattern: $\sum_n Y_n < 1 (= X)$ but is not too far away from 1 (in aggregate there are more workers than jobs but the difference is not too large), and the distribution of worker types is not too irregular.[35] We focus on this scenario because it resembles the real world situation: in labor

[35] The distribution of worker types being not too irregular means that in equilibrium $q_n^*$ will not vary too much across submarkets.

markets workers are usually on the long side.

Given the condition that in the initial equilibrium in each submarket $q_n^* > 1$ but not too far away from 1, the analysis in the previous subsection leads to the following results. If the initial equilibrium segmentation $\overline{P}$ does not change, then as $\alpha$ increases to $\alpha' > \alpha$ we have: (1) the wage in submarket 0 decreases, since $q_0^* > 1$; (ii) in submarket $n \geq 1$, there is an $\widetilde{x}_n \in (\overline{x}_n, x_{n+1}^*)$ such that $w_n'(\widetilde{x}_n, \overline{P}) = w_n(\widetilde{x}_n)$, and the wages of the types higher than $\widetilde{x}_n$ increase (since $q_n^*$ is not too far away from 1), and the wages of the types lower than $\widetilde{x}_n$ decrease (because $q_n^* > 1$).

**Proposition 10** *Suppose in the initial equilibrium, for all $n$, $q_n^* \geq 1$ but is not too far away from 1, and $q_0^*$ and $q_1^*$ are not too far apart. Suppose the contact rate $\alpha$ increases to $\alpha' > \alpha$. Denote all the equilibrium variables under $\alpha'$ by superscript $'$. Then, we have, (i) $x_n^{*\prime} > x_n^*$ for all $n$; (ii) for workers with $x \leq x_1^{*\prime}$, $w_e'(x) < w_e(x)$; for workers with $x \geq \widetilde{x}_N$, $w_e'(x) > w_e(x)$; (iii) in each submarket $n \geq 1$, the wage schedule becomes steeper: $\frac{\partial w_n'(x)}{\partial x} > \frac{\partial w_n(x)}{\partial x}$.*

Part (i) of Proposition 10 implies that an Internet-induced increase in search efficiency makes matching more assortative (fewer workers participating in higher submarkets). To understand this result, recall that if the segmentation does not change, an increase in the contact rate $\alpha$ makes the wage schedule in each submarket $n$ ($n \geq 1$) steeper, or the return of skills/abilities becomes higher. Moreover, in each submarket $n \geq 1$, the higher types now get higher wages, while the lower types get lower wages, than before.[36] This means that the lowest types in each submarket $n \geq 1$ now have incentives to switch to submarket $n-1$, where they will be among the highest types. Therefore, in the new equilibrium all the cutoff types increase.

Another way to understand the change in the equilibrium segmentation pattern is as follows. Essentially, as the contact rate increases, in any given submarket any type of worker is now counted more than one worker of that type under the initial contact rate. In other words, the effective number of any type of worker increases. Due to the negative externality imposed by higher type workers on lower type workers within any given submarket, naturally fewer workers will participate in higher submarkets; that is, matching becomes more assortative.

Part (iii) of Proposition 10 indicates that an Internet-induced increase in search efficiency unambiguously makes wages more unequal within each submarket $n \geq 1$. Again, this is because an increase in $\alpha$ increases the return to ability in each submarket. Part (ii) of Proposition 10 shows that an Internet-induced increase in search efficiency widens the support of the wage distribution: wages at the top end increase while wages at the low end decrease. Workers of the highest types (the highest types in submarket $N$) get higher wages than before for two reasons.

[36] In submarket 0 the wage decreases. But the reduction in $w_0$ is smaller than the reduction in $w_1(x_1^*)$. The condition that $q_0^*$ and $q_1^*$ are not too far apart ensures this is the case, since in submarket 1 type $x_1^*$ suffers from the negative externality imposed by the higher type workers. As a result, the lowest types in submarket 1 have incentives to switch to submarket 0.

First, an increase in $\alpha$ directly increases the return of their (high) ability. Second, they also benefit from the change in endogenous segmentation: some lowest types in submarket $N$ switch to submarket $N-1$, increasing the market tightness of submarket $N$. On the other hand, for similar reasons workers of the lowest types (those in submarket 0) get lower wages in the new equilibrium. Specifically, those workers suffer not only from the direct impact of an increase in $\alpha$, which reduces market tightness, but also from the adjustment in endogenous segmentation: submarket 0 will have more workers (switching from submarket 1), which further dampens the wage of the least productive workers.

Now we consider the middle types. Define $\Delta w_e(x) = w_e'(x) - w_e(x)$ as the change in wage for type $x$. The following proposition characterizes $\Delta w_e(x)$ for the middle types.

**Proposition 11** *Suppose the assumptions of Proposition 10 hold. Then we have the following properties. (i) For the nonswitching worker types in submarket $n$ ($n \geq 1$), $x \in [x_n^{*\prime}, x_{n+1}^*]$, $\Delta w_e(x)$ is increasing in $x$. Now suppose $\Delta\alpha \equiv \alpha' - \alpha$ is small. (ii) For the switching worker types (from submarket $n+1$ to submarket $n$), $x \in [x_{n+1}^*, x_{n+1}^{*\prime}]$, $\Delta w_e(x)$ is decreasing in $x$. (iii) Across submarkets the general pattern is that workers in higher submarkets benefit more: $\Delta w_e(x_n^{*\prime}) < \Delta w_e(x_{n+1}^{*\prime})$.*

Part (i) of Proposition 11 is easy to understand. It is due to the fact that in the new equilibrium the wage schedule is steeper in each submarket. Since the non-switching worker types in submarket $n$ remain in the same submarket, the higher types must benefit more (or lose less) than the lower types as $\alpha$ increases. To understand part (ii), note that the switching types (switch from submarket $n+1$ to $n$) changed submarkets. Whether the higher types benefit more (or get hurt less) than the lower types depends on the slope of the wage schedule in submarket $n+1$ in the initial equilibrium and the slope of the wage schedule in submarket $n$ in the new equilibrium. If the former is bigger than the latter, which is the case if the increase in $\alpha$ is small, then the higher types benefit less (or lose more) than the lower types. This is because in this case the return of ability becomes smaller among these types in the new equilibrium. Combining parts (i) and (ii), the following pattern emerges when the increase in $\alpha$ is small: in the new equilibrium, in submarket $n$ $\Delta w_e(x)$ increases in $x$ among the nonswitching worker types (lower types in the new segmentation), but it decreases in $x$ among the switching worker types (higher types in the new segmentation). Moreover, it is possible that $\Delta w_e(x)$ is positive for all nonswitching worker types, but it is negative for the highest switching worker types.

Part (iii) indicates that, if the increase in $\alpha$ is small, then the general pattern is that workers in higher submarkets gain more (or lose less). Since an increase in $\alpha$ can always be decomposed into several small enough increases, this general pattern holds for any increase in $\alpha$. Recall that the wage schedule is always continuous across submarkets in equilibrium, and an increase in $\alpha$ widens the support of the wage distribution and makes the wage schedules steeper in every submarket. Taken together, the general pattern is that all workers' wages in lower submarkets decrease, while almost all workers' wages in higher submarkets increase. Therefore, an Internet-induced increase in search efficiency increases wage inequality across submarkets.

Finally, we want to emphasize two points. First, the general pattern that wage inequality across submarkets increases in $\alpha$ is due to the adjustment in endogenous segmentation. Recall that if the initial equilibrium segmentation does not change, then as $\alpha$ increases in each submarket the highest types earn higher wages while the lowest types earn lower wages. It is the adjustment in endogenous segmentation that redistributes the gains and losses across submarkets. Since the direction of adjustment is for the lowest types in each submarket to switch to the adjacent lower submarket, workers in lower submarkets tend to lose while those in higher submarkets tend to benefit from this adjustment.

Second, although both an SBTC in the most productive submarket and an Internet-induced increase in search efficiency increase overall wage inequality, their impacts on the matching pattern are the opposite. In particular, while the former makes the matching pattern less assortative (more workers in higher submarkets), the latter leads to a more assortative matching (fewer workers in higher submarkets). This difference in predictions makes it possible to empirically distinguish the impacts of an SBTC and those of an Internet-induced increase in search efficiency.

**Example 7** *In the benchmark case, suppose $\alpha$ increases from 0.1 to 0.25. The changes in the equilibrium wage schedule are illustrated in Figure 10. We can see that the adjustments of all the cutoffs are significant.*[37] *The wages of all types higher than 2.76 increase, while the wages of all types below 2.76, which includes all types in submarkets 0, 1, 2, and 3, decrease. In absolute terms, the wage decrease in submarket 0 is insignificant (from 0.8533 to 0.8493), the wage decreases in submarket 1, 2, and 3 are relatively significant, and submarket 2 (the middle class) experiences the biggest wage drop.*

The reason that submarket 2 (the middle class) suffers the most is twofold. First, compared to submarket 0 and 1, $q_2^*$ is a lot bigger than $q_1^*$ and $q_0^*$ (roughly 3 versus 0.5).[38] This implies two things. First, fixing the initial equilibrium segmentation, the average types in submarkets 0 and 1 actually both benefit from an increase in $\alpha$, while the average type in submarket 2 becomes worse off. Second, the wages in submarkets 0 and 1 are not sensitive to changes in segmentation, but the wages in submarket 2 are sensitive. Taken together, the wage decreases in submarket 2 are bigger than those in submarket 0 and 1. Second, the main reason as to why the wage drops are bigger in submarket 2 than those in submarket 3 is that the ratio $q_2^*/q_1^*$ is significantly higher than the ratio $q_3^*/q_2^*$ (roughly 6 versus 1.5). Recall that both submarkets 2

[37] Recall that the adjustments of the cutoffs are small with a SBTC in submarket 4.

[38] Some of the conditions specified in Proposition 10 are not satisfied in this example. But the pattern of adjustments, both in terms of the wages and the matching pattern, is still the same as that predicted by Propositions 10 and 11.

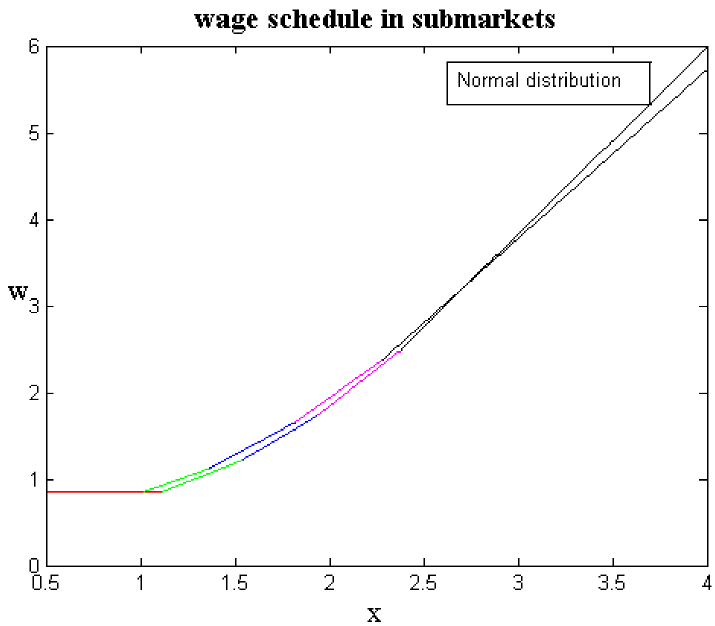


Figure 10: The Impact of an Increase in Search Efficiency

and 3 have higher types moving in and lower types moving out. But since the ratio $q_2^*/q_1^*$ is a lot higher than $q_3^*/q_2^*$, submarket 3 benefits more from lower types moving out than submarket 2 does. As a result, the wage drops are bigger in submarket 2.

This example actually shows that an Internet-induced increase in search efficiency does not only increase overall wage inequality, but also generates a pattern of wage polarization. To see this, note that the median worker type is around $x = 1.75$, who is always in submarket 2. Thus worker types within $[1.75, 4]$ belong to the upper tail, while those within $[0.5, 1.75]$ are in the lower tail. From Figure 10, we can see that an increase in $\alpha$ increases wage inequality in the upper tail: the wages of higher type workers increase, the wages of lower type workers decrease, and in general a higher type worker gains more or loses less than a lower type worker. In the lower tail wage inequality actually decreases: the wages of all types of workers decrease, but the wage reductions in general are bigger among higher types.

# 7 Conclusion and Discussion

This paper studies a targeted search/matching model with heterogeneous workers and heterogeneous firms. In terms of modeling, the main innovation is that we introduce targeted search: workers can choose in which submarket to participate beforehand, but search is random within each submarket. It falls somewhere in between random search and directed search, the two main search protocols in the labor search literature. In equilibrium, workers are endogenously segmented into different submarkets, and the segmentation/matching is weakly positively assortative. We show that a market equilibrium always exists and is unique.

An SBTC in the most productive submarket increases wages for all types of workers, and matching becomes less assortative. But quantitatively the positive impact is largely confined within the highest worker types, and wage inequality increases. When the number of jobs in the least productive submarket decreases, matching becomes less assortative as well, and wages of all types of workers decrease. But quantitatively again the negative impact is largely confined within the lowest worker types, leading to an increase in wage inequality. When a shock occurs to some middle submarket, the shock transmission quantitatively exhibits asymmetry: while the wages in the higher submarkets are significantly affected, the wages in the lower submarkets are barely affected. A combination of an SBTC in the most productive submarket and a negative trade shock in the second least productive submarket can generate a pattern of wage polarization.

The widespread use of the Internet over the last two decades could indeed have contributed to rising wage inequality. When an Internet-induced increase in search efficiency occurs, matching becomes more assortative, the wage schedule in each submarket becomes steeper, and the wages of the highest types increase while those of the lowest types decrease. Therefore, it not only increases wage inequality among workers having similar jobs, but also increases overall wage inequality across all types of workers. Moreover, our example also shows that it can generate a pattern of wage polarization.

In the rest of this section, we discuss how the main results of our model will change when we relax some of the assumptions.

**Free entry** In the model we have assumed that the number of jobs in each submarket is fixed and exogenously given. Here we briefly discuss how to incorporate free entry and endogenize the number of jobs. Like Moen (1997), we assume that firms are ex ante homogeneous before entry. If a firm decides to enter, then it pays a sunk cost of entry and draws a productivity type according to an exogenously fixed probability distribution. Thus the number of jobs in a particular submarket is always a fixed fraction of the total number of jobs. The only difference is that the total number of jobs is now endogenously determined by free entry: a firm's ex ante expected gross profit after entering must be equal to the entry cost.

The existence and uniqueness of the equilibrium, as well as the equilibrium properties, will not be qualitatively affected by free entry, as it just adds a free entry condition to determine the total number of jobs. However, free entry would certainly complicate the comparative statics analysis. For instance, an SBTC in some high submarket would also induce an adjustment in the total number of jobs. Nevertheless, we expect all the comparative statics results to hold qualitatively even with free entry. This is because the induced adjustment in the total number of jobs will be relatively small in magnitude, and it will not overturn the pattern of the impacts of the shock on different submarkets without free entry. The underlying reason is that the free entry condition only determines the total number of jobs, but not the fraction of jobs in each submarket. As a result, the comparison across submarkets will not be qualitatively affected by free entry. For instance, when an SBTC occurs in the most productive submarket, firms' value in each submarket increases, thus more firms will enter. This further increases workers' wages. However, it will not qualitatively change the comparison of wages across submarkets, as the fraction of jobs in each submarket is always the same.

**Urn ball matching technology** The urn ball matching technology greatly simplifies our analysis. In particular, it means that a worker's contact rate in any submarket, regardless of the worker's type and market tightness, is always fixed. It further implies that a worker's expected waiting time before getting employed in any submarket is always the same. As a result, each worker will choose the submarket which gives him the highest wage. With a more general matching technology, a worker's contact rate in submarket $n$ will negatively depend on $q_n$, the expected queue length. It implies that a worker's expected waiting time before getting employed in submarket $n$ is increasing in $q_n$. As a result, figuring out in which submarket a worker will choose to participate becomes more complicated, as his lifetime discounted utility depends on both the wage and the expected waiting time. In a nutshell, a more general matching technology introduces direct coordination/congestion friction among workers (they impose direct externalities among each other through the contact rate), which is absent under the urn ball matching technology. However, we believe that having direct coordination/congestion friction will not qualitatively affect our main results. This is because the direct externality due to the congestion friction works in the same direction as the indirect externality that has been captured in our model: a larger $q_n$ increases firms' contact rate and hence their continuation value, which through Nash bargaining reduces workers' wages and the attractiveness of submarket $n$ to workers. Quantitatively, we conjecture that introducing the direct congestion externality would reduce the differences of $q_n$ across submarkets in the equilibrium, as the impact of $q_n$ is magnified with direct congestion.

**The finiteness of firm types** This is mainly a modeling device, which makes the analysis tractable. In some sense, this assumption is also realistic, as anecdotally the labor market is vertically segmented into a finite number of sectors. Moreover, it is reasonable to think that workers are not able to categorize the submarkets in a very fine way. In essence, firm types being finite and each firm type constituting a distinctive submarket make the firm side homogeneous in each submarket. This feature greatly simplifies the analysis. To see this, consider the case that firm types are also continuous, and firms are exogenously partitioned into several submarkets according to their types. In this scenario, in each submarket both workers and firms are heterogeneous. With two-sided heterogeneity, the equilibrium in each submarket is hard to characterize (with the same difficulty as solving the equilibrium in the whole market with completely random search), and thus targeted search buys nothing. In a nutshell, targeted search combined with the finiteness of firm types separates the matching problem in the whole market with two-sided heterogeneity into a two-stage problem, with the problem in each stage being tractable. Specifically, in the second stage, in each submarket we solve a matching problem with one-sided heterogeneity and random search. And in the first stage, workers, anticipating the outcomes in the second stage, choose which submarket to enter. Within our framework, we can approximate the case that firm types are continuous, by increasing the number of firm types $N$ and letting it approaches infinity.[39]

[39] We can also make worker types finite. But this will bring inconvenience, as the marginal types of workers will play mixed strategies ( entering two adjacent submarkets with positive probabilities).

# Appendix

**Proof of Lemma 3.**

**Proof.** We first show that, for all $n \geq 1$, $w_n(x_n^*) \geq w_0 > b$. To see this, note that $w_1(x_1^*) = w_0$ by the indifference condition (10). Since $w_n(x)$ is increasing in $x$, we have $w_2(x_2^*) = w_1(x_2^*) > w_1(x_1^*) = w_0$. By induction, for all $n \geq 1$ we have $w_n(x_n^*) > w_0 > b$.

Next we show that, for any $n \geq 1$, $J_n(x_n^*) > V_n$ if type $n$ firms accept all workers with $x \in (x_n^*, x_{n+1}^*]$. To see this, inspect equation (9). Since $w_n(x_n^*) > b$, equation (9) implies that $J_n(x_n^*) > V_n$. Since $J_n(x)$ is increasing in $x$, this property also implies that $J_n(x) > V_n$ for any $x \in [x_n^*, x_{n+1}^*]$.

Finally, we rule out the possibility that a type $n$ firm can increase its $V_n$ by accepting only a strict subset of workers. Since $J_n(x)$ is increasing, we only need to worry about a firm accepting higher types but rejecting lower types. Suppose a type $n$ firm unilaterally deviates to only accepting workers whose types are within $[\widetilde{x}_n, x_{n+1}^*]$, $\widetilde{x}_n \in (x_n^*, x_{n+1}^*)$, and rejecting all types below $\widetilde{x}_n$. Note that this individual deviation will not affect the market wage schedule, $w_n(x)$, as it is pinned down by the market conditions. Denote $\widetilde{V}_n$ as a type $n$ firm' value deviating to this strategy, and $\widetilde{J}_n(x)$ and $\widetilde{q}_n < q_n$ are defined accordingly.[40] Suppose there is a $\widetilde{x}_n$ such that $\widetilde{V}_n > V_n$. Using the value functions of $V_n$ and $J_n(x)$, we can express the value of $\widetilde{V}_n$ as

$$r\widetilde{V}_n = \frac{\alpha \widetilde{q}_n E_{x \in [\widetilde{x}_n, x_{n+1}^*]}[\theta_n x - w_n(x)]}{r + \delta + \alpha \widetilde{q}_n}. \tag{13}$$

The value function of $V_n$ can also be written as

$$rV_n = \alpha \widetilde{q}_n [E_{x \in [\widetilde{x}_n, x_{n+1}^*]}[J_n(x)] - V_n] + \alpha (q_n - \widetilde{q}_n)[E_{x \in [x_n^*, \widetilde{x}_n]}[J_n(x)] - V_n].$$

Using the value function of $J_n(x)$, we can express $V_n$ as

$$rV_n = \frac{\alpha \widetilde{q}_n E_{x \in [\widetilde{x}_n, x_{n+1}^*]}[\theta_n x - w_n(x)]}{r + \delta + \alpha \widetilde{q}_n} + \alpha (q_n - \widetilde{q}_n)[E_{x \in [x_n^*, \widetilde{x}_n]}[J_n(x)] - V_n]. \tag{14}$$

By the expressions of (13) and (14), $\widetilde{V}_n > V_n$ implies that $E_{x \in [x_n^*, \widetilde{x}_n]}[J_n(x)] - V_n < 0$. Since $J_n(x_n^*) < E_{x \in [x_n^*, \widetilde{x}_n]}[J_n(x)]$, we have $J_n(x_n^*) < V_n$, a contradiction. Therefore, there is no $\widetilde{x}_n \in (x_n^*, x_{n+1}^*)$ such that an individual type $n$ firm can increase its $V_n$ by accepting only worker types above type $\widetilde{x}_n$. ■

**Proof of Lemma 4.**

**Proof.** Part (i). By (5), it is immediate that $w_n'(x') > w_n''(x')$ if and only if $V_n' < V_n''$.

Part (ii). As $\widehat{x}_1$ increases, the measure of workers active in submarket 0, $X_0$, increases. It follows from the steady-state equation that $q_0$ increases. By part (i) of Lemma 2, $\frac{\partial w_0}{\partial q_0} < 0$. Thus, $w_0$ is strictly decreasing in $\widehat{x}_1$. By part (i), it follows that $V_0$ is strictly increasing in $\widehat{x}_1$.

[40] Specifically, $\widetilde{q}_n = q_n \frac{F(x_{n+1}^*) - F(\widetilde{x}_n)}{F(x_{n+1}^*) - F(x_n^*)}$.

Part (iii). Suppose $\widehat{x}_N$ decreases to $\widehat{x}'_N < \widehat{x}_N$. The measure of workers active in submarket $N$, $X_N$, increases. It follows from the steady-state equation that $q_N$ increases to $q'_N$. For the corresponding $V_N$ and $V'_N$, we must have $V'_N \geq V_N$. This is because type $N$ firms can always accept only the original set of workers $x \geq \widehat{x}_N$, which guarantees $V'_N \geq V_N$. If type $N$ firms have incentives to accept type $\widehat{x}'_N$, then $V'_N \geq \theta_N \widehat{x}'_N + b$. By an argument similar to the proof of Lemma 3, it implies that $V'_N > V_N$.

Part (iv). Given that $[\widehat{x}_n, \widehat{x}_{n+1}] \sqsubseteq [\widehat{x}'_n, \widehat{x}'_{n+1}]$, $V'_N \geq V_N$ follows immediately since type $n$ firms can always accept only the original set of workers $[\widehat{x}_n, \widehat{x}_{n+1}]$. Now suppose $\widehat{x}_{n+1} < \widehat{x}'_{n+1}$ and $\widehat{x}'_n \leq \widehat{x}_n$. With the new set of participating workers' types, type $n$ firms can always accept only the higher type workers while rejecting the lower types such that the measure of accepted workers $X'_n = X_n$, and thus $q'_n = q_n$. Since $\widehat{x}_{n+1} < \widehat{x}'_{n+1}$, the new accepted set of workers are on average more productive than the original set. By equation (8), $V'_N > V_N$. Now consider the case that $\widehat{x}_{n+1} \leq \widehat{x}'_{n+1}$ and $\widehat{x}'_n < \widehat{x}_n$ and firms accept type $\widehat{x}'_n$ workers. This case is isomorphic to the proof in part (iii), thus a similar argument follows. ■

**Proof of Lemma 6.**

**Proof.** Since the results of Lemma 5 apply to $i = n-1$, given $\widehat{x}_{n+1}$ we only need to focus on $\widehat{x}_n$, or the segmentation between submarket $n$ and the rest of the lower submarkets, and the equilibrium indifference condition between submarkets $n-1$ and $n$. Denote a segmentation with $\widehat{x}_n$ and the corresponding $\{x_i^*\}_{i=1}^{n-1}$ induced by $\widehat{x}_n$ as $\overline{P}(\widehat{x}_n)$. From the wage equation (6), we can compute $w_n(\widehat{x}_n; \overline{P}(\widehat{x}_n))$ and $w_{n-1}(\widehat{x}_n; \overline{P}(\widehat{x}_n))$, which define two wage schedules as a function of $\widehat{x}_n$. Note that both wage schedules are continuous in $\widehat{x}_n$. Again, possibly there are two kinds of equilibria, with the equilibrium conditions explicitly listed below:

| | |
|---|---|
| Corner equilibrium: | $x_n^* = \underline{x}$ if $w_n(\underline{x}; \overline{P}(\underline{x})) \geq \overline{w}_{n-1}(\underline{x})$, |
| Interior Equilibrium: | $x_n^* \in (\underline{x}, \widehat{x}_{n+1})$ if there is an $\widehat{x}_n$ such that $w_n(\widehat{x}_n; \overline{P}(\widehat{x}_n)) = w_{n-1}(\widehat{x}_n; \overline{P}(\widehat{x}_n))$. |

Now we examine the properties of the wage schedules $w_n(\widehat{x}_n; \overline{P}(\widehat{x}_n))$ and $w_{n-1}(\widehat{x}_n; \overline{P}(\widehat{x}_n))$. As to $w_n(\widehat{x}_n; \overline{P}(\widehat{x}_n))$, $w_n = w_n(\underline{x}; \overline{P}(\underline{x}))$ when $\widehat{x}_n = \underline{x}$, it monotonically increases as $\widehat{x}_n$ increases, and it reaches $\overline{w}_n(\widehat{x}_{n+1})$ when $\widehat{x}_n = \widehat{x}_{n+1}$. Note that the monotonicity follows Lemma 4: as $\widehat{x}_n$ increases fewer workers are active in submarket $n$ and thus all wages in this submarket must increase. Regarding $w_{n-1}(\widehat{x}_n; \overline{P}(\widehat{x}_n))$, it starts at $\overline{w}_{n-1}(\underline{x})$ when $\widehat{x}_n = \underline{x}$, and it reaches $w_{n-1}(\widehat{x}_{n+1}; \overline{P}(\widehat{x}_{n+1}))$ when $\widehat{x}_n = \widehat{x}_{n+1}$. We do not know whether $w_{n-1}(\widehat{x}_n; \overline{P}(\widehat{x}_n))$ is increasing or decreasing in $\widehat{x}_n$, as there are two opposite effects.

Existence. We first show that an equilibrium $x_n^*$ must exist. To show this, consider two possible cases. Case 1: suppose $w_n(\underline{x}; \overline{P}(\underline{x})) \geq \overline{w}_{n-1}(\underline{x})$. Then by the equilibrium condition, the unique equilibrium is a corner equilibrium $x_n^* = \underline{x}$, with all workers participating in submarket $n$. Case 2: $w_n(\underline{x}; \overline{P}(\underline{x})) < \overline{w}_{n-1}(\underline{x})$. In this case, the corner equilibrium does not exist. However, an interior equilibrium exists. To see this, note that $w_{n-1}(\widehat{x}_{n+1}; \overline{P}(\widehat{x}_{n+1})) \leq \overline{w}_{n-1}(\widehat{x}_{n+1}) < \overline{w}_n(\widehat{x}_{n+1})$. Combining this with the condition that $w_n(\underline{x}; \overline{P}(\underline{x})) < \overline{w}_{n-1}(\underline{x})$, we have that, in the domain of $[\underline{x}, \widehat{x}_{n+1}]$, the wage schedule $w_{n-1}(\widehat{x}_n; \overline{P}(\widehat{x}_n))$ starts above the wage schedule $w_n(\widehat{x}_n; \overline{P}(\widehat{x}_n))$, but it ends up below $w_n(\widehat{x}_n; \overline{P}(\widehat{x}_n))$. Since both wage schedules are continuous in $\widehat{x}_n$, they must have an interior intersection, which is an interior equilibrium. This completes the proof of equilibrium existence.

Uniqueness. Note that we only need to establish the uniqueness in case 2 (interior equilibrium). Suppose there are two different interior equilibria $A$ and $B$, with $x^*_{nA} > x^*_{nB}$. By the presumption that the results of this lemma hold for $n-1$, the fact that $x^*_{nA} > x^*_{nB}$ implies that $x^*_{(n-1)A} \geq x^*_{(n-1)B}$, and $V^*_{(n-1)A} > V^*_{(n-1)B}$. Now consider submarket $n$. By Lemma 4, the fact that $x^*_{nA} > x^*_{nB}$ implies that $V^*_{nB} > V^*_{nA}$ (given that in equilibrium $B$ type $n$ firms accept worker type $x^*_{nB}$). By the monotonicity of the wage schedule and Lemma 4, we have $w^B_n(x^*_{nB}) < w^B_n(x^*_{nA}) < w^A_n(x^*_{nA})$. Given that in equilibrium $B$, type $n$ firms accept worker type $x^*_{nB}$, in equilibrium $A$ type $n$ firms are also willing to accept type $x^*_{nB}$, since $V^*_{nA} < V^*_{nB}$. The fact that type $x^*_{nB}$ workers choose type $n-1$ firms in equilibrium $A$ implies that $w^A_{n-1}(x^*_{nB}) > w^A_n(x^*_{nB})$. But by previous results, $w^A_n(x^*_{nB}) > w^B_n(x^*_{nB}) = w^B_{n-1}(x^*_{nB})$. Thus, we must have $w^A_{n-1}(x^*_{nB}) > w^B_{n-1}(x^*_{nB})$. By Lemma 4, this means that $V^*_{(n-1)A} < V^*_{(n-1)B}$, a contradiction. Therefore, there cannot be multiple equilibria.

Finally, we show that in the partial equilibrium $x^*_n$ is increasing in $\widehat{x}_{n+1}$. Consider two cutoffs, $\widehat{x}'_{n+1} > \widehat{x}_{n+1}$. By Lemma 4, $w_n(\underline{x}; \overline{P}'(\underline{x})) < w_n(\underline{x}; \overline{P}(\underline{x}))$. Thus, by the equilibrium condition of the corner equilibrium, with $\widehat{x}'_{n+1}$ the corner equilibrium becomes more difficult to exist. Therefore, if with $\widehat{x}_{n+1}$ the original equilibrium is a corner one, $x^*_n = \underline{x}$, then with $\widehat{x}'_{n+1}$ the new equilibrium is either the corner one or an interior one. In either case, $x^{*\prime}_n \geq x^*_n$. Now suppose, with $\widehat{x}_n$ the original equilibrium is an interior one, $x^*_n > \underline{x}$. Then $x^{*\prime}_n$ must be interior as well. Suppose to the contrary, $x^{*\prime}_n \leq x^*_n$. By Lemma 4, $\widehat{x}'_{n+1} > \widehat{x}_{n+1}$ and $x^{*\prime}_n \leq x^*_n$ imply that $V^{*\prime}_n > V^*_n$. Again by Lemma 4, this means that $w_n(x^*_n) > w'_n(x^*_n)$. By the presumption that the results of this lemma hold for $n-1$, the condition $x^{*\prime}_n \leq x^*_n$ implies $V^*_{n-1} \geq V^{*\prime}_{n-1}$, which by Lemma 4 implies that $w'_{n-1}(x^*_n) \geq w_{n-1}(x^*_n)$. By the indifference condition in the original equilibrium $w_{n-1}(x^*_n) = w_n(x^*_n)$, we have $w'_{n-1}(x^*_n) > w'_n(x^*_n)$. This means that with $\widehat{x}'_{n+1}$ type $x^*_n$ strictly prefers submarket $n-1$ to submarket $n$. Following the single-crossing property of Lemma 1, we reach the conclusion that with $\widehat{x}'_{n+1}$ type $x^{*\prime}_n$ strictly prefers submarket $n-1$ to submarket $n$, which contradicts to the fact that in equilibrium type $x^{*\prime}_n$ worker should be indifferent between submarkets $n-1$ and $n$. Therefore, we must have $x^{*\prime}_n > x^*_n$.

Given that $x^*_n$ is increasing in $\widehat{x}_{n+1}$, the proof that $V^*_n$ is strictly increasing in $\widehat{x}_{n+1}$ is similar to the proof that $V_1$ is strictly increasing in $\widehat{x}_2$ in Lemma 5, thus is omitted. ■

**Proof of Proposition 5.**

**Proof.** Part (i). We only need to show $x^{*\prime}_N < x^*_N$. This is because if this is the case, then Corollary 1 implies that $x^{*'}_n < x^*_n$ for all $n = 1, ..., N-1$. Suppose to the contrary, $x^{*\prime}_N \geq x^*_N$. Denote the hypothetical new segmentation as $\overline{P}'$. By Corollary 1, $x^{*\prime}_N \geq x^*_N$ implies that $x^{*\prime}_{N-1} \geq x^*_{N-1}$ and $V'_{N-1} \geq V^*_{N-1}$. Now consider submarket $N$. By previous results, we have

$w_N'(x_N^*;\overline{P}) > w_N(x_N^*;\overline{P})$. Moreover, it also implies that type $N$ firms will still accept type $x_N^*$ workers under $\theta_N'$. Now we compare $V_N'(\overline{P})$ and $V_N'(\overline{P}')$. Since $x_N^{*\prime} \geq x_N^*$ and firms are willing to accept type $x_N^*$ under $\theta_N'$, by Lemma 4 it must be the case that $V_N'(\overline{P}) \geq V_N'(\overline{P}')$ (fewer workers in submarket $N$ hurt firms). Denote $w_N'(x_N^*;\overline{P}')$ as the wage a type $x_N^*$ worker can get if he unilaterally deviates to submarket $N$ under $\theta_N'$ and segmentation $\overline{P}'$. Since $V_N'(\overline{P}) \geq V_N'(\overline{P}')$, by Lemma 4 we have $w_N'(x_N^*;\overline{P}') \geq w_N'(x_N^*;\overline{P}) > w_N(x_N^*;\overline{P})$. But the fact that in the equilibrium under $\theta_N'$ type $x_N^*$ workers choose type $N-1$ firms means that they must earn a higher wage by staying with type $N-1$ firms: $w_{N-1}'(x_N^*;\overline{P}') \geq w_N'(x_N^*;\overline{P}')$. Combining this with the previous result $w_N'(x_N^*;\overline{P}') > w_N(x_N^*;\overline{P})$, we have $w_{N-1}'(x_N^*;\overline{P}') > w_N(x_N^*;\overline{P}) = w_{N-1}(x_N^*)$. Applying Lemma 4, $w_{N-1}'(x_N^*;\overline{P}') > w_{N-1}(x_N^*)$ implies that $V_{N-1}' < V_{N-1}^*$. But this contradicts an earlier result that $V_{N-1}' \geq V_{N-1}^*$. Therefore, we must have $x_N^{*\prime} < x_N^*$.

Part (ii). By part (i), $x_n^{*'} < x_n^*$ for all $n$. Following Corollary 1, we have $V_n^{*\prime} < V_n^*$ for all $n \leq N-1$. First consider submarket $n$ with $n \leq N-1$. For worker types who stay in the same submarket $x \in [x_n^*, x_{n+1}^{*\prime}]$, we have $V_n^{*\prime} < V_n^*$, and Lemma 4 implies that $w_n'(x) > w_n(x)$. Now consider workers who switch from submarket $n$ to $n+1$: $x \in [x_{n+1}^{*\prime}, x_{n+1}^*]$. Since $V_n^{*\prime} < V_n^*$, in the new equilibrium under $\theta_N'$ a type $x$ worker could have earned wage $w_n'(x;\overline{P}') > w_n(x;\overline{P})$. The fact that he chooses type $n+1$ firms implies that $w_{n+1}'(x;\overline{P}') > w_n'(x;\overline{P}')$. Therefore, $w_{n+1}'(x;\overline{P}') > w_n(x;\overline{P})$. Thus we have proved that for all $x \leq x_N^*$, $w_e'(x) > w_e(x)$. Next, consider $x \geq x_N^*$. Recall that for type $x_N^*$, we have shown that $w_N'(x_N^*;\overline{P}') > w_{N-1}(x_N^*;\overline{P})$. By the equilibrium indifference condition, we have $w_N'(x_N^*;\overline{P}') > w_N(x_N^*;\overline{P})$. For any worker type $x > x_N^*$, by equation (5) we have

$$w_N(x) - w_N(x_N^*) = \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha}\theta_N(x - x_N^*).$$

A similar equation applies to the difference between $w_N'(x)$ and $w_N'(x_N^*)$. Thus,

$$[w_N'(x) - w_N(x)] - [w_N'(x_N^*) - w_N(x_N^*)] = \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha}(\theta_N' - \theta_N)(x - x_N^*) > 0.$$

Combining with $w_N'(x_N^*) - w_N(x_N^*) > 0$, we have $w_e'(x) - w_e(x) > 0$ for any $x > x_N^*$.

Part (iii). In the proof of parts (ii), we have shown that $V_n^{*\prime} < V_n^*$ for all $n \leq N-1$. Now we prove that $V_N^{*\prime} > V_N^*$. Given that in the new equilibrium type $N$ firms accept workers of type $x_N^{*'} < x_N^*$, it means that $V_N^{*\prime} > V_N'(\overline{P})$ (only accept workers whose $x \geq x_N^*$). Now compare $V_N'(\overline{P})$ and $V_N^*$. Note that they have the same $q_N$ and $E_{x_N}[x]$ in submarket $N$. By equation (8), we have

$$V_N^{*\prime}(\overline{P}) - V_N^* \propto (\theta_N' - \theta_N)E_{x_N}[x] > 0.$$

Therefore, $V_N^{*\prime} > V_N^*$.

Part (iv). By the steady state equations, $u_n$ and $q_n$ are both increasing in $X_n$. The results directly follow the change in the equilibrium segmentation pattern: $x_N^{*\prime} < x_N^*$ and $x_1^{*'} < x_1^*$. ■

**Proof of Proposition 6.**

**Proof.** Part (i). For $n \le N-1$, by equation (5), the equilibrium indifference conditions $w_n(x_n^*) = w_{n-1}(x_n^*)$ and $w_n'(x_n^{*\prime}) = w_{n-1}'(x_n^{*\prime})$ can be written as

$$\begin{aligned}(\theta_n - \theta_{n-1})x_n^* &= rV_n^* - rV_{n-1}^*,\\ (\theta_n - \theta_{n-1})x_n^{*\prime} &= rV_n^{*\prime} - rV_{n-1}^{*\prime}.\end{aligned}$$

From the above two equations and the result that $x_n^{*\prime} < x_n^*$, we have $\Delta V_n^* > \Delta V_{n-1}^*$.

Part (ii). Consider a worker with type $x \in [x_n^*, x_{n+1}^{*\prime}]$, $n \le N-1$. In both equilibria, this worker participates in submarket $n \le N-1$. Thus, by equation (5), the wage increase $\Delta w_e(x)$ can expressed as

$$\Delta w_e(x) = w_n'(x) - w_n(x) = \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha} r\Delta V_n^*,$$

which implies that $\Delta w_e(x)$ does not depend on $x$.

Part (iii). Consider a worker with type $x \in [x_{n+1}^{*\prime}, x_{n+1}^*]$, $n \le N-1$. This worker participates in submarket $n$ in the initial equilibrium, and he participates in submarket $n+1$ in the new equilibrium. Thus, by equation (5), the wage increase $\Delta w_e(x)$ can expressed as

$$\Delta w_e(x) = w_{n+1}'(x) - w_n(x) = \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha}[(\theta_{n+1} - \theta_n)x - (rV_{n+1}^{*\prime} - rV_n^*)],$$

which is clearly increasing in $x$.

Part (iv). First consider a worker with type $x \in [x_N^{*\prime}, x_N^*]$. By equation (5), the wage increase $\Delta w_e(x)$ can expressed

$$\Delta w_e(x) = w_N'(x) - w_{N-1}(x) = \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha}[(\theta_N' - \theta_{N-1})x - (rV_N^{*\prime} - rV_{N-1}^*)],$$

which is clearly increasing in $x$. Next consider a worker with type $x \ge x_N^*$. His wage increase can be expressed as

$$\Delta w_e(x) = w_N'(x) - w_N(x) = \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha}[(\theta_N' - \theta_N)x - (rV_N^{*\prime} - rV_N^*)],$$

which is also increasing in $x$, as $\theta_N' > \theta_N$. ■

**Proof of Proposition 7**.

**Proof.** Part (i). We only need to show $x_1^{*\prime} < x_1^*$. This is because if this is the case, then Corollary 1 implies that $x_n^{*'} < x_n^*$ for all $n = 2, ..., N$. Suppose to the contrary, $x_1^{*\prime} \ge x_1^*$. Denote the hypothetical new segmentation as $\overline{P}'$. By Corollary 1, $x_1^{*\prime} \ge x_1^*$ implies that $V_1'(\overline{P}') \le V_1^*$. Now consider submarket 0. Since $Y_0' < Y_0$ and $x_1^{*\prime} \ge x_1^*$, the steady state equation imply that $q_0' > q_0$. By wage equation (3), we have $w_0' < w_0$. This further implies that $w_1'(x_1^{*\prime}; \overline{P}') < w_1(x_1^{*\prime})$, since $w_1'(x_1^{*\prime}; \overline{P}') = w_0' < w_0 = w_1(x_1^*) \le w_1(x_1^{*\prime})$. Applying Lemma 4 in submarket 1, $w_1'(x_1^{*\prime}; \overline{P}') < w_1(x_1^{*\prime})$ means that $V_1'(\overline{P}') > V_1^*$. This contradicts an earlier result that $V_1'(\overline{P}') \le V_1^*$. Therefore, we must have $x_1^{*\prime} < x_1^*$.

Part (ii). By part (i), $x_n^{*'} < x_n^*$ for all $n$. Following Corollary 1, we have $V_n^{*\prime} > V_n^*$ for all $n \geq 1$. To compare the wages, first consider submarket $n$ with $n \geq 1$. For worker types $x \in [x_n^*, x_{n+1}^{*\prime}]$, who stay in the same submarket $n$, $V_n^{*\prime} > V_n^*$ and Lemma 4 implies that $w_n'(x) < w_n(x)$. Now consider workers who switch from submarket $n$ to $n+1$: $x \in [x_{n+1}^{*\prime}, x_{n+1}^*]$. Since $V_{n+1}^{*\prime} > V_{n+1}^*$, in the initial equilibrium with $Y_0$, by deviating to submarket $n+1$ a type $x$ worker could have earned wage $w_{n+1}(x; \overline{P}) > w_{n+1}'(x; \overline{P}')$. The fact that he chooses submarket $n$ in the initial equilibrium implies that $w_n(x; \overline{P}) > w_{n+1}(x; \overline{P})$. Therefore, $w_{n+1}'(x; \overline{P}') < w_n(x; \overline{P})$. Thus we have proved that for all $x \geq x_1^*$, $w_e'(x) < w_e(x)$. Regarding type $x_1^*$, we have $w_1'(x_1^*) < w_1(x_1^*) = w_0$. It immediately follows that for any $x \in [x_1^{*\prime}, x_1^*]$, $w_1'(x) \leq w_1'(x_1^*) < w_1(x_1^*) = w_0$, which means $w_e'(x) < w_e(x)$. Specifically, regarding type $x_1^{*\prime}$, we have $w_0' = w_1'(x_1^{*\prime}) < w_0$. Thus $w_0' < w_0$, which means $w_e'(x) < w_e(x)$ holds for $x \leq x_1^{*\prime}$. By Lemma 4, $w_0' < w_0$ implies that $V_0^{*\prime} > V_0^*$.

By the wage equation (3), $w_0' < w_0$ also implies that $q_0'^* > q_0^*$. The results that $u_0^{*\prime} < u_0^*$, $u_N^{*\prime} > u_N^*$, and $q_N^{*\prime} > q_N^*$ immediately follow the change of the segmentation pattern in part (i) and the steady state equations.

Part (iii). Denote $\Delta V_n^* \equiv V_n^{*\prime} - V_n^*$ and the wage decrease $\Delta w_e(x) \equiv w_e(x) - w_e'(x)$. By part (ii), $\Delta V_n^* > 0$ and $\Delta w_e(x) > 0$ for all $n$. For $n \geq 1$, by equation (5), the equilibrium indifference conditions $w_n(x_n^*) = w_{n-1}(x_n^*)$ and $w_n'(x_n^{*\prime}) = w_{n-1}'(x_n^{*\prime})$ can be written as

$$\begin{aligned}(\theta_n - \theta_{n-1})x_n^* &= rV_n^* - rV_{n-1}^*, \\ (\theta_n - \theta_{n-1})x_n^{*\prime} &= rV_n^{*\prime} - rV_{n-1}^{*\prime}.\end{aligned}$$

From the above two equations and the result that $x_n^{*\prime} < x_n^*$, we have $\Delta V_n^* < \Delta V_{n-1}^*$.

Now consider a worker with type $x \in [x_n^*, x_{n+1}^{*\prime}]$, $1 \leq n \leq N-1$. In both equilibria, this worker participates in submarket $n \leq N-1$. Thus, by equation (5), $\Delta w_e(x)$ can expressed as

$$\Delta w_e(x) = w_n(x) - w_n'(x) = \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha} r \Delta V_n^*,$$

which implies that $\Delta w_e(x)$ does not depend on $x$. As to worker $x \leq x_1^{*\prime}$, he participates in submarket 0 in both equilibria. Thus $\Delta w_e(x) = w_0 - w_0'$, which is constant.

Now consider a worker with type $x \in [x_n^{*\prime}, x_n^*]$, $1 \leq n \leq N$. This worker participates in submarket $n-1$ in the initial equilibrium, and he participates in submarket $n$ in the new equilibrium. Thus, by equation (5), the wage decrease $\Delta w_e(x)$ can be expressed as

$$\Delta w_e(x) = w_{n-1}(x) - w_n'(x) = \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha}[-(\theta_n - \theta_{n-1})x + (rV_n^{*\prime} - rV_{n-1}^*)],$$

which is clearly decreasing in $x$. ■

**Proof of Proposition 8.**

**Proof.** Part (i). By Corollary 1, it is enough to show $x_j^{*\prime} > x_j^*$ and $x_{j+1}^{*'} < x_{j+1}^*$. First, suppose $x_j^{*\prime} \leq x_j^*$ and $x_{j+1}^{*'} \geq x_{j+1}^*$. Then Lemma 4 and $Y_j' < Y_j$ imply that $V_j' > V_j^*$. Since $x_j^{*\prime} \leq x_j^*$, Corollary 1 implies that $V'_{j-1} \leq V^*_{j-1}$, which by Lemma 4 we have $w'_{j-1}(x^*_j) \geq w_{j-1}(x^*_j)$. It follows that $w'_j(x^*_j) \geq w_j(x^*_j)$ as $x^{*\prime}_j \leq x^*_j$. Thus by Lemma 4 we have $V'_j \leq V^*_j$. A contradiction.

Next, suppose $x^{*\prime}_j > x^*_j$ and $x^{*'}_{j+1} \geq x^*_{j+1}$. By Corollary 1, $V'_{j-1} > V^*_{j-1}$ and $V'_{j+1} \leq V^*_{j+1}$. Similar to the proof in the previous case, $V'_{j-1} > V^*_{j-1}$ and $x^{*\prime}_j > x^*_j$ imply that $w'_{j-1}(x^{*\prime}_j) < w_{j-1}(x^{*\prime}_j)$ and $w'_j(x^{*\prime}_j) < w_j(x^{*\prime}_j)$, thus $V'_j > V^*_j$. Similarly, $V'_{j+1} \leq V^*_{j+1}$ and $x^{*'}_{j+1} \geq x^*_{j+1}$ imply that $V'_j \leq V^*_j$. A contradiction.

Combining the previous two cases, we must have $x^{*'}_{j+1} < x^*_{j+1}$. Now suppose $x^{*\prime}_j \leq x^*_j$. By Corollary 1, $V'_{j-1} \leq V^*_{j-1}$ and $V'_{j+1} > V^*_{j+1}$. Similar to the proof in the previous case, $V'_{j-1} \leq V^*_{j-1}$ and $x^{*\prime}_j \leq x^*_j$ imply that $V'_j \leq V^*_j$, while $V'_{j+1} > V^*_{j+1}$ and $x^{*'}_{j+1} < x^*_{j+1}$ imply that $V'_j > V^*_j$. A contradiction. Therefore, we must have $x^{*\prime}_j > x^*_j$ and $x^{*'}_{j+1} < x^*_{j+1}$.

Parts (ii) and (iii). The proof of most claims are similar to those of parts (ii) and (iii) in Proposition 7, and thus is omitted. We only show the new result: for $x < x^{*\prime}_{j+1}$, $\Delta w_e(x)$ is weakly increasing in $x$. Let $n \leq j$. For workers with type $x \in [x^{*\prime}_n, x^*_{n+1}]$, they choose submarket $n$ in both equilibria. In this case, $\Delta w_e(x) = w_n(x) - w'_n(x) \propto rV^{*\prime}_n - rV^*_n$, which is independent of $x$. For workers with type $x \in [x^*_{n+1}, x^{*\prime}_{n+1}]$, they choose submarket $n+1$ in the initial equilibrium but choose submarket $n$ in the new equilibrium. The wage decrease can be expressed as

$$\Delta w_e(x) \equiv w_{n+1}(x) - w'_n(x) = \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha}[(\theta_{n+1} - \theta_n)x - (rV^*_{n+1} - rV^{*\prime}_n)],$$

which is increasing in $x$. ■

**Proof of Proposition 9**.

**Proof.** Part (i). By Corollary 1, it is enough to show $x^{*'}_{j+1} > x^*_{j+1}$. First, suppose $x^{*'}_{j+1} \leq x^*_{j+1}$ and $x^{*\prime}_j \geq x^*_j$. Then Lemma 4 and $\theta'_j > \theta_j$ imply that $w'_j(x^{*\prime}_j) > w_j(x^{*\prime}_j)$. Since $x^{*\prime}_j \geq x^*_j$, Corollary 1 implies that $V'_{j-1} \geq V^*_{j-1}$, which by Lemma 4 we have $w'_{j-1}(x^{*\prime}_j) \leq w_{j-1}(x^{*\prime}_j)$. It further implies $w'_j(x^{*\prime}_j) \leq w_j(x^{*\prime}_j)$. A contradiction.

Next suppose $x^{*'}_{j+1} \leq x^*_{j+1}$ and $x^{*\prime}_j < x^*_j$. By Corollary 1, $V'_{j-1} < V^*_{j-1}$ and $V'_{j+1} \geq V^*_{j+1}$. Applying Lemma 4, we have $w'_{j-1}(x^{*\prime}_j) > w_{j-1}(x^{*\prime}_j)$ and $w'_j(x^{*\prime}_j) > w_j(x^{*\prime}_j)$, and similarly $w'_{j+1}(x^{*\prime}_{j+1}) \leq w_{j+1}(x^{*\prime}_{j+1})$ and $w'_j(x^{*\prime}_{j+1}) \leq w_j(x^{*\prime}_{j+1})$. The inequalities $w'_j(x^{*\prime}_j) > w_j(x^{*\prime}_j)$ and $w'_j(x^{*\prime}_{j+1}) \leq w_j(x^{*\prime}_{j+1})$, by the wage equation, translate into

$$\begin{aligned}(\theta'_j - \theta_j)x^{*\prime}_j &> rV^*_j - rV^{*\prime}_j, \\ (\theta'_j - \theta_j)x^{*\prime}_{j+1} &\leq rV^*_j - rV^{*\prime}_j.\end{aligned}$$

A contradiction, since $x^{*'}_{j+1} > x^{*\prime}_j$. Therefore, we must have $x^{*'}_{j+1} > x^*_{j+1}$.

Part (ii) and (iii). The proofs are similar to those of Proposition **8**, thus are omitted. ■

**Match fitness in the horizontal dimension.**

For any worker-firm pair, there is a possibility that the worker does not like or is not fit for the job, which is unrelated to wage payment. In particular, a worker $i$ derives a match value $\varepsilon_{ij}$ when matched with firm $j$. The match value $\varepsilon_{ij}$ can be either 0 (the worker is fit for the job) or $-\infty$ (the worker is not fit for the job and will never accept it). We assume that $\varepsilon_{ij}$ is i.i.d. across all matching pairs and is independent of workers' types and firms' types. $\Pr[\varepsilon_{ij} = 0] = \lambda \in (0,1)$, which is common knowledge. The assumption that the horizontal match value $\varepsilon_{ij}$ can be either 0 or $-\infty$ is a vast simplification, which makes the model tractable.

Before a worker $i$ applies to firm $j$, he receives an informative signal (either good or bad) regarding the horizontal match value. If the match value is 0, he receives the good signal for sure. If the match value is $-\infty$, he receives the bad signal with probability $\gamma \in (0,1)$ and a good signal with probability $1-\gamma$, where $\gamma$ captures the accuracy of the signals. One can consider this as a pre-search screening process of a worker before applying for jobs. By viewing the job descriptions of a particular job, the worker can get some rough idea as to whether this job is suitable for him. According to the information structure of the signal, a worker will never apply or contact a firm whose matching signal is bad. For a firm with a good matching signal, with probability $\frac{\lambda}{\lambda+(1-\lambda)(1-\gamma)} \equiv s \in (0,1)$ the job is suitable. The parameter $s$ is the success rate of turning a meeting into a suitable match. Note that $s$ is increasing in $\gamma$. Once a worker and a firm meet, the worker immediately finds out the match value or whether the job is suitable for him.

To summarize, if a worker searches in submarket $n$, he only applies for vacancies whose matching signals are good. Those with a bad matching signal are ruled out in the pre-search screening process, and thus do not affect the effectiveness of his search intensity.[41] As a result, a worker's contact rate is still $\alpha$, but his successful contact rate (of encountering suitable jobs) is $\alpha s$. Correspondingly, for a type $n$ firm, its contact rate is $\alpha q_n$, and its successful matching rate is $\alpha s q_n$, depending only on the market tightness of submarket $n$. This is because the horizontal aspect of match fitness is i.i.d., though different workers are targeting a different subset of firms.[42]

The Internet increases the accuracy of signals $\gamma$ by allowing firms to post more detailed job descriptions online at virtually zero additional cost. As a result, workers are able to rule out more unsuitable jobs from the very beginning. In other words, the pre-search screening process becomes more efficient. An increase in $\gamma$ is translated into an increase in $s$. However, since the successful contact rate is $\alpha s$, in term of modeling an increase in $\alpha$ and an increase in $s$ are equivalent. To make the model more parsimonious, we choose not to introduce the additional parameter $s$ into the model.

**Proof of Proposition 10.**

**Proof.** Part (i). Recall that, if under $\alpha'$ the initial equilibrium segmentation $\overline{P}$ remains the

[41] Note that for a particular worker, the proportion of type $n$ firms generating the good signal is $\lambda+(1-\lambda)(1-\gamma)$. Essentially, he will only search among those firms.

[42] Given the signals, in submarket $n$ a worker is targeting at $[\lambda+(1-\lambda)(1-\gamma)]v_n$ firms, and each type $n$ firm is in $[\lambda+(1-\lambda)(1-\gamma)]u_n$ workers' targeted set, so for each individual firm its expected queue length is still $u_n/v_n$.

same, then by the analysis in subsection 6.1 we have (i) $w_0'(\overline{P}) < w_0$; (ii) in each submarket $n \geq 1$, $w_{n+1}'(x_{n+1}^*; \overline{P}) < w_{n+1}(x_{n+1}^*; \overline{P}) = w_n(x_{n+1}^*; \overline{P}) < w_n'(x_{n+1}^*; \overline{P})$. Moreover, by the condition that $q_0^*$ and $q_1^*$ are not too far apart, $w_1'(x_1^*, \overline{P}) < w_0'(\overline{P})$ holds.

In step 1, we show that $x_1^{*\prime} > x_1^*$. Suppose to the contrary, $x_1^{*'} \leq x_1^*$. Denote the new equilibrium segmentation pattern as $\overline{P}'$. By Lemma 4, $x_1^{*'} \leq x_1^*$ implies that $w_0'(\overline{P}) \leq w_0'(\overline{P}')$. Combining with $w_1'(x_1^*; \overline{P}) < w_0'(\overline{P})$ and $w_1'(x_1^*; \overline{P}') \geq w_1'(x_1^{*\prime}; \overline{P}') = w_0'(\overline{P}')$, we reach the conclusion that $w_1'(x_1^*; \overline{P}') > w_1'(x_1^*; \overline{P})$. By Lemma 4, this implies that $V_1'(\overline{P}') < V_1'(\overline{P})$. To ensure that $x_1^{*\prime}$ is the marginal type between submarkets 0 and 1 in the new equilibrium, it must be the case that $x_2^{*\prime} < x_2^*$. To see this, suppose $x_2^{*\prime} \geq x_2^*$. Given that $[x_1^*, x_2^*] \sqsubseteq [x_1^{*\prime}, x_2^{*\prime}]$, by Lemma 4 it must be the case that $V_1'(\overline{P}') \geq V_1'(\overline{P})$. A contradiction. Therefore, we must have $x_2^{*\prime} < x_2^*$, and $V_1'(\overline{P}') < V_1'(\overline{P})$.

Now consider submarkets 1 and 2. The condition $V_1'(\overline{P}') < V_1'(\overline{P})$ implies that $w_1'(x_2^*; \overline{P}') > w_1'(x_2^*; \overline{P})$. The fact that $x_2^{*\prime} < x_2^*$ means that $w_2'(x_2^*; \overline{P}') > w_1'(x_2^*; \overline{P}')$. Thus, we have $w_2'(x_2^*; \overline{P}') > w_1'(x_2^*; \overline{P})$. But $w_2'(x_2^*; \overline{P}) < w_1'(x_2^*; \overline{P})$. Therefore, $w_2'(x_2^*; \overline{P}') > w_2'(x_2^*; \overline{P})$. By Lemma 4, this implies that $V_2'(\overline{P}') < V_2'(\overline{P})$, which again means that $x_3^{*\prime} < x_3^*$.

Applying a similar argument by induction (up to submarket $N-1$), we have $x_N^{*\prime} < x_N^*$, $V_{N-1}'(\overline{P}') < V_{N-1}'(\overline{P})$, and $w_N'(x_N^*; \overline{P}') > w_N'(x_N^*, \overline{P})$. Now consider submarket $N$. By Lemma 4, $x_N^{*\prime} < x_N^*$ implies that $V_N'((\overline{P}') > V_N'(\overline{P})$ and $w_N'(x_N^*, \overline{P}') < w_N'(x_N^*, \overline{P})$. A contradiction. Therefore, we must have $x_1^{*\prime} > x_1^*$.

In step 2, we show $x_2^{*\prime} > x_2^*$. Suppose to the contrary, $x_2^{*\prime} \leq x_2^*$. Since $x_1^{*\prime} > x_1^*$ by step 1, by Lemma 4 it implies $V_1'(\overline{P}') < V_1'(\overline{P})$. Applying the same logic as in the previous step, it implies that $x_3^{*\prime} < x_3^*$ and $w_2'(x_2^*; \overline{P}') > w_2'(x_2^*; \overline{P})$. By induction up to submarket $N-1$, we have $x_N^{*\prime} < x_N^*$, $w_N'(x_N^*; \overline{P}') > w_N'(x_N^*; \overline{P})$. But in submarket $N$, $x_N^{*\prime} < x_N^*$ implies that $V_N'((\overline{P}') > V_N'(\overline{P})$ and $w_N'(x_N^*; \overline{P}') < w_N'(x_N^*; \overline{P})$. A contradiction. Therefore, we must have $x_2^{*\prime} > x_2^*$.

Using a similar argument as in step 2, we can show $x_n^{*\prime} > x_n^*$ for all $n \leq N$.

Part (ii). Since $x_1^{*\prime} > x_1^*$, by Lemma 4 $w_0'(\overline{P}') < w_0'(\overline{P})$. Since $w_0'(\overline{P}) < w_0$, we have $w_0'(\overline{P}') < w_0$. Thus for any $x \leq x_1^*$ (choose submarket 0 in both equilibria), $w_e'(x) < w_e(x)$. For any $x \in (x_1^*, x_1^{*\prime}]$, $w_0'(\overline{P}') < w_0 = w_1(x_1^*) \leq w_1(x)$, and hence $w_e'(x) < w_e(x)$. Now consider submarket $N$. Since $x_N^{*\prime} > x_N^*$, by Lemma 4 $V_N'(\overline{P}') < V_N'(\overline{P})$ and $w_N'(x; \overline{P}') > w_N'(x; \overline{P})$ for any $x \geq x_N^{*\prime}$. Thus for any $x \geq \widetilde{x}_N$, $w_N'(x; \overline{P}') > w_N'(x; \overline{P}) > w_N(x)$. That is, $w_e'(x) > w_e(x)$.

Part (iii). It has been shown in the analysis in subsection 6.1. ■

**Proof of Proposition 11**.

**Proof.** Part (i). For worker types $x \in [x_n^{*\prime}, x_{n+1}^*]$, $n \geq 1$, they choose submarket $n$ in both equilibria. Let $z = \frac{\beta(r+\delta+\alpha)}{r+\delta+\beta\alpha}$ and $z' = \frac{\beta(r+\delta+\alpha')}{r+\delta+\beta\alpha'}$. Note that $z' > z$. By equation (5),

$$\frac{\partial \Delta w_e(x)}{\partial x} = (z' - z)\theta_n > 0.$$

Part (ii). For worker types $x \in [x_{n+1}^*, x_{n+1}^{*\prime}]$, they choose submarket $n+1$ in the initial equilibrium but choose submarket $n$ in the new equilibrium. Thus $\Delta w_e(x) = w_n'(x) - w_{n+1}(x)$. By equation (5),

$$\frac{\partial \Delta w_e(x)}{\partial x} = z'\theta_n - z\theta_{n+1}.$$

Given that $\Delta\alpha$ is small, $z' - z$ is also small. Thus $\frac{\partial \Delta w_e(x)}{\partial x} < 0$.

Part (iii). By equation (5),

$$\begin{aligned}
\Delta w_e(x_{n+1}^{*\prime}) - \Delta w_e(x_n^{*\prime}) &= [w_{n+1}'(x_{n+1}^{*\prime}) - w_{n+1}(x_{n+1}^{*\prime})] - [w_n'(x_n^{*\prime}) - w_n(x_n^{*\prime})] \\
&= [z'\theta_n - z\theta_{n+1}](x_{n+1}^{*\prime} - x_{n+1}^{*}) + (z' - z)\theta_n(x_{n+1}^{*} - x_n^{*\prime}).
\end{aligned}$$

Given that $\Delta\alpha$ is small, $x_{n+1}^{*\prime} - x_{n+1}^{*}$ is small relative to $x_{n+1}^{*} - x_n^{*\prime}$. Thus $\Delta w_e(x_{n+1}^{*\prime}) - \Delta w_e(x_n^{*\prime}) > 0$. $\blacksquare$

## References


[1] Acemoglu, D. "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence," *American Economic Review*, 1999, 89, 1259-1278.

[2] Albrecht, J., and Vroman, S. "A Matching Model of Endogenous Skill Requirements," *International Economic Review*, 2002, 43(1), 283-305.

[3] Antonczyk, D., DeLeire, T., and Fitzenberger, B. "Polarization and Rising Wage Inequality: Comparing the U.S. and Germany," 2010, IZA working paper.

[4] Autor, D., Katz, L., and Kearney, M. "The Polarization of the U.S. Labor Market," *American Economic Review*, 2006, 96(2), 189-194.

[5] Autor, D., Katz, L., and Kearney, M. "Trends in U.S. Wage Inequality: Re-Assessing the Revisionists," *Review of Economics and Statistics*, 2008, 90(2), 300-323.

[6] Autor, D., Levy, F., and Murnane, R. "The Skill Content of Recent Technological Change: An Empirical Exploration," *The Quarterly Journal of Economics*, 2004, 118(4), 1279-1333.

[7] Battistin, E., Blundell, R., and A. Lewbel. "Why Is Consumption More Log Normal than Income? Gibrat's Law Revisited." *Journal of Political Economy*, 2009, 117(6), 1140-1154.

[8] Becker, G. "A Theory of Marriage: Part I," *Journal of Political Economy*, 1973, 81, 813-846.

[9] Bethune, Z., Choi, M., and R. Wright. "Frictional Goods Market: Theory and Applications," 2016, working paper, University of Wisconsin.

[10] Burdett, K., and Coles, M. "Marriage and Class," *The Quarterly Journal of Economics*, 1997, 112, 141-168.

[11] Cheremukhin, A., P. Restrepo-Echavarria, and A. Tutino. "A Theory of Targeted Search," 2014, Federal Reserve Bank of St. Louis, working paper.

[12] Diamond, P. "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, 1982, 90(5), 881-894.

[13] DiNardo, J., Fortin, N., and Lemieux, T. "Labor Market Institutions and the Distribution of Wages, 1973-1992: A Semiparametric Approach," *Econometrica*, 1996, 64(4), 1001-1044.

[14] Dustmann, C., Ludsteck, J., and Schonberg, U. "Revisiting the German Wage Structure," *The Quarterly Journal of Economics*, 2009, 124(2): 843-881.

[15] Eeckhout, J. and Kircher, P. "Sorting and Decentralized Competition," *Econometrica*, 2010, 78(2), 539-574.

[16] Goos, M., and Manning, A. "Lousy and Lovely Jobs: The Rising Polarization of Work in Britain," *Review of Economics and Statistics*, 2007, 89(1): 118-133.

[17] Herrnstein, R. and C. Murray. *The Bell Curve: Intelligence and Class Structure in American Life*. 1994, The Free Press, New York.

[18] Jacquet, N., and Tan, S. "On the Segmentation of Markets," *Journal of Political Economy*, 2007, 115(4), 639-664.

[19] Katz, L. and Autor, D. "Changes in the Wage Structure and Earnings Inequality." 1999, Ashenfelter, O. and D. Card (eds), *Handbook of Labor Economics*, 3A: 1463-1555, North Holland, Amsterdam.

[20] Lemieux, T. "Increased Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill," *American Economic Review*, 2006, 96(3): 461-498.

[21] Lester, B. "Information and Prices with Capacity Constraints," *American Economic Review*, 2011, 101(4), 1591-1600.

[22] Menzio, G., and S. Shi. "Block Recursive Equilibria for Stochastic Models of Search on the Job," *Journal of Economic Theory*, 2010a, 145(4), 1453-1494.

[23] Menzio, G., and S. Shi. "Directed Search on the Job, Heterogeneity and Aggregate Fluctuations," *American Economic Review*, 2010b, 100(2), 327-332.

[24] Moen, E. "Competitive Search Equilibrium," *Journal of Political Economy*, 1997, 105(2), 385-411.

[25] Montgomery, J.D. "Equilibrium Wage Distribution and Interindustry Wage Differentials," *Quarterly Journal of Economics*, 1991, 106, 163-79.

[26] Mortensen, D. "Property Rights, and Efficiency in Mating, Racing, and Related Games," *American Economic Review*, 1982, 72(5), 968-979.

[27] Mortensen, D. and C. Pissarides. "Unemployment Responses to 'Skill-Biased' Technology Shocks: The Role of Labor Market Policy," *Economic Journal*, 1999, 109(2), 242-265.

[28] Mortensen, D. and C. Pissarides. "New Developments in Models of Search in the Labor Market," 1999b, in *Handbook of Labor Economics* (O. Ashenfelter & D. Card, eds.). Amsterdam: North-Holland.

[29] Peters, M. "Ex Ante Price Offers in Matching Games: Non-Steady State," *Econometrica*, 1991, 59, 1425-54.

[30] Pissarides, C. *Equilibrium Unemployment Theory*, 1990, Oxford: Blackwell.

[31] Rogerson, R., Shimer, R., and Wright, R. "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, 2005, XLIII, 959-988.

[32] Shi, S. "Frictional Assignment: I. Efficiency," *Journal of Economic Theory*, 2001, 98, 111-130.

[33] Shi, S. "A Directed Search Model of Inequality with Heterogenous Skills and Skill-Biased Technology," *Review of Economic Studies*, 2002, 69(2), 467-491.

[34] Shimer, R. "The Assignment of Workers to Jobs in an Economy with Coordination Frictions," *Journal of Political Economy*, 2005, 113(5), 996-1025.

[35] Shimer, R., and Smith, L. "Assortative Matching and Search," *Econometrica*, 2000, 68(2), 343-369.

[36] Xu, Y., and H. Yang. "Targeted Search with Horizontal Differentiation in the Marriage Market," 2016, working paper.

[37] Yang, H. "Targeted Search and the Long Tail Effect," *Rand Journal of Economics*, 2013, 44(4), 733-756.