# Pre-Communication in a Coordination Game with Incomplete Information

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**Abstract**

We study several pre-communication protocols in a coordination game with incomplete information. Under decentralized decision making, we show that informative communication can be sustained in equilibrium, yet miscoordination arises with positive probabilities. Moreover, the equilibrium takes a partitional structure and messages are rank ordered, with higher messages becoming increasingly imprecise. Compared to centralized decision making (a mediator without commitment), decentralization leads to more informative communication when the miscoordination cost is high, and decentralization performs better when the miscoordination cost is neither too low nor too high. We also stduy the case in which the mediator is able to commit to a decision rule beforehand.

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# I Introduction

This paper studies a simple two-player game with both players having private information. The game is a variant of the battle of the sexes. Specifically, both players simultaneously choose between two actions, say $A$ and $B$. Each player has his own favored action: while player 1 favors action $A$, player 2 favors the other action $B$ (each player gets an intrinsic payoff when he chooses his preferred action). How much a player prefers his own favored action relative to the other action is his own private information. Moreover, other things equal, both players would like to coordinate their actions by choosing the same action: if they choose different actions, then both of them suffer a cost of miscoordination. Unlike the battle of the sexes, in our game the two miscoordination outcomes are asymmetric. In the "positive" miscoordination (outcome $AB$) where each agent chooses his own favored action, each agent gets his intrinsic payoff by playing his preferred action in addition to suffering the miscoordination loss. On the other hand, in the "negative" miscoordination (outcome $BA$) where each agent chooses an action favored by the other player, each agent does not get any intrinsic payoff but only suffers the miscoordination loss.

Many real world examples fit the game we just described. The classical example is the battle of the sexes. A couple would rather spend the night together than apart, but the wife prefers going to an opera concert while the husband would rather go to a boxing match. But how much the wife prefers the opera and how much the husband prefers the boxing match are private information. In another example, two firms simultaneously choose technology standards. Each firm prefers choosing their own current technology as the technology standard, but the intensity of its own-standard preference is each firm's private information. On the other hand, if two firms end up with different technology standards then both of them will suffer a miscoordination loss. As a final example, consider two divisions of a U.S. multinational company independently decide to open new offices in Europe. Each division has two choices, locating its own new office in either London or Paris. The two divisions prefer different locations (one favors London and the other favors Paris), but both of them suffer a miscoordination loss if they choose different locations.

Because players suffer from miscoordination, they have incentives to coordinate their actions. This means that they want to communicate with each other before playing the game. However, since each player prefers to coordinate on the action that he favors, each player would "exaggerate" the degree to which he prefers his own action (type). The main goal of this paper is to study how pre-communication will take place and how it will affect the outcome of the game. Specifically, we model pre-communication as "cheap talk," which means that the messages are costless, nonbinding and nonverifiable. Moreover, each player's message is regarding his own type. Our research questions are as follows. Can pre-communication be informative, and how informative it can be? Does pre-communication help players obtain higher expected payoffs? How does the magnitude of the miscoordination loss affect the informativeness of the equilibrium pre-communication? Will introducing a mediator improve players' payoffs?

Due to the coordination nature of the game, there are multiple equilibria even in the basic game, and adding pre-communication significantly expands the set of equilibria. We thus focus on a class of plausible equilibria, which we call symmetric partition equilibria (simply equilibria hereafter). Specifically, two players play symmetric strategies and have the same set of messages; moreover, the type space is partitioned into connected intervals, with a higher type sending a weakly higher message. The messages are rank ordered**.** In particular, if two players send different messages, then both players play (coordinate on) the action that is favored by the player who sends the higher message. If two players send the same message, then they play a symmetric Bayesian equilibrium in the continuation game: each player plays his favored action if his type is higher than a cutoff and plays the other action otherwise.[1]

We show that there exist a class of informative equilibria with different numbers of messages/partition elements. In the most informative equilibrium, the number of messages is infinite. In particular, the partition becomes very fine when a player's type approaches zero. Moreover, the size of the partition element increases as a player's type becomes higher, or communication becomes noisier when players claim to be higher types. This is because, given that messages are rank ordered, each type of agent has an incentive to send higher

[1] The justification of this class of equilibria is provided in Section 4.

messages. To counter this tendency of exaggeration, the probabilities of miscoordination when both agents sending the same message (serving as punishments) must be higher when the message becomes higher, which implies that the sizes of partition elements get larger for higher messages (or higher messages are noisier). Informative pre-communication improves players' ex ante expected payoff, and this benefit is increasing in the number of partition elements. However, the marginal gain in payoff by adding an additional message is decreasing in the number of messages. Even in the most informative equilibrium, miscoordination arises with positive probabilities.

As the miscoordination cost increases, the size of the partition elements in the most informative equilibrium becomes more even. However, this improved communication is outweighed by the direct negative impact of the increase in the miscoordination cost. As a result, players' expected payoff decreases in the miscoordination cost. We also found that, as the miscoordination cost increases, players benefit more from pre-communication in the following sense. When the miscoordination cost is low (high), players' expected payoff in a two-partition or three-partition equilibrium is already very close to (still far away from) that under the most information equilibrium, which means that adding more messages almost does not help (helps a lot).

We then introduce a mediator, whose objective is to maximize two agents' joint payoffs. We first study the case that the mediator is not able to commit to a decision rule beforehand. This case corresponds to centralized decision making in an organization, while the baseline model (without mediator) can be interpreted as decentralized decision making. We establish that, under centralization in equilibrium each agent has at most two messages. In particular, when the miscoordination cost is high, only babbling equilibrium exists. Therefore, the number of messages can be sustained under centralization is a lot less than under decentralization. The main reason is that the presence of a mediator plus no commitment means that inefficient miscoordination will never occur, which implies that no credible and rich punishments can be created to sustain more than two messages under centralization.

Decentralized communication is more informative than centralized communication when the miscoordination cost is high. However, when the miscoordination cost is low, the comparison is subtler. Even though a lot more messages are sustained under decentralization, the two-message communication under centralization is relatively more informative. These results are quite different from those in Alonso, et al. (2008) (ADM, thereafter) and Rantakari (2008), where centralized communication is always more informative than decentralized communication. In terms of agents' expected payoff, centralization performs relatively better when the miscoordination cost is either sufficient small or sufficiently high, and decentralization outperforms centralization when the miscoordination cost is intermediate. This result also differs from those in ADM and Rantakari (2008), where decentralization performs better if and only if the cost of miscoordination is sufficiently low.

Finally, we consider the case that the mediator is able to commit to a decision rule beforehand. For simplicity, we focus on two-partition symmetric equilibrium in which each agent only has two messages. In the optimal mechanism, we show that the mediator punishes the agents only by choosing the positive miscoordination outcome $AB$, and she does so only when both agents send the high message and the cost of miscoordination is sufficiently small. Our results indicate that, the lack of creditable punishment instead of centralization itself is the key to explain the collapse of informative communication under centralization.

**Related literature** Our paper is related to several strands of literature. The first strand studies how pre-communication works in coordination games. Farrell (1987) pioneers this line of research.[2] In his model, two firms first announce their intentions to enter or not (cheap talk) and then proceed to play in a battle-of-the-sexes type entry game. He shows that adding pre-communication increases the frequency of coordination and hence leads to higher ex ante payoffs for both firms.[3] Our model differs from that of Farrell in two main aspects. First, he deals with a complete information coordination game, but we study an incomplete information one. Second, Farrell's model is more tailored to study firms' entry behavior, by contrast our model covers a more general coordination game, which potentially has more extensive applications in the real world. Following Farrell, Banks and Calvert (1992) analyze

[2] An extensive discussion of the informativeness of cheap talk in coordination and other games alike can be found in Farrell and Rabin (1996).

[3] Cooper et al. (1989) and Cooper et al. (1992) provide experimental evidence on the role of preplay cheap talk in several versions of bilateral coordination games including that of Farrell.

pre-communication in a classic battle-of-the-sexes game with asymmetric information, which is closely related to our paper.[4] The main difference between their model and ours is that in their model the agents have two discrete types while in our model the types of the agents are continuous. With continuous types, the construction of cheap talk equilibria in our model is more aligned with Crawford and Sobel's (1982) classical cheap talk model, and it is more involved than the communication equilibria in Banks and Calvert's two-type model.[5]

The second strand is the literature on communication within organizations and delegation (Melumad and Shibano, 1991; Dessein 2002; Harris and Raviv, 2005; Goltsman et al., 2009). In this literature, the most closely related papers to ours are ADM and Rantakari (2008). For instance, in ADM there are two divisional managers, each of whom has private information regarding the local condition of his own division. Under centralization, both managers communicate simultaneously to the CEO of the firm and then the CEO makes decisions for both divisions. Alternatively, under decentralization the two managers first communicate with each other and then each of them makes decision for his own division. Our baseline model corresponds to decentralization in ADM, while the scenario of a mediator without commitment in our model corresponds to centralization in ADM. However, there are two important differences. First, in ADM each agent's decision is continuous, while in our model each agent's action is binary. Second, in ADM agents have quadratic loss payoff functions, while in our model an agent's payoff is additive in his own intrinsic preference and the mis-coordination loss. These differences lead to different incentives to communicate and different values of communication, which are reflected in the following qualitatively different results across two models. First, the constructions of cheap talk equilibria are different. Second, in our setting decentralization could lead to more informative communication than centralization, while never arises in ADM. Third, the comparison of the relative performance between

[4] Matthews and Postlewaite (1989) and Farrell and Gibbons (1989) both study pre-communication in two-person double auctions. Baliga and Morris (2002) analyze the role of cheap-talk in two player games with one-sided private information. The games analyzed in their paper have both features of coordination and spillovers.

[5] Another difference is that, in their setting with a mediator, they can use the Revelation Principle to fully characterize all incentive compatible mechanisms. However, with types being continuous in our model, though the Revelation Principle applies, it is impossible to fully characterize all incentive compatible mechanisms.

centralization and decentralization is also different across two papers. These differences will be elaborated in later sections.

Finally, our paper is also related to a recent small literature on cheap talk in the setting of project choice (Rantakari, 2014; Rantakari, 2016; Li et al., 2016; Li, 2016). The common features of these papers are that there are multiple senders, who have non-overlapping private information, and the receiver's decision is binary (which project to implement).[6] While the feature of binary action is also shared by the current model, the difference is that in the current model there are two actions to take (one for each agent), while in other models there is only a single decision to make.

The rest of the paper is organized as follows. Section II describes the model. In Section III, we study two benchmarks: the first best outcome and the coordination game without preplay communication. Section IV studies the augmented game with pre-communication. In Section V we introduce a mediator without commitment. Section VI considers the case in which the mediator is able to commit to a decision rule beforehand. Finally, we conclude in Section VII.

# II The model

Two agents, 1 and 2, simultaneously decide their own actions. Each agent has two available actions: $A$ and $B$. Agent 1 intrinsically prefers action $A$, while agent 2 intrinsically prefers action $B$. In particular, if agent $i$ chooses his preferred action, then he gets a benefit $\theta_i$. We assume that $\theta_i$ is uniformly distributed on $[0, 1]$.[7] Moreover, $\theta_1$ and $\theta_2$ are independent from each other. We further assume that the realization of $\theta_i$ is agent $i$'s private information. Two agents also benefit from coordination (or suffer from miscoordination). Specifically, if two agents choose different actions, then both of them suffer a miscoordination loss $c > 0$. The payoff matrix is described below.

[6] McGee and Yang (2013) also study a setting with multiple senders who have non-overlapping private information, but the receiver's decision is continuous.

[7] The assumption of uniform distribution is standard in the literature of cheap talk with multiple senders, e.g. ADM.

| Agent 1\ Agent 2 | $A$ | $B$ |
|---|---|---|
| $A$ | $(\theta_1, 0)$ | $(\theta_1 - c, \theta_2 - c)$ |
| $B$ | $(-c, -c)$ | $(0, \theta_2)$ |

Finally, we assume that, other than the realizations of $\theta_1$ and $\theta_2$, all the aspects of the game are common knowledge.

This is a coordination game with private information. It actually resembles the game of the battle of the sexes (e.g., Gibbons, 1992; p11). Specifically, agent 1 is the wife and agent 2 is the husband, while actions $A$ and $B$ are opera and boxing, respectively.[8] As we mentioned earlier in the Introduction, many other real world situations correspond to our game.

We want to point out that our game differs from the standard game of the battle of the sexes in two ways. First, rather than a game of complete information, in our game how much the wife prefers opera and how much the husband prefers boxing are uncertain and private information. Second, while in the standard game of the battle of sexes both agents suffer equally from either case of miscoordination,[9] in our game both agents get different payoffs in two different scenarios of miscoordination. In particular, in the case of "positive" miscoordination where each agent chooses his/her own preferred action (outcome $(A, B)$, or both agents "stick"), on top of suffering the miscoordination loss $c$ each agent $i$ gets $\theta_i$. Note that this "positive" miscoordination might lead to a higher joint payoff than the perfectly coordinated outcomes $(A, A)$ or $(B, B)$ (if $\min\{\theta_1, \theta_2\} \geq 2c$), as both agents get the benefit from choosing their own preferred actions. On the other hand, in the case of "negative" miscoordination where each agent chooses his/her less preferred action (outcome $(B, A)$, or both agents "yield"), each agent's payoff is just $-c$, the miscoordination loss. Note that if $\theta_i \geq c$ $(c < 1)$, then choosing his own favored action is agent $i$'s dominant strategy.

To justify the second feature of our game, one can think of $\theta_i$ as agent $i$'s intrinsic payoff by playing his/her preferred action, regardless of whether actions are coordinated between the two agents. In the context of the battle of the sexes, the wife always gets a payoff $\theta_1$

[8] If we add $c$ to all the payoffs in the payoff matrix, then the payoffs look more like those in the game of the battle of the sexes ($c$ becomes the coordination benefit enjoyed by both agents when they choose the same action). But this is just a normalization, which would not qualitatively affect our results.

[9] Translating into our setting, it means that both agents get $-c$ whenever miscoordination occurs.

if she chooses opera, and the husband always gets a payoff $\theta_2$ by choosing boxing. In the example of adopting the industrial standard, firm $i$'s always gets a payoff $\theta_i$ if his preferred (or his own) standard is adopted as the industrial standard.

In this game, we are interested in whether allowing pre-play communication in the form of cheap talk would help. The timing of the game thus goes as follows. First, $\theta_1$ and $\theta_2$ are realized and learned privately by agent 1 and agent 2, respectively. Then two agents play a two-stage game. In the first stage, the two agents simultaneously send messages to each other regarding their own private information. In the second stage, the two agents simultaneously choose actions.

# III Two Benchmarks

We first study two benchmarks: the first-best outcome and the Bayesian Nash equilibrium (BNE) for the second stage game without pre-communication in the first stage.

## III(i) The first best outcome

Suppose there is a social planner/mediator, who has full information about $\theta_1$ and $\theta_2$. The mediator chooses the outcome to maximize the social surplus. The joint first-best ex ante social surplus can be computed as:

$$E\pi_{fb}(c) = E_{\theta_1,\theta_2}[\max\{\theta_1, \theta_2, \theta_1 + \theta_2 - 2c\}].$$

Note that, when $0 < c < \frac{1}{2}$, the outcome of "positive" miscoordination, $AB$, can be socially optimal (if both $\theta_1$ and $\theta_2$ are bigger than $2c$). Thus, the joint first-best social surplus is

$$\begin{aligned} E\pi_{fb}(c) &= 2\int_0^{2c}\int_{\theta_1}^1 \theta_2 d\theta_2 d\theta_1 + 2\int_{2c}^1\int_{2c}^1 (\theta_2 - c)\, d\theta_2 d\theta_1 \\ &= 2c + (2c-1)^2 - \frac{8}{3}c^3. \end{aligned}$$

On the other hand, when $c \geq \frac{1}{2}$ the outcome of "positive" miscoordination, $AB$, can never be socially optimal.[10] Thus, we have

[10]More precisely, $\max\{\theta_1, \theta_2\} \leq 1 \leq 2c$ when $c \geq 1/2$.

$$E\pi_{fb}(c) = 2\int_0^1 \int_{\theta_1}^1 \theta_2 d\theta_2 d\theta_1 = \frac{2}{3}.$$

Notice that, when $c \geq \frac{1}{2}$, the joint first-best social surplus is 2/3, which is the expected value of the first-order statistic of two uniformly distributed variables.

## III(ii) Equilibrium without Pre-Communication

Now we study the BNE for the second stage game without pre-communication. This equilibrium also corresponds to the equilibrium in the two-stage game where in the communication stage both agents babble (transmit no information).

In this benchmark case, agent $i$'s strategy is an action rule $d_i(\theta_i)$: $[0,1] \to \{A, B\}$. We first establish that in BNE both agents must adopt cutoff strategies. That is, agent $i$ chooses his preferred action if and only if $\theta_i \geq \widehat{\theta}_i$, where $\widehat{\theta}_i$ is the cutoff or threshold for agent $i$. To see this, consider agent 1. Suppose agent 1 holds the belief that agent 2 will play $B$ with probability $p_2(B)$. Then, agent 1's expected payoff by choosing $A$ is

$$\pi_1(A) = [1 - p_2(B)]\theta_1 + p_2(B)(\theta_1 - c) = \theta_1 - p_2(B)c,$$

while the expected payoff by choosing $B$ is

$$\pi_1(B) = [1 - p_2(B)](-c) + p_2(B)(0) = [p_2(B) - 1]c.$$

Comparing these two payoffs, we have

$$\pi_1(A) \geq \pi_1(B) \Leftrightarrow \theta_1 \geq [2p_2(B) - 1]c \equiv \widehat{\theta}_1. \tag{1}$$

Therefore, agent 1's equilibrium strategy must be of cutoff form. Observing (1), we see that agent 1's cutoff depends on $p_2(B)$, the probability that agent 2 plays $B$. In particular, the cutoff is increasing in $P_2(B)$. This result is quite intuitive: agent 1's expected payoff by playing his favored action, relative to that of playing the other action, is increasing in his type $\theta_1$.

In general, there could be multiple equilibria. To see this, suppose $c > 1$. It is straightforward to check that the following constitutes a BNE: agent 1's cutoff is 1 (always play $B$), and agent 2's cutoff is 0 (always play $B$). This is due to the coordination nature of the game: if agent 2 always play $B$, then agent 1's best response is to always play $B$ as well if the cost of miscoordination $c$ is big enough. However, when $c > 1$ there is another BNE in which both agents always choose $A$. Notice that neither these two equilibria $(A, A)$ and $(B, B)$ is symmetric, as one agent is always favored over the other. Moreover, both agents' action choices do not depend on their private information. One problem with adopting either of the above equilibria is that both players still face the problem of which equilibrium to play because the agents still differ in their preferences towards these asymmetric equilibria. Since the agents are ex ante identical and the essence of the coordination problem is how identical players achieve symmetrical coordinated outcomes (Farrell, 1987), in the following analysis we will restrict our attention to symmetric equilibria, in which both agents adopt the same strategy, or have the same cutoff $\widehat{\theta}$.

Symmetry of equilibrium requires that $p_2(B) = 1 - \widehat{\theta}$. Now (1) becomes

$$(1 - 2\widehat{\theta})c = \widehat{\theta} \Rightarrow \widehat{\theta} = \frac{c}{2c+1},$$

which is not only smaller than $c$, but also less than 1/2. The underlying reason for $\widehat{\theta} < 1/2$ is that agent $i$ intrinsically prefers his own favored action.[11] The ex ante expected payoff (before agents learn their types) of each agent, $E\pi_{D,\phi}$, can be computed as

$$\begin{aligned} E\pi_{D,\phi}(c) &= \widehat{\theta}^2 (-c) + \left(1 - \widehat{\theta}\right) \widehat{\theta} \frac{1+\widehat{\theta}}{2} + \left(1 - \widehat{\theta}\right)^2 \left(\frac{1+\widehat{\theta}}{2} - c\right) \\ &= \frac{-4c^3 - c^2 + 2c + 1}{2\left(2c+1\right)^2}. \end{aligned}$$

Note that in the symmetric BNE, from ex ante point of view both the "positive" miscoordination and the "negative" miscoordination will arise with positive probabilities.

[11] If $\widehat{\theta} \geq 1/2$, meaning that the other agent yields with a probability more than 1/2, then agent $i$'s indifference type between sticking and yielding is strictly below 1/2.

# IV Equilibrium with Pre-Communication (Decentralization)

Now we go back to our main focus: the two stage game with pre-communication. Denote $M = R_+$ as each agent's message space. Agent $i$'s strategy in this game includes a communication rule in the first stage and an action rule in the second stage. In particular, agent $i$'s communication rule, denoted as $\mu_i(m_i|\theta_i)$, specifies the probability with which agent $i$ sends a message $m_i \in M$ given that his type is $\theta_i$. Agent $i$'s action rule, denoted as $d_i(m_i, m_j, \theta_i)$, specifies the probability with which agent $i$ plays his favored action given that his type is $\theta_i$ and the messages sent in the first stage are $(m_i, m_j)$. Given the notations, the payoff function of agent $i$ thus can be written as $u_i(d_i, d_j, \theta_i)$. Denote $g_i(\theta_j \mid m_j)$ as agent $i$'s updated belief (in the beginning of the second stage) about $\theta_j$ given agent $j$'s message $m_j$.[12] Our solution concept is Perfect Bayesian Equilibrium (PBE), which requires:

($i$) Given agents' updated beliefs $g_i(\theta_j \mid m_j)$ and $g_j(\theta_i \mid m_i)$, the action rules $d_i(m_i, m_j, \theta_i)$ and $d_j(m_i, m_j, \theta_j)$ constitute a BNE in the second stage game.

($ii$) Given the equilibrium action rules $d_i(m_i, m_j, \theta_i)$ and $d_j(m_i, m_j, \theta_j)$ in the second stage and agent $j$'s communication rule $\mu_j(m_j|\theta_j)$, for each $i$, agent $i$'s communication rule $\mu_i(m_i|\theta_i)$ is optimal.

($iii$) The belief function of agent $i, g_i(\theta_j \mid m_j)$ is derived from agent $j$' communication rules $\mu_j(m_j|\theta_j)$ according to Bayes rule whenever possible.

As in most cheap talk games, in our game there are multiple equilibria. For instance, each agent babbling in the pre-communication stage and playing the symmetric equilibrium in the second stage as in the benchmark case is always an equilibrium. In addition, when $c \geq 1$ there is a simple asymmetric equilibrium. Specifically, in the very beginning (before agents learn their types), a favored agent is randomly selected with probability 1/2, and in the second stage both agents play the favored action of the favored agent. In this asymmetric equilibrium, each agent gets an ex ante payoff of 1/4. Essentially, this asymmetric equilibrium completely avoids miscoordination, but at the same time it might end up choosing the inefficient outcome by completely ignoring agents' private information.

[12] Recall that $\theta_1$ and $\theta_2$ are independent from each other.

However, in general it is hard to characterize the set of all possible PBE. The main difficulty is that, due to the coordination nature of the game, the continuation game in the second stage has multiple equilibria. In the following analysis, we will mainly focus on a particular class of symmetric PBE, which we call symmetric partition PBE. Specifically, in the first stage both agents adopt the same partition communication rule: the type space $[0,1]$ is partitioned into $N$ connected intervals with a sequence of partition points $\{a_1, a_2, ..., a_{N-1}\}$, and each agent $i$ sends message $m_n$ if and only if $\theta_i \in [a_{n-1}, a_n]$. In the second stage, if two agents send different messages, then both agents adopt the action which is favored by the agent who sends a higher message. If two agents send the same message $m_n$, then they play a symmetric BNE conditional on the updated belief: both agents' type are within $[a_{n-1}, a_n]$.

We focus on this class of symmetric partition PBE for the following reasons. First, partition equilibria has been typical in most cheap talk games, starting from Crawford and Sobel (1982). Second, symmetric strategies are natural as the two agents are ex ante identical. Third, the whole point of pre-communication is to indicate how much each agent prefers his own favored action, thus miscoordination in actions could be avoided. Therefore, it is natural that when two agents send different messages they coordinate on the action which is favored by the agent who sends a higher message.[13] At the end of this section, we will argue that symmetric partition PBE are very likely the most efficient PBE.

The next lemma demonstrates that we shall focus on partition equilibria where $a_{N-1} < c$, or all the interior message cutoff points are strictly below $c$.

**Lemma 1** *There is no symmetric partition equilibrium with $a_{N-1} \geq c$.*

The intuition behind Lemma 1 is as follows. If $a_{N-1} \geq c$, then the marginal type $a_{N-1}$ of agent 1 has a dominant strategy in second stage game: he will always choose $A$. But this means that he cannot be indifferent between sending message $m_{N-1}$, and $m_N$, as sending the higher message can increase agent 2's probability of choosing $A$.

Now we can rule out the fully revealing symmetric partition PBE; that is, each agent fully reveals his type. To see this, first consider the case that $c \geq 1$ (no type of agents

[13]Note that under $(m_n, m_{n'})$, $n > n'$ (agent 1's message is higher), the other BNE in which both agents choose agent 2's favored action $B$ might still exist.

has a dominant strategy in the second stage game). Note that, under the fully revealing communication rule, with probability one two agents send different messages. As a result, miscoordination will never arise in the candidate equilibrium. But then any type of agent $i$ (except for type 0) will have an incentive to claim to be the highest possible type, type 1, as it can increase the chance that his favored coordinated outcome will be implemented. Therefore, the fully revealing equilibrium cannot be incentive compatible. A slightly modified argument also applies to the case when $c < 1$.

In order to characterize symmetric partition PBE, we first characterize the symmetric BNE in the second stage continuation game when both agents send the same message $m_{n+1}$. Note that in this continuation game it is common knowledge that both agents' types are uniformly distributed within $[a_n, a_{n+1}]$. As in the benchmark case, in symmetric BNE each agent $i$ plays his favored action if and only if his own state $\theta_i \geq x_{n+1}$, where $x_{n+1}$ is the cutoff. To find the equilibrium cutoff $x_{n+1}$, consider agent 1 whose type is $x_{n+1}$. His expected payoff from choosing $A$ is

$$\frac{x_{n+1} - a_n}{a_{n+1} - a_n} x_{n+1} + \frac{a_{n+1} - x_{n+1}}{a_{n+1} - a_n} (x_{n+1} - c) = x_{n+1} - \frac{a_{n+1} - x_{n+1}}{a_{n+1} - a_n} c.$$

On the other hand, if he chooses $B$ then his expected payoff is

$$\frac{x_{n+1} - a_n}{a_{n+1} - a_n} (-c) .$$

Equating these two payoffs, we thus obtain

$$x_{n+1} = \frac{a_{n+1} + a_n}{a_{n+1} - a_n + 2c} c. \tag{2}$$

It is straightforward to check that both boundary conditions $x_1 = \frac{a_1}{a_1 + 2c} c > 0$ and $x_N = \frac{1 + a_{N-1}}{1 - a_{N-1} + 2c} c < 1$ are satisfied.

Equation (2) can be reformulated as

$$\frac{a_{n+1} - x_{n+1}}{x_{n+1} - a_n} = \frac{c + x_{n+1}}{c - x_{n+1}} > 1. \tag{3}$$

Equation (3) shows that $x_{n+1}$ is closer to $a_n$ than $a_{n+1}$. In other words, in the continuation game when both agents send the same message $m_{n+1}$, each agent will stick with a probability bigger than 1/2. This property arises because of agent $i$'s intrinsic payoff from choosing his favored action: if the other agent sticks and yields with the same probability 1/2, then agent $i$'s indifference action type is strictly below $(a_{n+1} + a_n)/2$.

Based on the above result, an equilibrium is characterized by the message cutoffs $\{a_n\}$ and the action cutoffs $\{x_n\}$. Moreover, when both agents send the same message $m_n$ in the first stage: each agent $i$ chooses his favored action (stick) if $\theta_i \in [x_n, a_n]$, and yield if $\theta_i \in [a_{n-1}, x_n]$. Notice that $x_n < c$ for all $n \leq N$, as otherwise type $x_n$ would have a dominant strategy in sticking. The equilibrium structure is illustrated in the following figure.

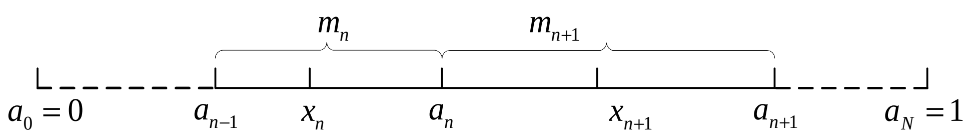



Illustration of the Partition Equilibrium

We are now ready to characterize the message cutoffs for the communication stage. In particular, consider a marginal type of agent 1 with $\theta_1 = a_n$. This type should be indifferent between sending messages $m_{n+1}$ and $m_n$. His expected payoff for agent 1 when he sends message $m_n$ is

$$\begin{aligned}\pi_1(a_n, m_n) &= (a_n - a_{n-1})\left(a_n - c\frac{a_n - x_n}{a_n - a_{n-1}}\right) + a_n a_{n-1} \\ &= (a_n - a_{n-1})\left(-c\frac{a_n - x_n}{a_n - a_{n-1}}\right) + a_n^2.\end{aligned}$$

With probability $(a_n - a_{n-1})$ agent 2 also sends message $m_n$, and in this case agent 1 will stick as his type $a_n$ is higher than the action cutoff $x_n$; thus agent 1 gets $a_n - c\frac{a_n - x_n}{a_n - a_{n-1}}$. With probability $a_{n-1}$ agent 2 sends a message lower than $m_n$ and agent 1 gets $a_n$. With the remaining probability, agent 2 sends a message higher than $m_n$ and agent 1 gets 0. Thus, if he sends the lower message and it is matched, then the agent prefers to choose his preferred action. Similarly, his expected payoff when sending message $m_{n+1}$ is

$$\pi_1(a_n, m_{n+1}) = (a_{n+1} - a_n)\left(-c\frac{x_{n+1} - a_n}{a_{n+1} - a_n}\right) + a_n^2.$$

With probability $(a_{n+1} - a_n)$ agent 2 sends message $m_{n+1}$, and in this case agent 1 will yield as his type $a_n$ is below the action cutoff $x_{n+1}$; thus agent 1 gets an expected payoff of $-c\frac{x_{n+1}-a_n}{a_{n+1}-a_n}$. With probability $a_n$ agent 2 sends a message lower than $m_{n+1}$ and agent 1 obtains $a_n$. With the remaining probability, agent 2 sends a message higher than $m_{n+1}$ and agent 1 gets 0. Thus, if he sends the higher message and it is matched, he will yield, which pins down the payments. The indifference condition yields

$$x_{n+1} + x_n = 2a_n, \text{for all } n = 1, 2, ..., N. \tag{4}$$

The above equation says that, in order to make the marginal type indifferent between sending two adjacent messages, a message cutoff point must lie exactly in the middle of two adjacent action cutoff points.

The intuition for this result is as follows. First, whether agent 1 sends message $m_n$ or $m_{n+1}$ matters only if agent 2's message is $m_n$ or $m_{n+1}$ (agent 1's two messages will induce the same outcome if either agent 2's message is strictly lower than $m_n$ or is strictly higher than $m_{n+1}$). Second, given agent 2's message, whether agent 1 sends message $m_n$ or $m_{n+1}$ does not affect agent 1's action choice. For instance, when agent 2 sends message $m_n$, type $a_n$ of agent 1 always chooses action $A$ no matter whether he sends message $m_n$ or $m_{n+1}$. As a result, whether agent 1 gets benefit $\theta_1 = a_n$ does not depend on the message he sends. Third, whether agent 1 sends message $m_n$ or $m_{n+1}$ will affect agent 2's action choice, thus affecting the possibility of incurring the miscoordination loss. Specifically, if agent 1 sends message $m_n$, then miscoordination will occur if and only if agent 2's type is within $[x_n, a_n]$ (agent 2 sends message $m_n$ as well and chooses action $B$, agent 1 chooses $A$, and the outcome is $AB$). On the other hand, if agent 1 sends message $m_{n+1}$, then miscoordination will occur if and only if agent 2's type is within $[a_n, x_{n+1}]$ (agent 2 sends message $m_{n+1}$ as well and chooses action $A$, agent 1 chooses $B$, and the outcome is $BA$). Thus, to make the marginal type of agent 1 indifferent between sending messages $m_n$ and $m_{n+1}$, the probability of incurring the miscoordination loss when he sends message $m_n$ has to be the same as that when he sends message $m_{n+1}$. Therefore, $a_n - x_n = x_{n+1} - a_n$, or $a_n$ has to be the middle point between $x_n$ and $x_{n+1}$.

The message cutoffs $\{a_n\}$ and the action cutoffs $\{x_n\}$ are recursively defined by equations (3) and (4), with boundary conditions $a_0 = 0$ and $a_N = 1$. Before studying the properties of this difference equation system, we first prove that the proposed strategies along with the cutoffs indeed constitute a symmetric equilibrium, by showing a single crossing property.

**Lemma 2** *The proposed strategies with the cutoffs defined by equations (3) and (4) constitute a symmetric equilibrium.*

The underlying reason for Lemma 2 is that a higher type agent benefits more than a lower type agent from sending a higher message relative to sending a low message. This is because a higher type has a higher intrinsic payoff by choosing his own favored action. Consequently a higher type agent has a (at least weakly) stronger incentive to send a higher message.[14]

Now we shift our attention back to the equations that characterize the cutoffs. Actually, equations (3) and (4), with boundary conditions $a_0 = 0$ and $a_N = 1$, form a first-order difference equation systems with $\{a_n\}$ and $\{x_n\}$ being the endogenous variables. Substituting $\{x_n\}$ by $\{a_n\}$, we obtain the following nonlinear second-order difference equation in $\{a_n\}$

$$(5)\quad a_n^3 - a_n^2\left(a_{n-1} + a_{n+1}\right) + a_n\left(-2c^2 + ca_{n-1} - ca_{n+1} + a_{n-1}a_{n+1}\right) + c^2\left(a_{n-1} + a_{n+1}\right) = 0.$$

Unfortunately, there is no analytical solution to the above difference equation (5). When $N = 2$, after defining $a_1 \equiv a$ the difference equation becomes

$$(6)\qquad f(a) \equiv -a^3 + a^2 + 2ac^2 + ac - c^2 = 0.$$

Observing (6), we see that $f(a)$ is continuous in $a$. Moreover, $f(0) = -c^2 < 0$ and $f(c) = c^3 + c^2 > 0$. Therefore, there exists a cutoff $a \in (0, c)$ such that $f(a) = 0$. Furthermore, $f'(a) = 2(c^2 - a^2) + a + c > 0$ since $a < c$. Therefore, there is a unique $a \in (0, c)$ such that $f(a) = 0$. In addition, since $f(\frac{1}{2}) = \frac{1}{4} + \frac{c}{2} > 0$, it follows that $a < \frac{1}{2}$.

To study the properties of the difference equation, we go back to the original first-order difference equation system, and simplify the notation by defining

$$\Delta x_{n+1} \equiv x_{n+1} - a_n \text{ and } \Delta y_{n+1} \equiv a_{n+1} - x_{n+1}.$$

[14] It is interesting to observe that, in equilibrium, not only the marginal type $a_n$ is indifferent between sending messages $m_n$ and $m_{n+1}$, but all types within $[x_n, x_{n+1}]$ are indifferent between sending messages $m_n$ and $m_{n+1}$. On the other hand, an agent type with $\theta_i \in [x_{n+1}, a_{n+1}]$ strictly prefers sending message $m_{n+1}$ to message $m_n$.

Note that $[a_n, a_{n+1}]$ (the message partition element $n+1$) is now partitioned into $\Delta x_{n+1}$ (the left partition element) and $\Delta y_{n+1}$ (the right partition element) by $x_{n+1}$. The difference equation system now can be reformulated as

$$\frac{\Delta y_{n+1}}{\Delta x_{n+1}} = \frac{c + x_{n+1}}{c - x_{n+1}} > 1, \tag{7}$$

$$\Delta x_{n+2} = \Delta y_{n+1}. \tag{8}$$

The next lemma demonstrates that there exists a unique solution to the above difference equation system.

**Lemma 3** *For a given $N$ and $c$, there exists a unique solution to the difference equation system defined by equations (3) and (4), with boundary conditions $a_0 = 0$ and $a_N = 1$.*

From (7) and (8), three observations are in order. First, the ratio of the size of the right partition element to that of the left partition element is increasing in $x_{n+1}$, hence in $n$. This is because a higher type agent has a stronger incentive to stick (hence get his intrinsic payoff $\theta_i$) in the continuation game when both agents send the same message. As a result, not only the size of the message partition elements $a_{n+1} - a_n$ increases in $n$, but also it increases at an increasing rate. Second, the above mentioned ratio converges to 1 when $x_{n+1}$ approaches 0. Intuitively, a type 0 agent has no intrinsic payoff thus he does not favor sticking over yielding; in other words, he has no incentive to exaggerate his type. This property implies that the message partition elements $a_{n+1} - a_n$ increases very slowly around 0, which further implies that the number of partition elements $N$ could be infinity. Finally, the above mentioned ratio is decreasing in $c$, the cost of miscoordination. This is because, as $c$ increases miscoordination becomes more costly and the agents' incentive to stick decreases in order to avoid miscoordination.

Another way to understand the equilibrium partition pattern is as follows. Given that the messages are rank ordered, and the agent sending a relatively higher message will have his favored coordinated outcome implemented, each agent has an incentive to exaggerate his own type. In order to restore incentive compatibility, there must be costs of sending higher messages, and these costs come from the possibility of miscoordination when both agents send the same message. Moreover, a higher type agent has a stronger incentive to exaggerate, as his intrinsic payoff of playing his own favored action is higher. As a result, full incentive compatibility requires that the costs of sending a higher message be higher. To achieve this, miscoordination when both agents send the same message must become more likely as the message becomes higher, which implies that for higher messages the sizes of partition elements get larger (or higher messages are noisier). On the other hand, when an agent's type approaches zero, his incentive to exaggerate also approaches zero; as a result, the corresponding low messages become infinitely accurate.

By the same logic, as the cost of miscoordination increases, other things equal, the punishment of miscoordination becomes more severe. This means that to provide the same amount of punishment, now the probabilities of miscoordination when two agents send the same message can be reduced, which implies that the sizes of equilibrium partition elements now can be smaller, or communication becomes more informative.

The following proposition summarizes the results we have so far.

**Proposition 1** *Given $c$ and the number of message partition elements $N$, in the game with pre-communication there is a unique symmetric partition equilibrium, with message cutoffs $\{a_n\}$ and action cutoffs $\{x_n\}$ following equations (3) and (4). In the unique equilibrium, the size of the partition element increases at an increasing rate. Moreover, as the cost of miscoordination $c$ increases, the rates at which the size of the partition element increases decrease, or the size of the partition elements becomes more even.*

For a given $N$ and $c$, we denote the ex ante expected payoff of an agent with pre-communication as $E\pi_D(N,c)$. It can be computed as follows.

$$E\pi_D(N,c) = \Sigma_{n=1}^{n=N}\left((x_n - a_{n-1})^2(-c) + (a_n - x_n)^2\left(\frac{a_n + x_n}{2} - c\right) + (x_n - a_{n-1})(a_n - x_n)\frac{a_n + x_n}{2}\right) + \Sigma_{n=1}^{n=N}(a_n - a_{n-1})\,a_{n-1}\frac{a_n + a_{n-1}}{2}. \quad (9)$$

In the above expression, the first term is the ex ante expected payoff if two agents are tied with sending the same message, and the second term is the ex ante expected payoff if the agent in question sends a message higher than the other agent does.

**Lemma 4** *Given c, denote* $(a, x)$ *and* $(a', x')$ *as the cutoff points in a N-partition equilibrium and a* $(N+1)$*-partition equilibrium, respectively. Then we have: (i)* $a'_1 < a_1 < a'_2 < a_2 ... < a'_{N-1} < a_{N-1} < a'_N$*; and (ii)* $\Delta x'_{n+1} < \Delta x_n$ *and* $\Delta y'_{n+1} < \Delta y_n$ *for all* $n \leq N$*.*

Lemma 4 indicates that, as the number of partition elements $N$ increases, not only the range of meaningful communication expands ($a_{N-1}$ increases), but also the partition elements become more even. As a result, communication becomes more informative and the probability of having socially inefficient miscoordination decreases.[15] Therefore, we have the following proposition.[16]

**Proposition 2** *In the most informative equilibrium, the number of message partition elements N is infinite. Moreover, the size of message partition element approaches zero when an agent's type approaches 0.*

The equilibria in our model and those in ADM and Rantakari (2008) share some similar features. For instance, equilibria are partitional and the most informative equilibrium has an infinite number of partition elements. This is because the conflict between coordination and choosing the actions favoring agents themselves are present both in our model and their models. However, as mentioned earlier, a key difference between our model and those of ADM and Rantakari is that in their models actions are continuous, while in our model they are binary. Moreover, agents' payoff functions are also different: in ADM the payoff functions are quadratic loss functions, while in our model an agent's payoff is additive in his own intrinsic preference and the miscoordination loss. Continuous actions plus quadratic loss payoff functions in ADM imply that agents always suffer from miscoordination loss, but the magnitudes vary continuously with the difference between the two actions. In our model,

[15] Both $a_{N-1}$ and $x_N$ are smaller than $c$, thus any miscoordination is socially inefficient.

[16] When $N$ goes to $\infty$, the equilibrium partition must converge. To see this, define $a_{N-1}(N)$ as the largest message cutoff in the $N$-partition equilibrium. By Lemma 1, $a_{N-1}(N) \leq c$ for any $N$, and by Lemma 4 $a_{N-1}(N)$ is increasing in $N$. Therefore, $a_{N-1}(N)$ must converge as $N$ goes to infinity. This also implies that all other message cutoffs converge, as the difference equation system holds for any $N$.

binary actions imply that agents either suffer from miscoordination or do not suffer at all. These differences lead to different equilibrium construction. Specifically, in our model agents' messages are rank ordered: the agent sending a higher ranked message gets his way (has his preferred coordinated outcome implemented), and agents suffer from miscoordination only when they send the same message, which serve as a punishment for exaggerating. This rank-ordered feature of messages is not present in ADM, instead coordination loss of varying degrees is always suffered regardless of the messages sent in the communication stage.[17]

In the symmetric partition equilibria, two agents play a symmetric BNE in the second stage game whenever they send the same message. Since the symmetric BNE involves with costly miscoordination, one might wonder whether introducing a public randomization device could help. In particular, whenever two agents send the same message, a public randomization device randomly picks a favored agent with probability 1/2, and then two agents play the coordinated outcome favored by the favored agent. This candidate equilibrium is more ex post efficient, as it gets rid of costly miscoordination. However, no informative communication will occur in this kind of equilibrium (only babbling equilibrium exists), exactly because the public randomization device gets rid of the possibility of miscoordination. Without the possibility of costly miscoordination in any scenario, any type of agent has an incentive to exaggerate his own type to the extreme by sending the highest message, as the cost or the punishment of sending the highest message no longer exists.

We conjecture that the symmetric partition equilibria are very likely the most efficient equilibria among all possible PBE. The underlying logic can be best illustrated for the case that $c \geq 1$, under which the socially efficient outcome must be one of the two coordinated outcomes, $AA$ and $BB$. In any PBE with precommunication, for any message pair sent in the 1st stage, in the continuation game there are at most three possible equilibria: $AA$, $BB$, and a cutoff strategy equilibrium with higher type agents choosing his favored action. Note that the cutoff strategy equilibrium is inefficient as miscoordination occurs with positive probabilities. Under symmetric partition equilibria agents' messages are rank ordered; when two agents send different messages the socially efficient outcome is implemented: both agents

[17] The difference equations characterizing the equilibria are also different. In ADM the difference equation has analytical solutions, but the one in our model does not have analytical solutions.

coordinate on the action favored by the agent whose type is higher. The inefficient cutoff strategy equilibrium is chosen only when two agents send the same message, which serves as necessary punishments to counter agents' incentives to exaggerate their types. In any other PBE with different communication rules, for instance some low types and some high types pooled together to send a message while some middle types send another different message, the rank order of the messages becomes less precise about the rank order of agents' types. As a result, the socially efficient outcome -the coordinated outcome favored by the higher type agent- will less likely to be implemented. Therefore, symmetric partition equilibria are very likely more efficient than other PBE.

To study the effect of the number of partition elements $N$ on the ex ante expected payoff, we resort to numerical analysis because there is no analytical solution for the difference equation system. Table 1 compares each agent's expected payoff across equilibria with different $N$. Since the expected payoffs are influenced by the miscoordination cost, we calculate the percentage gain using the equilibrium payoff without precommunication ($\pi_{D,\phi}$) as a benchmark. In particular, we compute the following ratio $(\pi_D^N - \pi_{D,\phi})/(\pi_D - \pi_{D,\phi})$, where $\pi_D^N$ is the expected payoff with $N$ partition elements and $\pi_D$ is the expected payoff when $N$ goes to infinity.[18] Two observations are in order. First, although the expected payoff increases with $N$, the marginal increase decreases quickly as $N$ increases. Taking $c = 1/2$ as an example, the marginal gain (in term of percentage) from the two-partition equilibrium to the three-partition equilibrium is about 16%, but it decreases to 7% moving from the three-partition equilibrium to the four-partition equilibrium. The intuition for this result is that, with more partition elements the probability of miscoordination decreases, but the reduction in the overall probability of miscoordination becomes smaller as $N$ increases.[19] Second, with a bigger miscoordination cost $c$, as $N$ increases the marginal gain in payoff decreases more slowly. For instance, when $c = 1/5$ the payoff difference between the two-partition equilibrium and ten-partition equilibrium is about 25%. In contrast, when $c = 1$ this difference increases to 36%. This observation indicates that the agents benefit more from more informative

[18] Since the expected payoff converges quickly as the number of partition $N$ increases, we use $N = 20$ to approximate the converged payoff.

[19] As $N$ increases, the overall probability of miscoordination quickly converges to that of the most informative equilibrium.

| N\c (%) | 1/5 | 1/2 | 4/5 | 1 |
|---|---|---|---|---|
| 2 | 75.44 | 67.38 | 63.01 | 61.13 |
| 3 | 89.47 | 83.80 | 80.62 | 79.10 |
| 4 | 93.86 | 90.44 | 88.11 | 87.00 |
| 5 | 96.49 | 93.70 | 92.08 | 91.23 |
| 10 | 99.99 | 98.65 | 98.19 | 97.93 |

Table 1: Ex ante Payoff and the Number of Partition Elements

communication as the cost of miscoordination $c$ increases. This is because more informative communication can reduce the overall probability of miscoordination, which is more valuable under a bigger $c$. More precisely, under a bigger $c$ the range of potential meaningful communication is larger (recall that in equilibrium $a_{N-1} < c$). Moreover, a bigger $c$ reduces the rates at which the size of partition element increases, which means that an increase in $N$ becomes more valuable.

We conclude this section by studying the effects of the cost of miscoordination $c$ on each agent's ex ante expected payoff. The results are demonstrated in Figure 1. As it reveals, the expected payoff under the most informative equilibrium decreases with the cost of miscoordination $c$.[20] Intuitively, as the cost of miscoordination increases, it directly hurts the agents. However, it also makes the size of the partition elements more even, which reduces the overall probability of miscoordination and thus indirectly benefits agents. Our numerical simulation shows that overall the direct effect dominates.[21]

[20] In the numerical simulation, we choose $N = 100$. This is because the expected payoff quickly converges when $N > 20$.

[21] We want to point out that when the cost of miscoordination is relatively small ($c < \frac{1}{5}$), even the two-partition equilibrium can achieve a majority gain of the first-best social surplus, similar to a finding in Macfee (2002) in the context of assortative matching. In particular, when $c = 1/10$, the ratio of the expected payoff under the two-partition equilibrium to the first-best payoff is 99%, and it is about 94% when $c = 1/5$. However, the efficiency of the two-partition equilibrium declines quickly as the cost of miscoordination increases further.

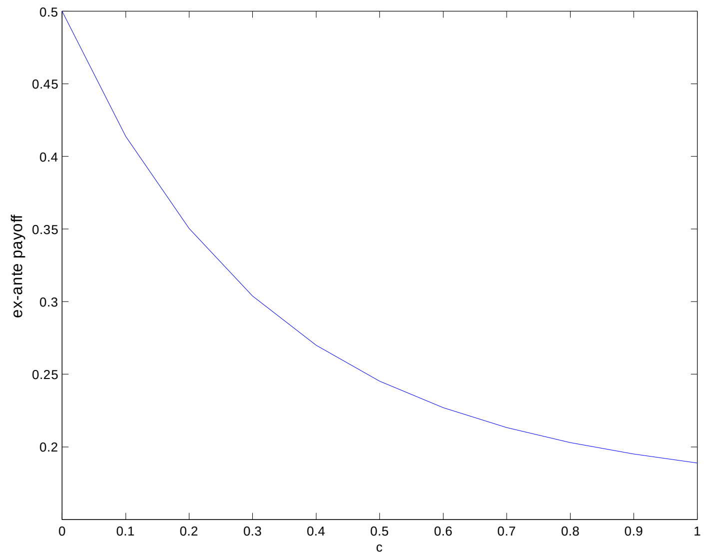



Figure 1: The Relatioship between Ex ante Payoff and $c$

# V Mediator without Commitment (Centralization)

In this section we introduce a mediator/social planner, whose objective is to maximize the ex ante social surplus (the joint payoff of the two agents), and study how it affects pre-communication and the equilibrium outcome. Moreover, the mediator is not able to commit to a decision rule beforehand; that is, given any message pair $(m_1, m_2)$, the mediator will choose the outcome that maximizes the social surplus based on the information revealed in the messages. This scenario corresponds to centralization in ADM. In particular, one can think of the two agents as two divisional managers of a company and the mediator as the CEO of the company, who cares about the joint payoff of the two divisions. To ease exposition, we sometimes call this scenario centralization, and the scenario of the baseline model decentralization.

## V(i) A Benchmark

We first study a benchmark case, under which there is no pre-communication from the agents to the mediator. We study this case because the introduction of a mediator (who has the final decision rights) can potentially get rid of the possibility of undesirable miscoordination. Specifically, in this case the expected payoff for the mediator from choosing either outcome $AA$ or $BB$ is $\frac{1}{2}$. On the other hand, the expected payoff of choosing outcome $AB$ is $1-2c$. Therefore, if $c \leq \frac{1}{4}$, then it is optimal for the mediator to choose outcome $AB$ and the joint social surplus is $1-2c$; if $c > \frac{1}{4}$, then it is optimal for the mediator to choose either $AA$ or $BB$ (or randomizing between the two outcomes with any probability) and the joint social surplus is $\frac{1}{2}$. Denote the expected payoff of each agent under mediation when there is no pre-communication as $E\pi_{C,\phi}(c)$, we thus have

$$E\pi_{C,\phi}(c) = \begin{cases} \frac{1}{2} - c & \text{if } c < 1/4 \\ \frac{1}{4} & \text{if } c \geq 1/4 \end{cases}.$$

## V(ii) Mediated Pre-Communication

As before, we focus on symmetric partitional equilibria. That is, two agents have the same set of messages, and they follow the same communication rule which is partitional: send message $m_n$ if and only if $\theta_i \in [a_{n-1}, a_n]$. In the mediator's decision rule, he treats two agents in a symmetric way.

We first characterize the mediator's optimal decision rule. Denote $\overline{m}_n$ as the mediator's posterior about $\theta_i$ after agent $i$ sends message $m_i$: $\overline{m}_n \equiv E[\theta_i|m_n] = \frac{a_{n-1}+a_n}{2}$. Given a message pair $m_n$ and $m_{n'}$, $n' \geq n$, the mediator chooses outcomes to maximize the joint payoff: $\max\{\overline{m}_n, \overline{m}_{n'}, \overline{m}_n + \overline{m}_{n'} - 2c\}$, which correspond to $AA, BB$, and $AB$, respectively. It is immediate that her optimal decision is as follows:

$$d^*(m) = \begin{cases} p_{AB} = 1; \text{ if } \overline{m}_n \geq 2c \\ p_{AA} = 1 \text{ (agent 1 sends } m_{n'}), \, p_{BB} = 1 \text{ (agent 2 sends } m_{n'}); \text{ if } \overline{m}_n < 2c \text{ and } n \neq n' \\ p_{AA} = p_{BB} = 1/2; \text{ if } n = n' \end{cases}.$$

When $c \geq 1/2$, the above optimal decision rule implies that the outcome $AB$ will never be chosen (no potential punishment). As a result, each type of agent $i$ will claim to be the highest types or send the highest message $m_N$. Therefore, in this case only the babbling equilibrium exists, and the mediator chooses both $AA$ and $BB$ with probability 1/2.

Now consider the case that $c < 1/2$. In the following lemma we show that there exists no equilibrium with three or more partition elements.

**Lemma 5** *Suppose $c < 1/2$ and the mediator is not able to commit. Then the most informative symmetric partitional equilibrium can have at most two partition elements.*

To understand the intuition for Lemma 5, we first note that the mediator will never implement the socially undesirable (negative miscoordination) outcome $BA$. Second, without commitment the mediator's optimal decision is not rich enough. In particular, essentially there are only two types of outcomes: the mediator will choose the outcome $AB$ if the lower message's posterior is bigger than $2c$, and otherwise chooses one of the two perfectly coordinated outcomes $AA$ and $BB$. To sustain incentive compatibility, sending different messages should lead to different types of outcomes. Thus the fact that in total there are only two possible types of outcomes exactly means that at most two meaningful equilibrium messages can be sustained.

Now we characterize the two-partition equilibrium when $c < 1/2$. Denote the messages as $m_1$ and $m_2$, and the message cutoff type as $a$. By previous results, it must be the case that $\overline{m}_1 < 2c$, $\overline{m}_2 > 2c$, or $a < 4c$ and $1 + a > 4c$. Given the mediator's optimal decision rule $d^*(m)$, consider the marginal type $a$ of agent 1. His expected payoff of sending $m_1$ and $m_2$ can be computed as follows:

$$a^2 + (1-a)(a-c) = \pi_1(a, m_2) = \pi_1(a, m_1) = \frac{1}{2}a^2$$

$$\Rightarrow f(a) \equiv \frac{1}{2}a^2 + (1-a)(a-c) = 0. \tag{10}$$

By (10), we can see that $a < c$. It can be readily verified from (10) that $f(0) < 0$, $f(c) > 0$, and $f'(a) > 0$. Thus there is a unique $a \in (0, c)$ satisfying (10). Finally, the equilibrium has to satisfy $1 + a > 4c$. Using (10), this condition is equivalent to $c \leq \widehat{c} = \frac{3}{4} - \frac{\sqrt{3}}{4} \simeq 0.317$. That is, if $c \geq \widehat{c}$, then only babbling equilibrium exists.

We denote each agent's ex ante expected payoff under mediation without commitment as $E\pi_D(c)$. Based on the previous results, it can be computed as

$$E\pi_D(c) = \begin{cases} a^2\left(\frac{a}{4}\right) + (1-a)^2\left(\frac{1+a}{2} - c\right) + (1-a)\,a\left(\frac{1+a}{2}\right) & \text{if } c \leq \frac{3}{4} - \frac{\sqrt{3}}{4} \\ \frac{1}{4} & \text{if } c > \frac{3}{4} - \frac{\sqrt{3}}{4} \end{cases},$$

where $a$ is defined in equation (10). The results in this subsection are summarized in the following proposition.

**Proposition 3** *Suppose the mediator is not able to commit. If* $c > \frac{3}{4} - \frac{\sqrt{3}}{4}$*, then only babbling equilibrium exists. If* $c \leq \frac{3}{4} - \frac{\sqrt{3}}{4}$*, then there is a unique two-partition equilibrium, with the cutoff satisfying (10); moreover, equilibria with three or more partition elements do not exist.*

Recall that in the baseline model without mediator (decentralization), there is an infinite number of messages in the most informative equilibrium. Thus more equilibrium messages can be sustained under decentralization than under centralization. The reason that more messages can be sustained under decentralization (despite the fact that there are only four possible outcomes) but not under centralization is that, under decentralization miscoordination occurs with positive probabilities whenever both agents send the same message (regardless of which message), which serves as punishments for exaggerating one's own type. In contrast, under centralization miscoordination occurs only when it is socially desirable, which limits the punishments for agents' exaggeration and as a result, at most two equilibrium messages can be sustained.

Based on the result of Proposition 3, we conclude that, when $c > \frac{3}{4} - \frac{\sqrt{3}}{4} \simeq 0.317$, , only babbling equilibrium exists under centralization. When $c \leq 0.317$, although only two messages can be sustained under centralization while an infinite number of messages can be sustained under decentralization, the partition seems more even under centralization than under centralization. For instance, when $c = 0.3$, under centralization the cutoff is 0.256, while under decentralization with 30 messages the highest cutoff ($a_{29}$) is 0.197.[22] The reason that the highest marginal type under decentralization is lower than the marginal type under centralization is as follows. Under centralization, although the marginal type will suffer from miscoordination with a positive probability if he sends the high message, he will not suffer from miscoordination if he sends the low message. In contrast, under decentralization, although the highest marginal type suffers from more damaging miscoordination compared to that under centralization (recall that centralization gets rid of the damaging negative miscoordination) if he sends the higher message ($m_N$), he still suffers from the possibility of miscoordination if he sends the lower message ($m_{N-1}$) (but the probability is lower than the case if he sends the higher message). Consequently, the difference in the expected

[22] The other messages are very accruate, but the highest message is very noisy, with all types above 0.197 pooled together. The same pattern holds for $c$ smalller than 0.3.

miscoordination costs suffered between sending two adjacent messages is actually higher under centralization than under decentralization, implying that the agents have a stronger incentive to send the highest message under decentralization than under centralization.

We want to point out that the above result is quite different from that in ADM and Rantakari (2008), where communication is always more informative under centralization than under decentralization. Recall that in their models the agents' actions are continuous. Consequently, under centralization the trade-off between adaptation and coordination always exists and the CEO's optimal decisions varies with agents' messages continuously. Centralized communication performs better than decentralized communication because the agents' incentives are more aligned with the CEO than between themselves. In our model, the agents' actions are binary, and communication is more informative under decentralization than under centralization when the cost of miscoordination is high enough. Therefore, our study demonstrates that the reason for communication and the value of communication differ significantly across binary action setting and continuous action setting.

In Figure 2, we compare the ex ante expected payoff for each agent under the most informative equilibrium in the baseline model ($\pi_D$) with that under centralization ($\pi_C$).

From the figure we see that, at $c = \frac{3}{4} - \frac{\sqrt{3}}{4} \simeq 0.317$, the expected payoff under centralization has a discontinuous jump. This is because, as mentioned earlier, a two-partition equilibrium can be sustained when $c \leq \frac{3}{4} - \frac{\sqrt{3}}{4}$, but only babbling equilibrium exists when $c > \frac{3}{4} - \frac{\sqrt{3}}{4}$, and agents suffer a payoff loss when informative communication becomes infeasible.[23]

Comparing the payoffs under decentralization and centralization, we observe the following pattern. When the miscoordination cost $c$ is sufficiently small ($c \leq 0.317$), centralization performs slightly better than decentralization. When $c$ is intermediate ($0.317 < c < 0.48$), decentralization performs better than centralization. However, when $c$ is large ($c > 0.48$), then centralization is again better than decentralization.

[23]When $c = \frac{3}{4} - \frac{\sqrt{3}}{4}$, the mediator is indifferent between choosing $AB$ and $AA$ or $BB$ when both agents send the high message. However, in this case, only the following equilibrium exists: outcome $AB$ is chosen for sure when both agents send the high message. If the probability of choosing $AB$ is strictly less than 1, then the message cutoff $a$ will decrease, and choosing $AB$ is no longer optimal for the mediator (since $\overline{m}_2$ decreases with $a$), which destroys equilibrium.

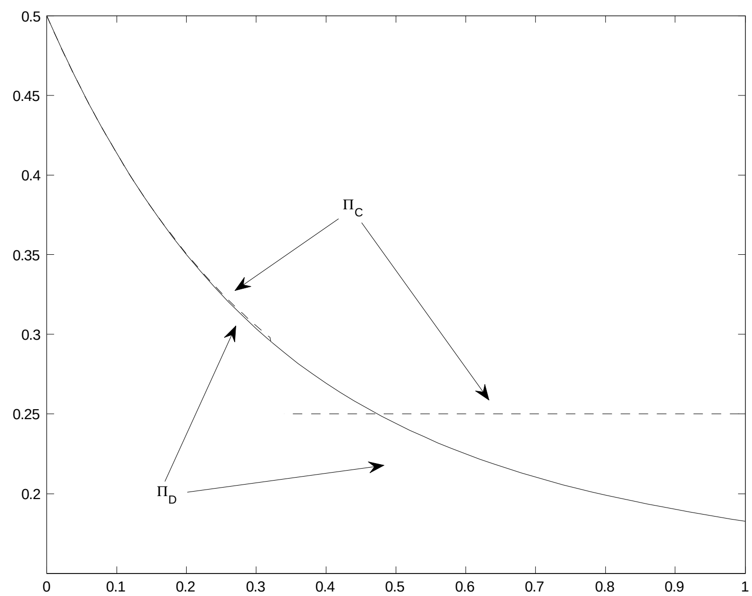



Figure 2: Payoff Comparision between Centralization and Decentralization

To understand this pattern, recall that on the one hand communication under decentralization is different from that under centralization, which we call the "communication effect." This effect favors decentralization when the miscoordination cost is high ($\geq 0.317$), but it favors centralization when the miscoordination cost is low ($\leq 0.317$). On the other hand, the presence of the mediator can get rid of the damaging "negative" miscoordination outcome, while miscoordination occurs with positive probabilities under decentralization. We call this the "coordination effect," which always favors centralization. When the cost of miscoordination is sufficiently low ($\leq 0.317$), the coordination effect is insignificant. At the same time, the communication effect is insignificant as well, as communication is very noisy in both scenarios since the cost of exaggerating is low. As a result, decentralization and centralization almost achieve the same payoff, though centralization performs slightly better (both the coordination effect and the communication effect favor centralization). In the other extreme, when the cost of miscoordination is sufficiently high, the coordination effect favors centralization while the communication effect favors decentralization. However, the coordination effect dominates as miscoordination is very costly; and as a result, again centralization, which completely ignores private information, performs better than decentralization. The more interesting case is when the miscoordination cost is intermediate. In this situation the coordination effect is intermediate, while the communication effect favoring decentralization is strong, as only babbling equilibrium exists under centralization. And thus decentralization performs better than centralization.

The above pattern differs from the comparison of centralization and decentralization in ADM and Rantakari (2008). For instance, ADM shows that decentralization is better when coordination is not important but centralization performs better when coordination is important.[24] However, we show that centralization performs better only when coordination is either sufficiently unimportant or sufficiently important in the current setting. This further demonstrates that the incentive to communicate and the value of communication in our model are different from those in a continuos actions setup such as ADM.

# VI Mediator with Commitment (Mechanism Design)

In this section, we adopt a mechanism design approach to study the communication equilibria when the mediator is able to commit to a pre-announced decision rule. Specifically, the mediator first announces a decision rule $d(m_1, m_2)$, which maps a message pair $(m_1, m_2) \equiv m$ into a probability distribution over four possible outcomes: $p(m) = (p_{AA}(m), p_{BB}(m), p_{AB}(m), p_{BA}(m))$. Then, the two agents simultaneously communicate to the mediator regarding their own types. And finally, the mediator chooses the actions for both players according to the pre-announced decision rule $p(m)$.

Our purposes of studying mediated pre-communication with commitment are twofold. First, we want to show that if the mediator can commit a decision rule beforehand, then even a simple two-message equilibrium is able to deliver an outcome that dominates the most informative decentralized solution; second and more important, we want to illustrate the point that the collapse of communication under centralization is not due to central-

[24] Our model corresponds to the case that $\lambda = 1$ in their model. That is, each agent only cares about his own payoff.

ization itself but due to the fact that under centralization the mediator cannot commit to choosing miscoordinated outcomes to punish the agents, which eliminates their incentives to communicate.

There is a literature on how mediated pre-communication can improve efficiency in games (e.g., Forges, 1990; Myerson, 1991; Banks and Calvert, 1992; Ben-Porath, 2003; Gerardi, 2004; Krishna, 2007; Bergemann and Morris, 2011; Vida and Forges, 2013). In those papers, agents all have a finite number of types. Moreover, the mediator usually only recommends (either privately or publicly) actions for each agent, and the recommendations are not binding. As a result, an obedience constraint for each agent (has an incentive to follow the mediator's recommended action) needs to be added. Having a finite number of types makes those models tractable, as they can use the Revelation Principle to identify the set of incentive compatible mechanisms. In our model, there are multiple agents and each agent's type is continuous. Although the Revelation Principle still applies, it is very hard to identify the set of incentive compatible mechanisms in our setting.[25]

Due to this difficulty and for the purpose of our analysis, here we assume that the mediator is able to choose the outcome of the game (hence drop the obedience constraints) and restrict our attention to a simple class of (direct) mechanisms, which we call two-partition symmetric mechanisms. In particular, each agent has two messages, $h$ and $l$, and each agent plays the same following strategy: sends message $h$ if his own type is higher than a cutoff $a$, and sends message $l$ otherwise. If two agents send different messages, then the mediator chooses the coordinated outcome favored by the agent who sends message $h$. If two agents send the same message $l$ ($h$), then the decision rule is $p^l$ ($p^h$). Moreover, $p^l_{AA} = p^l_{BB}$ and $p^h_{AA} = p^h_{BB}$; that is, the mediator treat two agents symmetrically if they send the same message. The decision rule is summarized below.

$$p(m) = \begin{cases} p_{AA} = 1 \text{ if } m_1 = h, m_2 = l \\ p_{BB} = 1 \text{ if } m_1 = l, m_2 = h \\ p^l_{AB}, p^l_{BA}, p^l_{AA} = p^l_{BB} \text{ if } m_1 = m_2 = l \\ p^h_{AB}, p^h_{BA}, p^h_{AA} = p^h_{BB} \text{ if } m_1 = m_2 = h \end{cases}$$

We will characterize the optimal two-partition symmetric mechanism that maximizes the ex ante social surplus. In theory, we could also study $N$-partition symmetric mechanisms.

[25] To the best our knowledge, nobody has done that in the existing literature.

But it turns out to be much more complicated, as there will be too many design variables. Later on, we will show that the optimal two-partition mechanism can already achieve a majority of the social surplus when the cost of miscoordination $c$ is small, relative to the first best outcome.

Given the symmetric requirement, the decision rule is characterized by four variables: $p^h_{AB}$, $p^h_{BA}$, $p^l_{AB}$, and $p^l_{BA}$, as $p^h_{AA} = p^h_{BB} = (1 - p^h_{AB} - p^h_{BA})/2$ and $p^l_{AA} = p^l_{BB} = (1 - p^l_{AB} - p^l_{BA})/2$. Given this mechanism, the two agents play a Bayesian game in the communication stage. We first characterize the marginal type of agents who is indifferent between sending messages $h$ and $l$. Suppose agent 2 sends message $l$ with probability $a$, and consider type $\theta_1$ of agent 1. His expected payoff by sending message $h$, $\pi_1(h, \theta_1)$, is

$$\pi_1(h, \theta_1) = a(\theta_1) + (1 - a)[p^h_{AA}\theta_1 + p^h_{AB}(\theta_1 - c) + p^h_{BA}(-c)],$$

while his expected payoff of sending message $l$, $\pi_1(l, \theta_1)$, is

$$\pi_1(l, \theta_1) = a[p^l_{AA}\theta_1 + p^l_{AB}(\theta_1 - c) + p^l_{BA}(-c)].$$

Taking the difference, we have

$$\frac{\partial[\pi_1(h, \theta_1) - \pi_1(l, \theta_1)]}{\partial\theta_1} > 0.$$

Thus in equilibrium agent 1 (and agent 2 as well by symmetry) will adopt a cutoff strategy. To solve the equilibrium cutoff $a$, we equate $\pi_1(h, \theta_1)$ and $\pi_1(l, \theta_1)$ and impose symmetry $\theta_1 = a$, which yields the following equation

$$\begin{aligned}(11)\qquad &\left(\frac{p^h_{BA} + p^l_{BA} - p^h_{AB} - p^l_{AB}}{2}\right)a^2 + \\ &\left(\frac{1 + p^h_{AB} - p^h_{BA}}{2} + p^h_{AB}c + p^h_{BA}c + p^l_{AB}c + p^l_{BA}c\right)a - \left(p^h_{AB} + p^h_{BA}\right)c = 0\end{aligned}$$

In the optimal two-partition symmetric mechanism, the mediator chooses $p^l_{AB}, p^l_{BA}, p^h_{AB}$ and $p^h_{BA}$ to maximize the following social surplus, subject to (11):

$$\begin{aligned}(12)\qquad f\left(p^l_{AB}, p^l_{BA}, p^h_{AB}, p^h_{BA}\right) \equiv\ &\frac{1 + p^l_{AB} - p^l_{BA}}{2}\frac{a^3}{2} - \left(p^l_{AB} + p^l_{BA}\right)ca^2 + \frac{a - a^3}{2} \\ &+ \frac{1 + p^h_{AB} - p^h_{BA}}{2}\frac{(1 + a)(1 - a)^2}{2} - \left(p^h_{AB} + p^h_{BA}\right)c(1 - a)^2\end{aligned}$$

**Lemma 6** *In the optimal two-partition symmetric mechanism: (i)* $p^l_{BA} = p^h_{BA} = 0$*; (ii)* $a < 2c$*; (iii)* $p^l_{AB} = 0$.

The intuition for Lemma 6 is as follows. As for part (i), recall that the reverse miscoordination outcome $BA$ is very costly for both agents. On the other hand, the positive miscoordination outcome $AB$ is not that costly, since both agents get their intrinsic payoffs. As a result, to reduce agents' incentives to exaggerate their own types, it is more efficient to use outcome $AB$ alone to provide the necessary punishments. By part(i), part (ii) follows naturally. Since $p^h_{BA} = 0$, the punishment for sending the high message is not that severe. As a result, when an agent's type is bigger than $2c$, sending the high signal becomes a dominant strategy. Thus, $a < 2c$. This further implies that the cutoff point $a$ is decreasing in $p^l_{AB}$ but increasing in $p^h_{AB}$. These results are intuitive. As $p^l_{AB}$ increases, the punishment for sending the low signal increases, and thus the agents have a stronger incentive to exaggerate or $a$ decreases. On the other hand, as $p^h_{AB}$ increases, the punishment for sending the high signal increases, and thus agents have a weaker incentive to exaggerate or $a$ increases. Now the intuition for part (iii) becomes clear. Given that each agent has a natural tendency to exaggerate his own type, sending message $l$ should be encouraged. To achieve this, the punishment of sending message $l$ ($p^l_{AB}$) should be minimized. Moreover, this punishment of miscoordination is costly for the social surplus. Therefore, in the optimal mechanism there should be no punishment if both agents send the same message $l$.

Based on the results of Lemma 6, the programming problem is further reduced to

$$\max_{p^h_{AB}, a} \frac{a(1-a)}{4} + \frac{(1-a)^2(1+a)}{4} p^h_{AB} - (1-a)^2 p^h_{AB} c + \frac{1}{4},$$

$$\text{where } \frac{-p^h_{AB}}{2} a^2 + \left( \frac{1+p^h_{AB}}{2} + p^h_{AB} c \right) a - p^h_{AB} c = 0. \quad (13)$$

**Lemma 7** *In the optimal two-partition symmetric mechanism,* $p^h_{AB} = 1$ *if* $c < \frac{1}{2}$*,* $p^h_{AB} = 0$ *if* $c > \frac{1}{2}$*, and any* $p^h_{AB} \in [0, 1]$ *is optimal if* $c = \frac{1}{2}$.

Lemma 7 indicates that, when $c > 1/2$, in the optimal mechanism both agents babble. This is because in this case it is too costly for the mediator to choose the outcome $AB$.[26]

[26] Recall that it is socially inefficient to implement outcome $AB$ whenever $1 < 2c$.

On the other hand, when $c < 1/2$, it is not that socially costly to implement the outcome $AB$. Moreover, since $a < c < 1/2$, a higher cutoff point $a$ makes the two partition elements more even, which increases the probability of choosing the socially efficient outcome. As a result, the optimal $p^h_{AB}$ is 1, which achieves the highest possible $a$. When $c = 1/2$, the direct cost of implementing $AB$ (which is socially inefficient when both agents send message $h$), exactly equals to the indirect benefit of implementing $AB$ (increases $a$ such that information transmission becomes more informative). As a result, any $p^h_{AB}$ between 0 and 1 is optimal.

We denote each agent's ex ante expected payoff under the optimal mechanism as $E\pi_M(c)$. Based on the previous results, it can be computed as

$$E\pi_M(c) = \begin{cases} a^2\left(\frac{a}{4}\right) + (1-a)^2\left(\frac{1+a}{2} - c\right) + (1-a)\,a\left(\frac{1+a}{2}\right) & \text{if } c < 1/2 \\ \frac{1}{4} & \text{if } c \geq 1/2 \end{cases},$$

where $a$ is defined in equation (13) with $p^h_{AB} = 1$.

The following proposition summarizes the above results.

**Proposition 4** *In the optimal two-partition symmetric mechanism,* $p^l_{BA} = p^l_{AB} = p^h_{BA} = 0$*;* $p^h_{AB} = 1$ *if* $c < \frac{1}{2}$*,* $p^h_{AB} = 0$ *if* $c > \frac{1}{2}$*, and any* $p^h_{AB} \in [0,1]$ *is optimal if* $c = 1/2$*. In other words, the mediator only uses the outcome AB to punish agents, and she will do so only when both of agents send message h and the cost of miscoordination is sufficiently low.*

The relative efficiency of alternative equilibria and mechanisms in comparison with the first-best outcome are depicted in Figure 3. It is not surprising to observe that the expected payoff for an agent under the two-message mechanism ($\pi_M$, mediator with commitment) weakly dominates that under a mediator who cannot commit ($\pi_C$). More specifically, when $c \leq \frac{3}{4} - \frac{\sqrt{3}}{4} \approx 0.317$ and $c \geq \frac{1}{2}$, the expected payoffs under the two centralized arrangements are exactly the same. However, when $c \in (\frac{3}{4} - \frac{\sqrt{3}}{4}, \frac{1}{2})$, the agents achieve a higher expected payoff under the two-message mechanism than under a mediator without commitment. This is because in this range of $c$, while under a mediator without commitment communication completely collapses, under a mediator with commitment communication is still sustainable, and it is valuable. In addition, the two-message mechanism also dominates the most informative decentralized equilibrium ($\pi_M > \pi_D$) for all parameters, despite that there are only two messages in the mechanism. In particular, when $c < 1/2$ communication under the

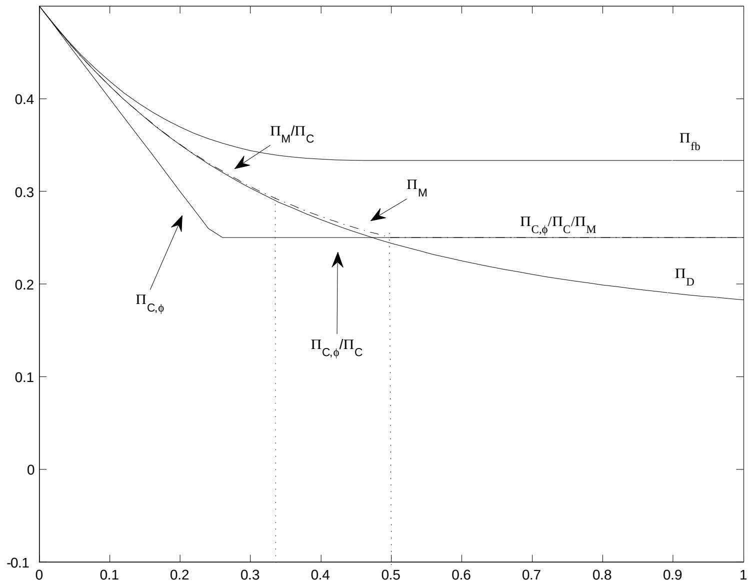



Figure 3: Payoff Comparison among Alternative Communication Protocols

two-message mechanism is actually more informative than that under the most informative decentralized equilibrium, despite the fact that the latter has an infinite number of messages (the comparison is similar to the comparison between two-partition equilibrium under centralization and the most informative decentralized equilibrium when $c$ is small, which we conducted in the previous section). Thus in this range of $c$ both the communication effect and the coordination effect favor the two-message mechanism. When $c > 1/2$, although there is no communication under the two-message mechanism, high miscoordination cost implies that for the mediator with commitment the gains from avoiding the damaging "negative" miscoordination outcome outweight the loss from less informative communication.

To further illustrate the efficiency of the two-message mechanism, we calculate the gains in expected payoff under the two-message mechanism using the payoff from the mediator without precommunication ($\pi_{C,\phi}$) as a benchmark. It turns out that the two-message mechanism can achieve a majority of gain in social surplus when the cost of miscoordination $c$ is small. In particular, we calculate the following ratio $(\pi_M - \pi_{C,\phi})/(\pi_{fb} - \pi_{C,\phi})$. This ratio is 97% when $c = 1/10$, it is about 71% when $c = 1/5$, and 45% when $c = 1/3$. However, this ratio becomes 0 when $c \geq 1/2$. This is simply because as $c$ increases, sustaining informative communication becomes more costly as it involves with a larger, socially inefficient miscoordination cost.

# VII Conclusion

In this paper, we pose the research question of whether pre-communication helps agents to coordinate in a coordination game with private information. We find that pre-communication does improve the frequency of coordination among the agents and thus increase their expected payoffs, but perfect coordination can never be achieved. The informativeness of cheap talk is related to the ratio of agents' realized type to the cost of miscoordination. In particular, communication is more informative if an agent's realized type is lower (weaker incentive to stick to his favored action) or the cost of miscoordination is higher. However, although higher cost of miscoordination leads to more informative communication, the expected payoff for each agent decreases with the cost of miscoordination. We also find that, when the cost of miscoordination is low, equilibrium with a small number of messages already achieves a high percentage of efficiency gain resulting from informative communication.

We further study how an impartial mediator would structure the communication and affect coordination outcome. When the mediator is not able to commit beforehand (centralization), at most two messages can be sustained in any equilibrium. In particular, when the miscoordination cost is high, centralized communication completely collapses and thus it is less informative than decentralized communication, which stands in contrast to the result in ADM. Moreover, the relative performance between centralization and decentralization in our model is also different from that of ADM. When the mediator is able to commit to a decision rule beforehand, in the optimal two-message mechanism we show that the agents are punished only when both of them send the high message and the cost of miscoordination is sufficiently small. Consequently, informative communication is restored for some range of the miscoordination cost when the mediator is able to maintain creditable punishment.

In the baseline model we have only considered simultaneous communication. How sequential communication and delegation will affect equilibrium and agents' payoff (similar questions are considered in Li et al. (2015))? The answer is that under either sequential communication or simple delegation, no informative communication can be sustained. To see this, first consider a simple delegation scheme in which agent 1 has the decision rights of picking actions for both agents. Given his payoff function, unless his type is 0, agent 1 will always pick his most-favored outcome AA, no matter what agent 2 reports about $\theta_2$. That is, agent 2's information transmission does not affect the final outcome, which is equivalent to no meaningful information transmission. Similar results occur in the context of sequential talk. Suppose agent 2 talks first and agent 1 second. Given agent 1's payoff function, no matter what agent 2's message is, agent 1 will always claim his type is higher than agent 2's, in order to have his favored outcome AA implemented. Thus agent 1 babbles. Given this, agent 2 who send message first, also has an incentive to always claim to be the highest possible type, in order to increase the probability that his most-favored outcome BB is chosen. Therefore, in equilibrium both agents babble. The main reason for communication being totally uninformative is that, under either sequential talk or delegation, two agents become asymmetric. Given that the two agents' interests are always orthogonal ex post, each agent has an incentive to exaggerate; but the asymmetric position of the two agents essentially get rid of the possibility of punishment. On the other hand, under simultaneous communication where two agents' positions are symmetric, the punishment of miscoordination is possible, which disciplines agents' incentives to exaggerate.

There are several directions for future research. First, suppose the mediator adopts sequential communication: first communicates with one agent publicly and then communicates with the other. Can sequential communication with a mediator improve the efficiency relative to simultaneous communication with a mediator? Second, will there be more information revealed if the agents are engaged in multi-rounds of simultaneous communication? In particular, imagine the two agents repeatedly send out binary information "high" or "low," with high indicating the end of the communication stage and the agents choosing their optimal actions, and "low" meaning we will communicate again with new beliefs. Finally, as we mentioned before, the mechanism design approach we take in this paper differs a lot from the standard approach in the previous literature. A promising research avenue would be to study whether there are other simple mechanisms that can achieve higher efficiency than the two-partition mechanism considered in our paper.

# References


[1] Alonso, Ricardo, Wouter Dessein and Niko Matoschek 2008, 'When does Coordination Require Centralization,' *American Economic Review*, 98(1), pp. 145-79.

[2] Baliga, Sandeep, and Stephen Morris 2002, 'Co-ordination, Spillovers, Cheap Talk,' *Journal of Economic Theory*, 105(2), pp. 450-68.

[3] Banks, Jeffrey S., and Randall L. Calvert 1992, 'A Battle-of-the-Sexes Game with Incomplete Information,' *Games and Economic Behavior*, 4, pp. 347-72.

[4] Ben-Porath,Elchanan 2003, 'Cheap Talk in Games with Incomplete Information' *Journal of Economic Theory*, 108, pp. 45–71

[5] Bergemann Dirk and Stephen Morris 2011, 'Correlated Equilibrium in Games with Incomplete Information,' *working paper.*

[6] Cooper Russell, Douglas V. DeJong, Robert Forsythe and Thomas W. Ross 1989, 'Communication in the Battle of the Sexes Game: Some Experimental Results,' *The RAND Journal of Economics*, 20(4), pp. 568-87

[7] Cooper Russell, Douglas V. DeJong, Robert Forsythe and Thomas W. Ross 1989, 'Communication in Coordination Games,' *The Quarterly Journal of Economics*, 107(2), pp. 739-71

[8] Crawford, Vincent, and Josel Sobel 1982, 'Strategic Information Transmission,' *Econometrica,* 50, pp. 143-51

[9] Dessein Wouter 2002, 'Authority and Communication in Organizations,' *Review of Economic Studies*, 69(4), pp. 811-38.

[10] Farrell, Joseph 1987, 'Cheap Talk, Coordination, and Entry,' *The RAND Journal of Economics*, 18(1), pp. 34-39.

[11] Farrell, Joseph 1996, and Matthew Rabin, 'Cheap Talk,' *The Journal of Economic Perspectives,* 10(3), pp. 103-18.

[12] Farrell, Joseph, and Robert Gibbons 1989, 'Cheap Talk Can Matter in Bargaining,' *Journal of Economic Theory*, 48(1), pp. 221-37.

[13] Forges, Francoise 1990, 'Universal Mechanisms,' *Econometrica*, 58, 1341–64.

[14] Gerardi, Dino 2004, 'Unmediated Communication in Games with Complete and Incomplete Information,'*Journal of Economic Theory*, 114, pp. 104–131.

[15] Gibbons, Robert 1992, *Game Theory for Applied Economists,* Princeton University Press.

[16] Goltsman, Maria, Johannes Horner, Gregory Pavlov, and Francesco Squintani, 'Mediation, Arbitration and Negotiation,' *Journal of Economic Theory*, 144, pp. 1397-1420.

[17] Krishna, R. Vijay 2007, 'Communication in Games of Incomplete Information: Two Players,' *Journal of Economic Theory*, 132, pp. 584–592.

[18] Li, Zhuozheng, Heikki Rantakari, and Huanxing Yang 2016, 'Competitive Cheap Talk,' *Games and Economic Behavior*, 96, pp. 65-89.

[19] Li, Zhouzheng, 'Project Selection with Sequential Search and Cheap Talk,' *working paper.*

[20] Matthews, Steven, and Andrew Postlewaite 1989, 'Pre-Play Communication in Two-Person Sealed- Bid Auctions,' *Journal of Economic Theory*, 48, pp. 245-52.

[21] Harris Milton, and Artur Raviv, 'Allocation of Decision-Making Authority,' *Review of Finance,* 9(3), pp. 353-83

[22] McGee, Andrew, and Huanxing Yang 2013. 'Cheap Talk with Two Senders and Complementary Information,' *Games and Economic Behavior,* 79, pp. 181-191.

[23] Mulumad, Nahum, and Toshiyuki Shibano 1991, 'Communication in Settings with No Transfers,' *The RAND Journal of Economics*, 22(1), pp. 173-98.

[24] Myerson, Roger B. 1991, *Game Theory*. Harvard University Press, Cambridge, Massachusetts.

[25] Rantakari, Heikki 2008, 'Governing Adaptation,' *Review of Economic Studies*, 75(4), pp. 1257-85.

[26] Rantakari, Heikki 2014, 'A Simple Model of Project Selection with Strategic Communication and Uncertain Motives,' *Journal of Economic Behavior and Organization*, 104, pp. 14-42.

[27] Rantakari, Heikki 2016, 'Soliciting Advice: Active versus Passive Principals,' *Journal of Law, Economics, and Organization*, forthcoming.

[28] Vida Peter and Francoise Forges 2013, 'Implementation of Communication Equilibria by Correlated Cheap Talk: The Two-Player Case,' *Theoretical Economics*, 8, pp. 95-123.

# Appendix

**Proof of Lemma 1.**

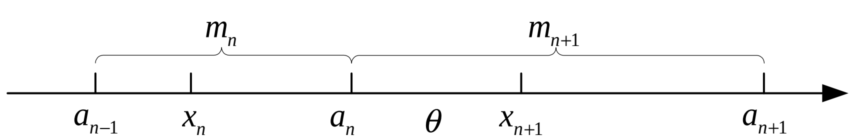
**Proof.** Suppose $a_{N-1} \geq c$, consider the marginal type of agent 1 with $\theta_1 = a_{N-1}$. Since $a_{N-1} \geq c$, the marginal type a has a dominant strategy, which is always playing his favored action $A$ regardless of the messages in the first stage. We can compute the marginal type's expected payoffs by sending message $m_{N-1}$ and by sending message $m_N$ as follows:

$$\pi_1(a_{N-1}, m_N) = (1 - a_N)(a_N - c) + a_N^2,$$
$$\pi_1(a_{N-1}, m_{N-1}) = (1 - x_{N-1})(a_N - c) + x_{N-1}a_{N-1}.$$

It can be readily verified that $\pi_1(a_{N-1}, m_N) > \pi_1(a_{N-1}, m_{N-1})$. Therefore, type $a_{N-1}$ cannot be indifferent between sending message $m_{N-1}$ and sending message $m_N$, and thus equilibria with $a_{N-1} \geq c$ do not exist.

**Proof of Lemma 2**

**Proof.**




Given that the marginal type $a_n$ is indifferent between sending messages $m_n$ and $m_{n+1}$, we only need to establish the following single crossing property

$$\frac{\partial[\pi(m_{n+1}, \theta) - \pi(m_n, \theta)]}{\partial \theta} \geq 0, \text{ for } \theta \in [a_n, a_{n+1}].$$

That is, a higher type agent has a stronger (at least weakly stronger) incentive to send a higher message. By symmetry, we shall prove the inequality holds for agent 1 without any loss of generality.

Consider a type $\theta$ of agent 1 with $\theta \in [a_n, a_{n+1}]$. If $\theta \in [a_n, x_{n+1}]$, when he sends message $m_{n+1}$, his expected payoff is

$$\pi(m_{n+1}, \theta) = a_n\theta - (a_{n+1} - a_n)\left(\frac{x_{n+1} - a_n}{a_{n+1} - a_n}c\right).$$

With probability $a_n$, agent 2 sends a message lower than $m_{n+1}$ and both agents coordinate on action $A$ so that agent 1 gets $\theta$. With probability $a_{n+1} - a_n$, both agents send message $m_{n+1}$ and agent 1 gets $-c$ with probability $\frac{x_{n+1}-a_n}{a_{n+1}-a_n}$.

If $\theta \in [x_{n+1}, a_{n+1}]$ and agent 1 sends message $m_{n+1}$, his expected payoff is

$$\begin{aligned}\pi(m_{n+1}, \theta) &= a_n\theta + (x_{n+1} - a_n)\theta + (a_{n+1} - x_{n+1})(\theta - c) \\ &= a_{n+1}\theta - (a_{n+1} - x_{n+1})\, c.\end{aligned}$$

With probability $a_n$, agent 2 sends a message lower than $m_{n+1}$ and both agents coordinate on action $A$ so that agent 1 gets $\theta$. With probability $a_{n+1} - a_n$, both agents send message $m_{n+1}$. When it happens, agent 1 gets $\theta$ with probability $\frac{x_{n+1}-a_n}{a_{n+1}-a_n}$ and $\theta$ $-c$ with probability $\frac{a_{n+1}-x_{n+1}}{a_{n+1}-a_n}$.

On the other hand, if $\theta \in [a_n, a_{n+1}]$ and agent 1 sends message $m_n$, his expected payoff is

$$\begin{aligned}\pi(m_n, \theta) &= a_{n-1}\theta + (x_n - a_{n-1})\theta + (a_n - x_n)(\theta - c) \\ &= a_n\theta - (a_n - x_n)\, c\end{aligned}$$

With probability $a_{n-1}$, agent 2 sends a message lower than $m_n$ and both agents coordinate on action $A$ so that agent 1 gets $\theta$. With probability $a_n - a_{n-1}$, both agents send message $m_n$. When it happens, agent 1 gets $\theta$ with probability $\frac{x_n-a_{n-1}}{a_n-a_{n-1}}$ and $\theta - c$ with probability $\frac{a_n-x_n}{a_n-a_{n-1}}$.

Combining the above results, we have,

$$\begin{aligned}&\frac{\partial[\pi(m_{n+1}, \theta) - \pi(m_n, \theta)]}{\partial\theta} = 0 \text{ for } \theta \in [a_n, x_{n+1}], \\ (14) \qquad &\frac{\partial[\pi(m_{n+1}, \theta) - \pi(m_n, \theta)]}{\partial\theta} = a_{n+1} - a_n > 0 \text{ for } \theta \in [x_{n+1}, a_{n+1}].\end{aligned}$$

Thus, overall, we have shown that $\frac{\partial[\pi(m_{n+1},\theta)-\pi(m_n,\theta)]}{\partial\theta} \geq 0$.

In addition, similarly we can show

$$(15) \qquad \frac{\partial[\pi(m_{n+1}, \theta) - \pi(m_n, \theta)]}{\partial\theta} = 0 \text{ for } \theta \in [x_n, a_n].$$

Equation (14) and Equation (15) together imply that all types within $[x_n, x_{n+1}]$ are indifferent between sending messages $m_n$ and $m_{n+1}$.

**Proof of Lemma 3.**

**Proof.** 1) (Existence) Note that the existence of equilibrium when $N = 2$ has been proved in the main text. Here we show that, given $c$ and $N \geq 2$, the difference equation system has a solution.

To that end, we first express all the message cutoffs $\{a_n\}$ and the action cutoffs $\{x_n\}$ as functions of $a_1$ (and only $a_1$ and $c$). Specifically, given $a_1$, by (2) we can compute

$$x_1 = \frac{a_1}{a_1 + 2c} c.$$

By equation (4), we have

$$x_2 = x_1 + 2(a_1 - x_1) = a_1 \frac{2a_1 + 3c}{a_1 + 2c}.$$

By equation (2), we can solve for $a_2$ as

$$a_2 = \frac{a_1 x_2 - 2c x_2 + a_1 c}{x_2 - c} = \frac{a_1^3 - 2c^2 a_1}{-c^2 + c a_1 + a_1^2} := f_2(a_1).$$

From the above expression, we can see that $f_2(\cdot)$ is continuous in $a_1$. Similarly, we can use equations (2) and (4) recursively to express $x_n$ and $a_n$ as a sole function of $a_1$ and $c$. Applying to $a_N$, we denote $a_N = f_N(a_1)$. By the recursive structure, $f_N(\cdot)$ is continuous in $a_1$.

Now, define

$$G(a_1) \equiv f_N(a_1) - 1.$$

Note that $a_1$ is a solution to the difference equation system if $G(a_1) = 0$. Moreover, $G(\cdot)$ is continuous in $a_1$, since $f_N(\cdot)$ is.

Suppose $a_1 = 0$. Then it is immediate that $x_1 = 0$ and $a_2 = 0$. By the recursive structure, $a_n = 0$ for any $n \leq N$. Therefore, we have

$$G(0) = f_N(0) - 1 = -1 < 0.$$

Now suppose $a_1 = 1$. By the recursive structure, it can be easily verified that $a_n > 1$ for any $2 \leq n \leq N$. Thus, we have

$$G(1) = f_N(1) - 1 > 0.$$

Given that $G(0) < 0$ and $G(1) > 0$, the continuity of $G(\cdot)$ means that there is a $a_1 \in (0,1)$ such that $G(a_1) = 0$. That is, the difference equation system has a solution.

2) (Uniqueness) To prove the uniqueness, fix $N \geq 2$. Suppose there are two different solutions to the difference equation system: $(a, x)$ and $(a', x')$. WLOG, suppose $a'_{N-1} > a_{N-1}$. By equation (2), $a'_{N-1} > a_{N-1}$ implies that $x'_N > x_N$. Therefore, $\Delta y'_N < \Delta y_N$. By (7), $x'_N > x_N$ implies that $\frac{\Delta y'_N}{\Delta x'_N} > \frac{\Delta y_N}{\Delta x_N}$. Since $\Delta y'_N < \Delta y_N$, it further implies that $\Delta x'_N < \Delta x_N$. Now by (8), it means that $\Delta y'_{N-1} < \Delta y_{N-1}$. Moreover, from the facts that $a'_{N-1} > a_{N-1}$ and $\Delta y'_{N-1} < \Delta y_{N-1}$, we get $x'_{N-1} > x_{N-1}$.

Now suppose $a'_n > a_n$ and $\Delta y'_n < \Delta y_n$, and thus $x'_n > x_n$, we want to show $a'_{n-1} > a_{n-1}$ and $\Delta y'_{n-1} < \Delta y_{n-1}$, and thus $x'_{n-1} > x_{n-1}$. By (7), $x'_n > x_n$ implies that $\frac{\Delta y'_n}{\Delta x'_n} > \frac{\Delta y_n}{\Delta x_n}$. Since $\Delta y'_n < \Delta y_n$, it further implies that $\Delta x'_n < \Delta x_n$. Now by (8), it means that $\Delta y'_{n-1} < \Delta y_{n-1}$. By the facts that $x'_n > x_n$ and $\Delta x'_n < \Delta x_n$, we get $a'_{n-1} > a_{n-1}$. Finally, the facts that $\Delta y'_n < \Delta y_n$ and $a'_{n-1} > a_{n-1}$ imply that $x'_{n-1} > x_{n-1}$.

Now by induction, we have $\Delta y'_1 < \Delta y_1$, $a'_1 > a_1$, and $x'_1 > x_1$. By (7), $x'_1 > x_1$ implies that $\frac{\Delta y'_1}{\Delta x'_1} > \frac{\Delta y_1}{\Delta x_1}$. Since $\Delta y'_1 < \Delta y_1$, it implies that $\Delta x'_1 < \Delta x_1$, which is equivalent to $x'_1 < x_1$. A contradiction.

**Proof of Lemma 4.**

**Proof.** Part (i). Fix $N \geq 1$. First, we show that $a'_N > a_{N-1}$. Suppose to the contrary, $a'_N \leq a_{N-1}$. Then by the same logic as in the proof for Lemma 3 (the uniqueness), we have $\Delta y_1 \leq \Delta y'_2$, $a_1 \geq a'_2$, and $x_1 \geq x'_2$. By (7), $x_1 \geq x'_2$ implies that $\frac{\Delta y'_2}{\Delta x'_2} \leq \frac{\Delta y_1}{\Delta x_1}$. Since $\Delta y_1 \leq \Delta y'_2$, it implies that $\Delta x_1 \leq \Delta x'_2$, which further implies that $x_1 < x'_2$. A contradiction. Therefore, $a'_N > a_{N-1}$.

By a similar logic, we can show that $a_{N-1} > a'_{N-1}$. Following the induction as in the proof for Lemma 3 (the uniqueness), we can show that $...a'_n < a_n < a'_{n+1} < a_{n+1}....$

Part (ii). The statement directly follows Part (i).

**Proof of Lemma 6**.

**Proof.** Part (i). To see $p^l_{BA} = 0$ in the optimal mechanism, by equation (11), we have

$$\frac{\partial a}{\partial p^l_{BA}} = \frac{-\left(\frac{a^2}{2} + ca\right)}{\left(p^h_{BA} + p^l_{BA}\right)(a+c) + \frac{1+p^h_{AB}-p^h_{BA}}{2} + \left(p^h_{AB} + p^l_{BA}\right)(c-a)} < 0,$$

$$\frac{\partial a}{\partial p^l_{AB}} = \frac{\left(\frac{a}{2} - c\right)a}{\left(p^h_{BA} + p^l_{BA}\right)(a+c) + \frac{1+p^h_{AB}-p^h_{BA}}{2} + \left(p^h_{AB} + p^l_{BA}\right)(c-a)}.$$

By the above equations,

$$\frac{\partial a}{\partial p^l_{AB}} = \frac{c - \frac{a}{2}}{c + \frac{a}{2}} \frac{\partial a}{\partial p^l_{BA}}.$$

Define the objective function as $F\left(p^l_{AB}, p^l_{BA}\right)$. Suppose $p^l_{BA} > 0$. We construct

$$\widetilde{p}^l_{BA} = p^l_{BA} - \Delta,$$
$$\widetilde{p}^l_{AB} = p^l_{AB} + \frac{c - \frac{a}{2}}{c + \frac{a}{2}}\Delta,$$

where $\Delta$ is a sufficiently small and positive number. By construction, $a\left(\widetilde{p}^l_{AB}, \widetilde{p}^l_{BA}\right) = a\left(p^l_{AB}, p^l_{BA}\right)$. We thus have

$$F\left(\widetilde{p}^l_{AB}, \widetilde{p}^l_{BA}\right) - F\left(p^l_{AB}, p^l_{BA}\right) \propto \frac{3ca^3}{2\left(c + \frac{a}{2}\right)},$$

which is positive. Therefore, in the optimal mechanism $p^l_{BA}$ cannot be positive.

Similarly, to see $p^h_{BA} = 0$ in the optimal mechanism, by equation (11), we have

$$\frac{\partial a}{\partial p^h_{AB}} = \frac{c - \frac{a}{2}}{c + \frac{a}{2}} \frac{\partial a}{\partial p^h_{BA}}.$$

Define the objective function as $F\left(p^h_{AB}, p^h_{BA}\right)$. Suppose $p^h_{BA} > 0$. We construct

$$\widetilde{p}^h_{BA} = p^h_{BA} - \Delta,$$
$$\widetilde{p}^h_{AB} = p^h_{AB} + \frac{c - \frac{a}{2}}{c + \frac{a}{2}}\Delta,$$

where $\Delta$ is a sufficiently small and positive number. By construction, $a\left(\widetilde{p}^h_{AB}, \widetilde{p}^h_{BA}\right) = a\left(p^h_{AB}, p^h_{BA}\right)$. We thus have

$$F\left(\widetilde{p}^h_{AB}, \widetilde{p}^h_{BA}\right) - F\left(p^h_{AB}, p^h_{BA}\right) \propto \frac{1+a}{4}\frac{2c}{c + \frac{a}{2}}\Delta - c\frac{-a}{c + \frac{a}{2}}\Delta \propto \left(\frac{3a+1}{a+2c}(a-1)^2 c\right),$$

which is positive. Therefore, in the optimal mechanism $p^h_{BA}$ cannot be positive.

Part (ii). Suppose to the contrary that $c \leq \frac{a}{2}$, which implies that $a - c \geq \frac{a}{2} > 0$. For the marginal type $a$ agent, the expected payoff of sending the high message and that of sending the low message are

$$\pi_1(a, H) = a^2 + (1-a)\left(p^h_{AB}(a-c) + \left(1 - p^h_{AB}\right)\frac{a}{2}\right),$$
$$\pi_1(a, L) = a\left(p^l_{AB}(a-c) + \left(1 - p^l_{AB}\right)\frac{a}{2}\right),$$

respectively (we have used the results in part (i) that both $p^h_{BA} = 0$ and $p^l_{BA} = 0$). Notice that in the parenthesis the payoffs are linear combinations of $a - c$ and $\frac{a}{2}$. It follows that the lower bound of $\pi_1(a, H)$ can be achieved at $p^h_{AB} = 0$, while the upper bound of $\pi_1(a, L)$ can be achieved at $p^l_{AB} = 1$. Therefore,

$$\begin{aligned}
&\pi_1(a, H) - \pi_1(a, L) \\
&\geq a^2 + (1-a)\left(\frac{a}{2}\right) - a(a-c) \\
&= (1-a)\left(\frac{a}{2}\right) + ac > 0,
\end{aligned}$$

which contradicts the fact that $a$ is the marginal type indifferent between sending the low message and sending the high message.

Part (iii). We now show that $p^l_{AB}$ cannot be positive in equilibrium. To see this, by equation (11), we have

$$\frac{\partial a}{\partial p^h_{BA}} = \frac{(1-a)\left(c + \frac{a}{2}\right)}{\left(\frac{a}{2} - c\right)a} \frac{\partial a}{\partial p^l_{AB}}.$$

Define the objective function as $F\left(p^h_{BA}, p^l_{AB}\right)$. Suppose $p^l_{AB} > 0$. We construct

$$\begin{aligned}
\widetilde{p}^l_{AB} &= p^l_{AB} - \Delta, \\
\widetilde{p}^h_{BA} &= p^h_{BA} + \frac{(1-a)\left(c + \frac{a}{2}\right)}{\left(\frac{a}{2} - c\right)a}\Delta,
\end{aligned}$$

where $\Delta$ is a sufficiently small and positive number. By construction, $a\left(\widetilde{p}^h_{AB}, \widetilde{p}^l_{AB}\right) = a\left(p^l_{AB}, p^l_{BA}\right)$. We thus have

$$\begin{aligned}
&F\left(\widetilde{p}^h_{BA}, \widetilde{p}^l_{AB}\right) - F\left(p^h_{AB}, p^l_{AB}\right) \\
&\propto a^2\left(c - \frac{a}{4}\right) + \left(\frac{(1-a)\left(c + \frac{a}{2}\right)}{\left(c - \frac{a}{2}\right)a}\right)(1-a)^2\left(\frac{(1+a)}{4} + c\right) > 0,
\end{aligned}$$

since $c > \frac{a}{2}$. Therefore, $p^l_{AB}$ cannot be positive in the optimal mechanism.

**Proof of Lemma 7.**

**Proof.** Now define the LHS of (13) as $G(a)$. It can be verified that $G'(a) > 0$, $G(0) < 0$, and $G(c) > 0$. Therefore, $a \in [0, c)$ for any $p^h_{AB}$. By previous results, $\frac{\partial a}{\partial p^h_{AB}} > 0$, or $a$ is increasing in $p^h_{AB}$.

By (13), choosing $p^h_{AB}$ is equivalent to choosing $a$. From (13), we can solve for $p^h_{AB}$ as a function of $a$. Substituting $p^h_{AB}$ in the objective function and simplifying, we get a new objective function $F(a)$, with the restriction that $a \in [0, \overline{a})$, where $\overline{a}$ is the solution to (13) when $p^h_{AB} = 1$. Specifically,

$$F(a) = \frac{1}{4} + \frac{1-2c}{4}\frac{a(1-a)}{2c-a}. \tag{16}$$

Observing (16), we notice that $\frac{a(1-a)}{2c-a} \geq 0$ since $2c > a$. It immediately follows that when $c > 1/2$, the optimal $a = 0$, which means that the optimal $p^h_{AB} = 0$. When $c = 1/2$, then any $a$ is optimal, or any $p^h_{AB} \in [0, 1]$ is optimal. Now consider the case that $c < 1/2$. Since $a < c$, it means that $a < 1/2$. For the term $\frac{a(1-a)}{2c-a}$, the numerator $a(1-a)$ is increasing in $a$ since $a < 1/2$; the denominator is decreasing in $a$. Therefore, $\frac{a(1-a)}{2c-a}$ is increasing in $a$ for $a \in [0, \overline{a})$. As a result, the optimal $a$ is the corner solution $a = \overline{a}$, which implies that the optimal $p^h_{AB} = 1$.

**Proof of Lemma 5**

**Proof.** We only need to rule out the existence of three-partition equilibria. First consider the case that $\overline{m}_1 \geq 2c$. Then by the monotonicity of $\overline{m}_n$ in $n$, $\overline{m}_n \geq 2c$ for all $n = 1, 2, 3$. This means that the mediator will always choose the outcome $AB$ for sure regardless of the messages. As a result, all the messages are outcome equivalent and the equilibrium is a babbling equilibrium. Second, consider the case that $\overline{m}_3 \leq 2c$. By a similar logic, in this case the outcome $AB$ is never chosen and either $AA$ or $BB$ is chosen. As a result, in this case both agents will always send the highest message $m_3$, or the equilibrium is a babbling equilibrium.

Now consider the case that $\overline{m}_1 < 2c$ and $\overline{m}_3 > 2c$. Suppose $\overline{m}_2 < 2c$. Then outcome $AB$ is chosen if and only if both agents send message $m_3$. In this case all types of agent $i$

with $\theta_i \leq a_2$ will have an incentive to send message $m_2$, in order to increase the chance that his favored coordinated outcome is chosen. That is, the two low messages are essentially combined into a single message. Next suppose $\overline{m}_2 > 2c$. Then outcome $AB$ is chosen if the lower message is $m_2$. In this case the two high messages are outcome equivalent and are essentially combined into a single message.

Now the only case left is $\overline{m}_1 < 2c$, $\overline{m}_3 > 2c$, and $\overline{m}_2 = 2c$. This means that $a_2 > 2c$ and $a_1 < 2c$. To make $m_2$ different from $m_1$ and $m_3$, let $p \in (0, 1)$ be the probability that outcome $AB$ is chosen when both agents send $m_2$ (AA and BB are chosen with the same probability $(1 - p)/2$), and $q \in [0, 1]$ be probability that outcome $AB$ is chosen when one agent sends $m_3$ and the other sends $m_2$ (with probability $1 - q$ the favored coordinated outcome of the agent who sends $m_3$ is chosen). Now consider the marginal type $a_2$ of agent 1. If he sends message $m_2$, his expected payoff is

$$\pi_1(a_2, m_2) = a_1 a_2 + (a_2 - a_1)[p(a_2 - c) + \frac{1-p}{2} a_2] + (1 - a_2)q(a_2 - c).$$

If he sends message $m_3$, his expected payoff is

$$\pi_1(a_2, m_3) = a_1 a_2 + (a_2 - a_1)[q(a_2 - c) + (1 - q)a_2] + (1 - a_2)(a_2 - c).$$

Taking the difference, we have

$$\begin{aligned} &\pi_1(a_2, m_3) - \pi_1(a_2, m_2) \\ &= (a_2 - a_1)[-(q - p)c + \frac{1-p}{2} a_2] + (1 - a_2)(1 - q)(a_2 - c). \end{aligned}$$

To show that $\pi_1(a_2, m_3) - \pi_1(a_2, m_2) > 0$, it is sufficient to show that $-(q-p)c + \frac{1-p}{2} a_2 > 0$. Since $a_2 > 2c$, we have

$$-(q - p)c + \frac{1-p}{2} a_2 > -(q - p)c + (1 - p)c = (1 - q)c \geq 0.$$

Therefore, $\pi_1(a_2, m_3) - \pi_1(a_2, m_2) > 0$. But this contradicts to the fact that type $a_2$ of agent 1 should be indifferent between sending message $m_2$ and sending message $m_3$.