

# Cheap Talk with Two Senders and Complementary Information

Andrew McGee and Huanxing Yang\*  
Department of Economics, Ohio State University

November 2009

## Abstract

This paper studies a cheap talk model with two senders having partial and non-overlapping private information communicating with an uninformed receiver. The two senders' private information is complementary in the sense that the marginal impact of one sender's private information on the receiver's ideal action depends on the other sender's private information. When the two senders communicate simultaneously, their information transmission exhibits strategic complementarity: more information transmitted by one sender leads to more information transmitted by the other sender. When the two senders have like biases, it is always optimal for the receiver to delegate the decision rights to the sender with a smaller bias. When two senders have opposing biases, simultaneous communication is more likely to dominate delegation.

*JEL Classification:* D23, D72, D83, L23

*Keywords:* Cheap talk; Multiple senders; Delegation.

## 1 Introduction

Decision makers often seek advice from multiple experts who have expertise but whose interests may not be perfectly aligned with the decision maker's—an observation that motivated a growing literature on cheap talk with two senders (Gilligan and Krehbiel, 1989; Epstein, 1998; Krishna and Morgan, 2001a, KM hereafter). A common feature of these models is that the state of the world is one dimensional, and *both* senders (experts) *perfectly* observe the realized state. For example, a CEO must choose the size of a new factory to produce a new product, the optimal size of which depends on the profitability of the new product. For that purpose, the CEO consults two experts who both observe the profitability of the new product.

While in some situations it is reasonable to assume that both experts perfectly observe the realized state, more typically each expert might only observe *some aspects of but not the whole* of

---

\*We would like to thank Yaron Azrieli, Maria Goltsman, Wojciech Olszewski, Marco Ottaviani, Gregory Pavlov, and the seminar participants at Ohio State, the Econometric Society Summer Meetings (2009, Boston), and the Midwest Economic Theory Meetings (Fall 2009, Penn State) for helpful comments and suggestions.

the realized state. In a world emphasizing specialization, experts usually only have expertise in their own fields. In the factory size example, the two experts may be a marketing manager and a production manager respectively. Due to specialization, the marketing expert may only have knowledge about the demand for the new product, while the production expert may only have knowledge about the cost of production. *Both* pieces of information are essential to determine the profitability of the new product and the optimal size of the new factory.

Similar examples abound. For instance, consider a president who is deciding on a bailout plan for banks, the optimal size of which depends on how deep the banking crisis is and the constraints of the federal budget. The president consults one banking expert and one OMB budget expert, with the banking expert knowing only how serious the banking crisis is and the budget expert knowing only the availability of bailout funds. Alternatively, consider a military leader who in deciding how many troops to send to a war consults an intelligence expert and a field commander. The optimal number of troops depends on both the strength of the enemy and the strength of his own army. While the intelligence expert only knows the strength of the enemy, the field commander only knows the strength of his own forces.

This paper studies a cheap talk model in which two senders (experts) have *partial and non-overlapping* private information regarding the state of the world. To model partial and non-overlapping private information, we assume that the state of the world has two dimensions,  $\theta_1$  and  $\theta_2$ . Each expert  $i$  perfectly observes the realized state in dimension  $i$  ( $\theta_i$ ), but does not observe the realized state in dimension  $j$  ( $\theta_j$ ). The receiver's ideal decision is a function of the realized states,  $y^*(\theta_1, \theta_2)$ . We will focus on the case in which the marginal impact of information in dimension  $i$  on the ideal decision depends on the realized state in dimension  $j$  (i.e.,  $\partial^2 y^*(\theta_1, \theta_2) / \partial \theta_1 \partial \theta_2 \neq 0$ ). In this case we say that *two states are complementary*. Our leading example,  $y^*(\theta_1, \theta_2) = \theta_1 \theta_2$ , has an intuitive interpretation.<sup>1</sup> Consider the CEO example in which  $\theta_1$  is the demand size and  $\theta_2$  is the efficiency of production. In our leading example, the marginal impact of production efficiency on the optimal size of factory is larger when the market size is larger and vice versa.<sup>2</sup>

As in other cheap talk models, each expert has his own interests, which we model as a bias relative to the receiver's ideal decision. Adopting terminology from KM, we say that two experts have *opposing biases* when one expert wants to pull the decision to the left and the other to the right. Alternatively, two experts can have *like biases* if both want to pull the decision in the

---

<sup>1</sup>If  $\partial^2 y^*(\theta_1, \theta_2) / \partial \theta_1 \partial \theta_2 = 0$ , say  $y^*(\theta_1, \theta_2) = \theta_1 + \theta_2$ , then the cheap talk game with two-senders is qualitatively similar to standard cheap talk game with one sender. See Section 6 for a more detailed discussion.

<sup>2</sup>In the military example, let  $\theta_1$  be the weakness of one's own army and  $\theta_2$  be the strength of the enemy. In our leading example, the weaker one's own army is, the bigger is the marginal impact of the strength of the enemy on the optimal number of troops.

same direction, but possibly to different degrees.

We first study equilibrium information transmission when two experts send messages simultaneously. Equilibria are shown to be partition equilibrium in which each sender indicates only to which interval the realized state that he observes belongs, as in standard cheap talk models (Crawford and Sobel, 1982, CS hereafter). We focus on the most informative equilibrium. Interestingly, the information transmission of the two senders exhibits strategic complementarity: the more information that one sender transmits, the less the incentive the other sender has to distort information in the direction of his bias, and, hence, the more information he will transmit. As a result, a reduction in one sender's bias leads not only to more information transmitted by himself, but also induces the other agent to transmit more information. Intuitively, when sender  $i$  transmits more information, the other sender  $j$  faces greater uncertainty regarding the principal's conditional expectation of the other dimension of the state space ( $\theta_i$ ). Given that the two states are complementary, this magnifies the expected loss if sender  $j$  distorts his information; thus sender  $j$  has less incentive to distort his information.

When the experts communicate simultaneously, the equilibrium information transmission depends only on the absolute value of the senders' biases. Changing the sign of either sender's bias does not affect the information transmitted in equilibrium, implying that whether the two experts have like biases or opposing biases does not matter. Moreover, it is always better for the principal to consult two senders instead of consulting only one. These results are in contrast to those in KM, where they establish that the equilibrium information transmission depends on whether two senders have like biases or opposing biases and that it is better for the principal to only consult one expert in the case of like biases. The differences stem from the fact that in their model the experts' private information perfectly overlaps, while in our model the experts' private information is non-overlapping.

We then study the possibility of delegation in which the receiver delegates his decision rights to one of the senders. The agent to whom the decision rights are delegated first consults the other sender regarding his private information and then makes a decision. We show that it is always better for the receiver to delegate decision rights to the expert with a smaller bias in absolute value. The underlying reason for this result is that the effectiveness of communication between the two senders does not depend on which expert has the decision rights. Therefore, the decision rights should be delegated to the expert with the smaller bias to minimize the loss of control experienced by the receiver.

Finally, we compare delegation to simultaneous communication. Interestingly, whether delegation is optimal for the receiver depends on whether the experts have like biases or opposing

biases. When the experts have like biases and communication is informative under simultaneous communication, delegation dominates simultaneous communication for the receiver. This result still holds when two experts have opposing biases and the absolute value of the smaller bias is small enough. On the other hand, simultaneous communication dominates delegation if the two experts have opposing biases and the absolute value of the smaller bias is big enough. These results imply that we are more likely to observe delegation when the two experts bias toward the same direction relative to the principal. In a political context, delegation is more likely when either both experts are more liberal or both experts are more conservative than the principal.

The rest of the paper is organized as follows. The next subsection discusses related literature. Section 2 lays out the model. Simultaneous communication is studied in Section 3. In Section 4 we consider delegation and compare it to simultaneous communication. Section 5 discusses possible extensions and the robustness of our results. Section 6 concludes. All the longer proofs can be found in the Appendix.

## 1.1 Related Literature

Following the original work of CS on cheap talk, there is a growing literature on cheap talk with multiple senders. Gilligan and Krehbiel (1989) study a model in which two experts with symmetric opposing biases simultaneously communicate by submitting bills to the decision-making legislature. They show that the restrictive “closed rule,” in which amendments to bills are not permitted, is informationally superior to the “open rule” in which bills are freely amendable. Krishna and Morgan (2001b) reexamine the model and derive different results. Epstein (1998) generalizes the model of Gilligan and Krehbiel to the case where two experts have asymmetric opposing biases. KM (2001a) study a more general model in which the two experts, who can have like or opposing biases, communicate sequentially. Gick (2009) considers the model of KM with like biases by adding the twist that the receiver is able to commit to not best responding to the second sender. As observed in the introduction, a common feature of these models is that the state space is one dimensional and both senders perfectly observe the same realized state. In contrast, the two senders have partial and non-overlapping private information in our model.<sup>3</sup>

Austen-Smith (1993) considers a two-sender model with two experts imperfectly informed about the state. In his model each expert receives a noisy (binary) signal about the state, which is also binary. This formulation yields results very different from those of standard cheap

---

<sup>3</sup>For cheap talk models with one sender and multiple receivers, see Goltsman and Pavlov (2009).

talk models.<sup>4</sup> In his model, the two experts' signals are correlated and the combination of both still does not fully reveal the realized state. In our model, the two experts' signals are independent, but collectively they fully reveal the realized state. Li (2007) studies a model in which two experts perfectly observe the realized state, but each expert's bias is his own private information. Again in his model the number of states and signals is finite.

Our paper is also related to Alonso et. al (2008), who study strategic communication between a CEO and two division managers. Each manager has private information regarding the local conditions of his own division, and a decision needs to be made for each division. Furthermore, the decisions of the two divisions need to be coordinated, and each manager has a bias toward maximizing the profit of his own division. They compare two communication modes. In vertical communication (centralization), each manager communicates with the CEO simultaneously, and then the CEO makes decisions for each division. Under horizontal communication (decentralization), the two managers simultaneously communicate with each other and then each manager makes the decision for his own division. They show that even when the need of coordination is large, decentralization can be superior to centralization from the CEO's perspective. Our paper is different from theirs in that in our model there is only one decision to make instead of two. Furthermore, the need to communicate in their model results from the need to coordinate two decisions, while the need to communicate arises in our model because the optimal decision for the receiver depends on the private information of both experts. These differences affect the experts' incentives to communicate.

Battaglini (2002) studies a multidimensional cheap talk model with multiple senders. He concludes—that in contrast to one-dimensional cheap talk models with one sender—generically information can be fully revealed in equilibrium communication, and Ambrus and Takahashi (2008) provide further conditions under which fully revealing equilibria are possible in this setting. Our model differs from multidimensional cheap talk model in two regards. First, in our model each sender only observes the realized state in his own dimension, while in their models each sender observes the realized states in all dimensions. Second, in our model the decision is a one-dimensional variable while in theirs the decision is a two-dimensional vector. Based on these differences, fully revealing equilibria are impossible in our model.

Another work related to ours is Dessein (2002), who compares delegation to a one-sender cheap talk model as in CS.<sup>5</sup> Alonso and Matouschek (2007) endogenize the commitment power of the principal by developing an infinitely repeated delegation game. Our paper differs from

---

<sup>4</sup>For example, full revelation is possible even with a single agent, which is impossible with a richer signal space.

<sup>5</sup>Aghion and Tirole (1997) studies how delegation might boost agents' incentive to acquire information.

these studies in that we compare delegation *with cheap talk* to a cheap talk model with two senders (see Section 4 for more details). Harris and Raviv (2005), McGee (2008) and Chen (2009) study cheap talk models when both the receiver and sender have private information, and the possibility of delegation is considered in Harris and Raviv (2005) and McGee (2008). In McGee’s model, the receiver’s private information and the sender’s private information are complementary, a formulation similar to our model. In all these models, however, there is only one sender.

Since the work of CS, it is well-known that cheap talk models have multiple equilibria. There have been efforts made on equilibrium refinement (Matthews et. al, 1991; Chen et. al, 2008). We will follow a common practice in cheap talk models: whenever there are multiple equilibria, we will focus on the most informative equilibrium because it is Pareto dominant.

## 2 Model

To formalize the examples in the introduction, we provide a stylized model that can be applied to a broad range of institutional settings. Consider a decision maker (DM) who consults two experts  $i = 1, 2$ . The DM takes an action  $y \in R$ , and his utility depends on some underlying states of nature  $\theta_1$  and  $\theta_2$ . Each  $\theta_i$  is uniformly distributed on  $[0, A_i]$  with density  $1/A_i$ , and  $\theta_1$  and  $\theta_2$  are independent from each other. Expert  $i$  observes only the realized value of  $\theta_i$ . The DM does not observe the realization of either  $\theta_1$  or  $\theta_2$ , nor does expert  $i$  observe the realization of  $\theta_j$ ,  $j \neq i$ . This captures the fact that each expert is knowledgeable only in his own field. Note that both experts have private information, yet they are not overlapping in the sense that  $\theta_1$  and  $\theta_2$  are independent.

Expert  $i$  offers advice to the DM by sending message  $m_i \in [0, A_i]$ . In the basic model, we consider the case of simultaneous communication in which the two experts send messages simultaneously. After receiving messages  $m_1$  and  $m_2$ , the DM takes an action  $y(m_1, m_2)$ .

The utility functions are of quadratic loss form. Given realized states  $\theta_1$  and  $\theta_2$ , the ideal action for the DM is  $y^*(\theta_1, \theta_2)$ . Specifically, the utility function for the DM is

$$U^P(y, \theta_1, \theta_2) = -(y - y^*(\theta_1, \theta_2))^2.$$

For the most of the paper, we will use the specific functional form  $y^*(\theta_1, \theta_2) = \theta_1\theta_2$ . The robustness of our results with respect to different formulations will be discussed in Section 5. Note that  $\frac{\partial^2 y^*(\theta_1, \theta_2)}{\partial \theta_1 \partial \theta_2} > 0$ . As mentioned in the introduction, this condition implies that the two states are complementary: the marginal impact of state  $\theta_i$  on the ideal action depends on

the realized state  $\theta_j$ . This relationship leads to strategic interactions between the two experts' information transmission as we show in the next section.

Expert  $i$ 's ideal action is  $y^*(\theta_1, \theta_2) + b_i$ , where  $b_i$  is expert  $i$ 's bias relative to the DM. Bias  $b_i$ , which can be positive or negative, measures the degree to which the DM's and expert  $i$ 's interests are aligned. The utility function for expert  $i$  is

$$U^{A_i}(y, \theta_1, \theta_2, b_i) = -[y - (y^*(\theta_1, \theta_2) + b_i)]^2.$$

The biases are common knowledge. When  $b_1$  and  $b_2$  have the same sign, we say that the experts have like biases; otherwise, we say that they have opposing biases. Both experts and the DM are expected utility maximizers.

### 3 Simultaneous Communication

Under simultaneous communication, a strategy for expert  $i$  specifies a message  $m_i$  for each  $\theta_i$ , which is denoted as the communication rule  $\mu_i(m_i|\theta_i)$ . A strategy for the DM specifies an action  $y$  for each message pair  $(m_1, m_2)$ , which is denoted as decision rule  $y(m_1, m_2)$ . Let the belief function  $g(\theta_1, \theta_2|m_1, m_2)$  be the DM's posterior beliefs on  $\theta_1$  and  $\theta_2$  after hearing messages  $m_1$  and  $m_2$ . Since  $\theta_1$  and  $\theta_2$  are independent and expert  $i$  observes only  $\theta_i$ , the belief function can be decomposed into distinct belief functions  $g_1(\theta_1|m_1)$  and  $g_2(\theta_2|m_2)$ .

Our solution concept is Perfect Bayesian Equilibrium (PBE), which requires:

- (i) Given the DM's decision rule  $y(m_1, m_2)$  and expert  $j$ 's communication rule  $\mu_j(m_j|\theta_j)$ , for each  $i$ , expert  $i$ 's communication rule  $\mu_i(m_i|\theta_i)$  is optimal.
- (ii) The DM's decision rule  $y(m_1, m_2)$  is optimal given beliefs  $g_1(\theta_1|m_1)$  and  $g_2(\theta_2|m_2)$ .
- (iii) The belief functions  $g_i(\theta_i|m_i)$  are derived from the agents' communication rules  $\mu_i(m_i|\theta_i)$  whenever possible.

We first derive the DM's optimal decision rule  $\bar{y}(m_1, m_2)$ . Given  $m_1$  and  $m_2$ ,  $\bar{y}(m_1, m_2)$  maximizes  $-E[(y - \theta_1\theta_2)^2|m_1, m_2]$ . Since  $\theta_1$  and  $\theta_2$  are independent, and  $\theta_1|m_1$  and  $\theta_2|m_2$  are independent, it can be readily seen that

$$\bar{y}(m_1, m_2) = E[\theta_1|m_1]E[\theta_2|m_2]. \tag{1}$$

As in CS and Alonso *et.al* (2008), all PBE are interval equilibria. Specifically, the state space  $[0, A_i]$  is partitioned into intervals and expert  $i$  only reveals to which interval  $\theta_i$  belongs.

**Lemma 1** *All PBE in the communication game must be interval equilibria.*

Having established that all PBE must be interval equilibria, we now characterize them. Let  $N_i$ , which is a positive integer, be the number of partitions for agent  $i$ , and  $(a_{i,0}, a_{i,1}, \dots, a_{i,n}, \dots, a_{i,N_i})$  be the partition points with  $a_{i,0} = 0$  and  $a_{i,N_i} = A_i$ . Define random variable  $\bar{m}_i$  as the posterior of state  $\theta_i$  given message  $m_i$ ; that is,  $E[\theta_i | m_i] \equiv \bar{m}_i$ . Define  $\bar{m}_{i,n}$  as the receiver's posterior of  $\theta_i$  after receiving a message  $m_{i,n} \in (a_{i,n-1}, a_{i,n})$ . It follows that  $\bar{m}_{i,n} = (a_{i,n-1} + a_{i,n})/2$ . In state  $\theta_1 = a_{1,n}$ , agent 1 should be indifferent between sending a message that induces a posterior  $\bar{m}_{1,n}$  and a posterior  $\bar{m}_{1,n+1}$ , that is,  $E_{\theta_2}[U^{A_1} | \bar{m}_{1,n}, a_{1,n}] = E_{\theta_2}[U^{A_1} | \bar{m}_{1,n+1}, a_{1,n}]$ . More explicitly, this indifference condition can be written as

$$E_{\theta_2}[\{\bar{m}_2 \frac{a_{1,n} + a_{1,n-1}}{2} - (\theta_2 a_{1,n} + b_1)\}^2] = E_{\theta_2}[\{\bar{m}_2 \frac{a_{1,n} + a_{1,n+1}}{2} - (\theta_2 a_{1,n} + b_1)\}^2],$$

which can be simplified as

$$E[\bar{m}_2^2](a_{1,n+1} + 2a_{1,n} + a_{1,n-1}) - 4E[\theta_2 \bar{m}_2]a_{1,n} = 4b_1 E(\theta_2). \quad (2)$$

In the appendix we show that  $E[\theta_2 \bar{m}_2] = E[\bar{m}_2^2]$ . The indifference condition (2) can be further simplified as

$$(a_{1,n+1} - a_{1,n}) - (a_{1,n} - a_{1,n-1}) = \frac{E(\theta_2)}{E(\bar{m}_2^2)} 4b_1. \quad (3)$$

Similarly, the cutoff points  $a_{2,n}$  characterizing agent 2's partition equilibrium satisfy the indifference condition:

$$(a_{2,n+1} - a_{2,n}) - (a_{2,n} - a_{2,n-1}) = \frac{E(\theta_1)}{E(\bar{m}_1^2)} 4b_2. \quad (4)$$

Inspecting indifference conditions (3) and (4), we see that there is strategic interaction between the senders' information transmissions in equilibrium as the term  $\frac{E(\theta_j)}{E(\bar{m}_j^2)}$  appears in the condition that determines sender  $i$ 's cutoff points. In the quadratic-uniform case of CS's one-sender model, the indifference condition for cutoff points implies that the difference between the lengths of any two adjacent intervals, the incremental step size, is always  $4b$ . Conditions (3) and (4) show that the strategic interaction between the two senders changes the incremental step sizes: the effective incremental step size is  $\frac{E(\theta_j)}{E(\bar{m}_j^2)} 4b_i$ . Note that

$$E(\bar{m}_j^2) = (E(\bar{m}_j))^2 + \text{var}(\bar{m}_j) = (E(\theta_j))^2 + \text{var}[E(\theta_j | m_j)].$$

A bigger  $E(\bar{m}_j^2)$  (conditional variance of  $\theta_j$  given  $m_j$ ) means more information is transmitted by sender  $j$ . As one agent transmits more information, however, the effective incremental step size for the other agent decreases, which leads to more information being transmitted by the other agent. Therefore, the two agent's information transmission exhibits strategic complementarity.

Let  $\frac{E(\theta_i)}{E(\bar{m}_i^2)} \equiv x_i$ . Given  $a_{i,0} = 0$ , the solutions for difference equations (3) and (4) are:

$$a_{i,n} = a_{i,1}n + 2n(n-1)b_ix_j.$$

In the most informative equilibrium,  $N_i$  is the largest integer such that

$$2|b_i|x_jN_i(N_i-1) < A_i. \quad (5)$$

Given  $N_i$  and  $x_j$  and using the fact  $a_{1,N_i} = A_i$  we can solve for  $a_{i,1}$  and derive the following expression for expert  $i$ 's partitioning rule:

$$a_{i,n} = A_i \frac{n}{N_i} + 2b_jx_in(n-N_i). \quad (6)$$

**Lemma 2** (i)  $E(\bar{m}_i^2) = \frac{A_i^2}{3} - \frac{A_i^2}{12N_i^2} - \frac{b_i^2x_j^2(N_i^2-1)}{3}$ ; (ii)  $E(\bar{m}_i^2)$  is strictly increasing in  $N_i$ , strictly decreasing in  $x_j$  and  $b_i$ , and strictly increasing in  $E(\bar{m}_j^2)$ ; (iii)  $\frac{A_i^2}{4} \leq E(\bar{m}_i^2) \leq \frac{A_i^2}{3}$ .

Lemma 2 formally shows that the two agents' equilibrium information transmission exhibits strategic complementarity:  $E(\bar{m}_i^2)$  is increasing in  $E(\bar{m}_j^2)$  and vice versa. This result comes from the fact that two states are complementary: the ideal decision for DM given realized states is  $\theta_1\theta_2$ . Intuitively, agent  $i$ 's incentive to distort information in the direction of  $b_i$  is mitigated by his interest in matching the action chosen by the DM to the actual state. When agent 2 transmits more information,  $\bar{m}_2 (= E[\theta_2|m_2])$ , has a larger variance. For agent 1, distorting his information in the direction of  $b_1$  by the same amount now leads to a bigger expected loss in terms of matching the action chosen by the DM to the actual state, since he now is less certain about the DM's beliefs regarding  $\theta_2$ . This reduces agent 1's incentive to distort his information (his effective bias), leading him to transmit more information as well. Note that when the two states are additive ( $\frac{\partial^2 y^*(\theta_1, \theta_2)}{\partial \theta_1 \partial \theta_2} = 0$ ), this strategic interaction between two agents is absent.<sup>6</sup>

For PBE of the overall communication game, a babbling equilibrium always exists with  $N_1 = N_2 = 1$  and the DM ignoring the messages. Thus we do not need to worry about the

---

<sup>6</sup>Suppose  $y^*(\theta_1, \theta_2) = \theta_1 + \theta_2$ . Then the indifference condition for agent  $i$ 's partition points  $a_{i,n}$  is given by

$$(a_{i,n+1} - a_{i,n}) - (a_{i,n} - a_{i,n-1}) = 4b_i.$$

It is clear that each agent's information transmission is independent from the other's, and the two-sender model collapses to a one sender model.

existence of PBE. Straightforward calculation shows that the ex ante equilibrium payoffs for the DM,  $U_{ST}^P$ , and for agent  $i$ ,  $U_{ST}^{A_i}$  (where the subscripts denote simultaneous talk), are given by:

$$\begin{aligned} U_{ST}^P &= -E[(\bar{m}_1\bar{m}_2 - \theta_1\theta_2)^2] = -E(\theta_1^2)E(\theta_2^2) + E(\bar{m}_1^2)E(\bar{m}_2^2). \\ U_{ST}^{A_i} &= -E(\theta_1^2)E(\theta_2^2) + E(\bar{m}_1^2)E(\bar{m}_2^2) - b_i^2 \end{aligned} \quad (7)$$

By inspection, the most informative equilibrium on which we focus is also Pareto dominant.

**Proposition 1** *The most informative equilibrium is characterized by a pair of numbers of partition elements  $(N_1^*, N_2^*)$ . They satisfy*

$$E(\bar{m}_1^2) = \frac{E(\theta_1)}{x_1} \Leftrightarrow \frac{A_1^2}{3} - \frac{A_1^2}{12N_1^2} - \frac{b_1^2x_2^2(N_1^2 - 1)}{3} = \frac{A_1}{2x_1}, \quad (8)$$

$$E(\bar{m}_2^2) = \frac{E(\theta_2)}{x_2} \Leftrightarrow \frac{A_2^2}{3} - \frac{A_2^2}{12N_2^2} - \frac{b_2^2x_1^2(N_2^2 - 1)}{3} = \frac{A_2}{2x_2}; \quad (9)$$

and are the largest  $N_1$  and  $N_2$  such that

$$2|b_1|x_2N_1(N_1 - 1) < A_1; \quad 2|b_2|x_1N_2(N_2 - 1) < A_2. \quad (10)$$

Moreover,

$$\left\langle -\frac{1}{2} + \frac{1}{2}\left(1 + \frac{A_1A_2}{|b_i|}\right)^{1/2} \right\rangle \leq N_i^* \leq \left\langle -\frac{1}{2} + \frac{1}{2}\left(1 + \frac{4A_1A_2}{3|b_i|}\right)^{1/2} \right\rangle \quad (11)$$

**Proof.** By (7),  $E(\bar{m}_1^2)$  and  $E(\bar{m}_2^2)$  are maximized in the most informative equilibrium. By part (ii) of Lemma 2,  $E(\bar{m}_i^2)$  is increasing in  $N_i$ . Moreover,  $E(\bar{m}_i^2)$  is decreasing in  $x_j$  and  $x_j$  is decreasing in  $E(\bar{m}_j^2)$ . Thus  $E(\bar{m}_i^2)$  is increasing in  $N_j$  as well. Therefore,  $N_1$  and  $N_2$  should be maximized in the most informative equilibrium. The total length of the partition for each agent  $2|b_i|x_jN_i(N_i - 1)$  should be less than the length of the support of  $\theta_i$ ,  $A_i$ , which gives rise to condition (10). The bounds of  $N_i^*$  come from part (iii) of Lemma 2. The bounds of  $E(\bar{m}_i^2)$  imply that  $x_i \in [\frac{3}{2A_i}, \frac{2}{A_i}]$ . Then (11) follows. ■

**Corollary 1** *In the most informative equilibrium, a decrease in  $b_i$  results in not only an increase in  $E(\bar{m}_i^2)$  but also an increase in  $E(\bar{m}_j^2)$ .*

**Proof.** By Proposition 1,  $N_1^*$  and  $N_2^*$  are nonincreasing in  $b_i$ . The rest follows Lemma 2. ■

Corollary 1 can potentially be empirically tested: when one agent is replaced by a new agent whose interests are more aligned with those of the DM, more information will be transmitted by the other agent. This is illustrated in the following example.

**Example 1** Suppose  $A_1 = 10$  and  $A_2 = 4$ . Agent 1 has bias  $b_1 = 2$ , and agent 2 has bias  $b_2 = 1.15$ . Under simultaneous communication,  $N_1^* = 2$ ,  $N_2^* = 2$ ,  $E(\bar{m}_1^2) = 30.6009$ , and  $E(\bar{m}_2^2) = 4.9646$ . When  $b_2$  decreases to  $\frac{3}{4}$ ,  $N_1^* = 2$ ,  $N_2^* = 3$ ,  $E(\bar{m}_1^2) = 30.6456$ , and  $E(\bar{m}_2^2) = 5.1453$ . Note that  $E(\bar{m}_1^2)$  increases as  $b_2$  decreases.

In KM, equilibrium information transmission depends on whether the two agents have opposing or like biases. Specifically, if the two agents have like biases, then it is better for the DM to consult only one agent. When two agents have opposing biases, more information will generally be transmitted, and it is better for the DM to consult both agents. In our model, the following corollary shows that equilibrium information transmission does not depend on whether the two agents have opposing or like biases, and it is always better for the DM to consult two agents instead of one.

**Corollary 2** (i) It is always better for the DM to consult two agents instead of one. (ii) Fix all the other parameter values but change  $b_i$  to  $-b_i$ . Then  $(N_1^*, N_2^*)$ ,  $E(\bar{m}_i^2)$  and  $E(\bar{m}_j^2)$  remain the same, and each player's ex ante equilibrium payoff is unchanged.

**Proof.** Suppose the DM only consults one agent, say agent  $i$ . Then no information is transmitted by agent  $j$ .  $E(\bar{m}_j^2)$  reaches its lower bound  $\frac{A_j^2}{4}$ , and  $x_j$  reaches its upper bound  $\frac{2}{A_j}$ . It follows that the effective incremental step size for agent  $i$  is (weakly) bigger than when two agents are consulted. By Lemma 2, this implies that both  $E(\bar{m}_i^2)$  and  $E(\bar{m}_1^2)E(\bar{m}_2^2)$  are smaller when only agent  $i$  is consulted. This proves part (i).

Observing (8) and (9), we see that, fixing  $N_i$ ,  $E(\bar{m}_i^2)$  remains the same when  $b_i$  is replaced by  $-b_i$ . This means that  $E(\bar{m}_j^2)$  remains the same as well. Also note that condition (10) remains the same when the sign of  $b_i$  changes. Therefore,  $N_1^*$  and  $N_2^*$  will not change either. This proves part (ii). ■

The differences in the results stem from the fact that in KM the two agents have the same private information and they communicate sequentially, while in our model the two agents have partial and non-overlapping private information. When two agents have the same private information, their messages can potentially discipline each other. In our model this strategic effect is absent because agents have non-overlapping private information. Given that the agents' information transmission exhibits strategic complementarity and that they have non-overlapping private information, it is always better for the DM to consult two agents instead of one.

Whether two agents have like or opposing biases does not matter in our model because the interaction between the two agents' communication occurs only through the terms  $E(\overline{m}_1^2)$  and  $E(\overline{m}_2^2)$ . When  $b_i$  changes sign, only the direction of the partition of  $m_i$  reverses; the same amount of information is transmitted in equilibrium. One may still wonder whether two agents having like or opposing biases matters based on the following logic. Suppose both agents initially have positive biases. One may think that the agents have less incentive to overstate their information when both have positive biases because each agent expects the other to overstate his information so he needs to overstate his own information less to reach his ideal action. After agent 1's bias changes to  $-b_1$  and the agents' ideal actions are farther apart, one might think that agent 2 has a stronger incentive to overstate his own information as he expects agent 1 to understate  $\theta_1$ . This is not, however, what occurs in equilibrium. In equilibrium, the principal is not fooled, which means that neither agent can successfully understate or overstate his information. Anticipating this, whether one agent has an incentive to understate or overstate his information (as long as the absolute value of the bias is the same) will not affect the other agent's incentive to misrepresent his own information.

**Comparison** In some environments, the DM may have the freedom to change the assignment of agents to observe information in different dimensions. Suppose the two information dimensions have different underlying uncertainty, and the two agents have different biases. Without loss of generality, suppose  $A_1 > A_2$  and  $|b_1| = r|b|$  and  $b_2 = b$  with  $r > 1$ ; that is,  $\theta_1$  has a bigger variance and agent 1 has a bigger bias. The question naturally arises: to induce more effective overall communication, whether the DM should assign the agent with a smaller bias to observe the dimension with more uncertainty or should he assign the agent with a bigger bias to observe the dimension with more uncertainty. We call the first assignment positive assortative (PA) assignment (assigning agent 2 to observe  $\theta_1$  and agent 1 to observe  $\theta_2$ ) and the reverse assignment negative assortative (NA) assignment.

**Proposition 2** (i) *If  $r$  is big enough such that agent 1's communication is uninformative under both assignments, then both assignments yield the same ex ante payoff for the principal. (ii) if  $b \rightarrow 0$  but  $r|b| > 0$ , then both assignments yield the same ex ante payoff. (iii) Suppose both agents' communications are informative under either assignment, and let  $N_1^*$  and  $N_2^*$  ( $N_1^{*'} and  $N_2^{*'}$ ) be the equilibrium numbers of partitions under NA (PA) assignment. If  $N_1^* = N_2^{*'}$  and  $N_2^* = N_1^{*'}$ , then both assignments yield the same ex ante payoff.$*

Compared to NA assignment, PA assignment leads to more information transmitted regarding  $\theta_1$  (about which there is more underlying uncertainty), but less information transmitted regarding  $\theta_2$  (about which there is less underlying uncertainty). The different assignments might also affect the strategic complementarity in information transmission. Proposition 2 identifies the conditions under which assignments do not matter. Part (i) shows that when one agent's bias is big enough such that his communication is uninformative under both assignments, then assignments do not matter. Part (ii) shows that when one agent's bias is arbitrarily small such that his communication will be fully informative, then assignments do not matter either. Part (iii) considers the scenario when communication is informative but not fully revealing in both dimensions. It shows that assignments do not matter as long as the number of partitions just flips when we change the assignments. Though the latter condition cannot be proved for all the parameter values, it holds for most cases. To see this, first note that, by (11), the lower and upper bounds of  $N_i^*$  under the NA assignment are the same as those of  $N_j^*$  under PA assignment. Second, whenever communication is informative for agent  $i$ , the equilibrium number of partitions for this agent is very likely to reach the upper bound in (11). This is illustrated by the following numerical examples.

**Example 2** *Suppose  $A_1 = 7$  and  $A_2 = 5$  and that agent 1 has bias  $b_1 = 1.9$  and agent 2 has bias  $b_2 = 1.9$ . Under NA assignment, the lower and upper bounds for both  $N_1^*$  and  $N_2^*$  are 1 (1.70347) and 2 (2.02791) respectively. The equilibrium  $N_1^*$  and  $N_2^*$  are both 2, the upper bound. Note that the upper bound seems hard to achieve (2.02791 is only slightly above 2): for  $N_1^*$  to be 2,  $E(\overline{m}_2^2)$  has to be very close to its upper bound (when the message is fully revealing). The upper bound for  $N_2^*$ , however, is 2 as well. This means that even with just two partitions,  $E(\overline{m}_2^2)$  is already very close to its upper bound. This is indeed the case. The lower and upper bound for  $E(\overline{m}_2^2)$  are 6.25 and 8.33 respectively, while  $E(\overline{m}_2^2)$  with two partitions is 7.6139.*

The example above illustrates that even when an agent  $i$ 's equilibrium partition has only two elements, the conditional variance of  $\theta_i$  given  $m_i$  is close to its upper bound. This implies that the strategic complementarity in communication is already very strong and makes it very likely that in equilibrium agent  $j$ 's communication will achieve the upper bound for the number of partition elements  $N_j^*$ . Given that these upper bounds on the number of partition elements are very likely to be achieved, changing assignments will most likely just result in the number of partition elements switching (in which case part (iii) of Proposition 2 applies) as the following example demonstrates.

**Example 3** Suppose  $A_1 = 40$  and  $A_2 = 10$  and that agent 1 has bias  $b_1 = 1/2$  and agent 2 has bias  $b_2 = 1/8$ . Under NA assignment, the lower and upper bounds for  $N_1^*$  are 13 and 15 respectively, and those of  $N_2^*$  are 27 and 32 respectively. The equilibrium  $N_1^*$  and  $N_2^*$  are 15 and 32 respectively. Under PA assignment, the equilibrium  $N_1^{*'} and  $N_2^{*'}$  are 32 and 15 respectively.$

We tried many numerical examples in which communication from both agents is informative but not fully revealing. In all of these examples,  $N_1^*$  and  $N_2^*$  always reach the upper bound. Therefore, we conclude that in most cases assignments do not matter for overall information transmission.

## 4 Delegation

Though the DM has formal authority to make the decision, he may find it optimal to delegate decision rights to one of the agents. Given that there are two agents, two delegation arrangements need to be considered: delegating decision rights to agent 1 (D1 delegation) or delegating decision rights to agent 2 (D2 delegation). Under either delegation arrangement, the agent to whom decision rights are delegated first consults the other agent and then makes the decision. In the CEO example, if the marketing expert (manager) is given the decision rights, he first consults the production expert (manager) regarding production efficiency and then combines this information with his own information on market demand to decide on the new plant size. We answer two questions in this section. First, which delegation arrangement is optimal (D1 or D2 delegation)? Second, when does the DM have an incentive to delegate his decision rights?

### 4.1 Optimal delegation

First consider D1 delegation. In this case, agent 2 sends message  $m_2$  to agent 1, and then agent 1 makes the decision  $y(\theta_1, m_2)$ . Agent 1's optimal decision rule is  $\bar{y}(\theta_1, m_2) = \theta_1 \bar{m}_2 + b_1$ . Now the communication game between agent 2 and agent 1 is a one-sender cheap talk game with the receiver having private information, the setting studied by McGee (2008). It can be shown that all PBE are partition equilibria (see McGee for details). Let  $N_2$  be the number of partition elements and  $\{a_{2,0}, \dots, a_{2,n}, \dots, a_{2,N_2}\}$  be the cutoff points. In particular, given  $\bar{y}(\theta_1, m_2)$ , when  $\theta_2 = a_{2,n}$  agent 2 should be indifferent between sending a message immediately to the left of  $a_{2,n}$  and a message immediately to the right of  $a_{2,n}$ . This indifference condition can be explicitly written as:

$$E[\{\theta_1 \frac{a_{2,n} + a_{2,n-1}}{2} + b_1 - (\theta_1 a_{2,n} + b_2)\}^2] = E[\{\theta_1 \frac{a_{2,n} + a_{2,n+1}}{2} + b_1 - (\theta_1 a_{2,n} + b_2)\}^2]$$

$$\Leftrightarrow (a_{2,n+1} - a_{2,n}) - (a_{2,n} - a_{2,n-1}) = \frac{4E(\theta_1)}{E(\theta_1^2)}(b_1 - b_2) = \frac{3}{2A_1}4(b_1 - b_2)$$

In the most informative equilibrium, the (largest) number of partition elements is  $N_2^* = \langle -\frac{1}{2} + \frac{1}{2}(1 + \frac{4A_2A_1}{3|b_1-b_2|})^{1/2} \rangle$ , and

$$E(\bar{m}_2^2) = \frac{A_2^2}{3} - \frac{A_2^2}{12N_2^2} - \frac{(b_1 - b_2)^2(\frac{3}{2A_1})^2(N_2^2 - 1)}{3} \quad (12)$$

Note that under D1 delegation, the incremental step size of agent 2's equilibrium partition only depends on the difference in biases ( $|b_1 - b_2|$ ). As a result,  $N_2^*$ ,  $E(\bar{m}_2^2)$  and the equilibrium information transmission depend only on the difference between the biases of the two agents. The principal's equilibrium payoff under D1 delegation,  $U_{D1}^P$ , can be computed as

$$U_{D1}^P = -E[(\theta_1 \bar{m}_2 + b_1 - \theta_1 \theta_2)^2] = -E(\theta_1^2)E(\theta_2^2) + E(\theta_1^2)E(\bar{m}_2^2) - b_1^2 \quad (13)$$

Now consider D2 delegation. In this case, agent 1 first sends message  $m_1$  to agent 2, and then agent 2 makes the decision  $y(\theta_2, m_1)$ . Agent 2's optimal decision rule is  $\bar{y}(\theta_2, m_1) = \theta_2 \bar{m}_1 + b_2$ . Let  $N_1$  be the number of partition elements and  $\{a_{1,0}, \dots, a_{1,n}, \dots, a_{1,N_1}\}$  be the cutoff points of agent 1's equilibrium communication rule. In particular,  $a_{1,n}$  is characterized by

$$(a_{1,n+1} - a_{1,n}) - (a_{1,n} - a_{1,n-1}) = \frac{4E(\theta_2)}{E(\theta_2^2)}(b_2 - b_1) = \frac{3}{2A_2}4(b_2 - b_1).$$

In the most informative equilibrium,  $N_1^* = \langle -\frac{1}{2} + \frac{1}{2}(1 + \frac{4A_2A_1}{3|b_1-b_2|})^{1/2} \rangle$ , and

$$E(\bar{m}_1^2) = \frac{A_1^2}{3} - \frac{A_1^2}{12N_1^2} - \frac{(b_1 - b_2)^2(\frac{3}{2A_2})^2(N_1^2 - 1)}{3} \quad (14)$$

Comparing the expressions for  $N_1^*$  and  $N_2^*$ , we see that the number of partition elements is the same under D1 and D2 delegation. The principal's equilibrium payoff  $U_{D2}^P$  under D2 delegation is

$$U_{D2}^P = -E[(\theta_2 \bar{m}_1 + b_2 - \theta_1 \theta_2)^2] = -E(\theta_1^2)E(\theta_2^2) + E(\theta_2^2)E(\bar{m}_1^2) - b_2^2 \quad (15)$$

**Proposition 3** *Between D1 and D2 delegation, it is always optimal for the DM to delegate decision rights to the agent with a smaller bias in absolute value.*

**Proof.** Without loss of generality, suppose agent 1 has a smaller bias,  $|b_1| < |b_2|$ . We want to show that D1 delegation is better for the DM. From previous derivations, it is clear that  $N_1^*$  under D2 delegation is the same as  $N_2^*$  under D1 delegation. Let  $N^* = N_1^* = N_2^*$ . From (13) and (15),

$$U_{D1}^P - U_{D2}^P = E(\theta_1^2)E(\bar{m}_2^2) - E(\theta_2^2)E(\bar{m}_1^2) + (b_2^2 - b_1^2).$$

By (12) and (14),

$$\begin{aligned} U_{D1}^P - U_{D2}^P &= \frac{A_1^2}{3} \left[ \frac{A_2^2}{3} - \frac{A_2^2}{12N^{*2}} - \frac{3(b_1 - b_2)^2(N^{*2} - 1)}{4A_1^2} \right] \\ &\quad - \frac{A_2^2}{3} \left[ \frac{A_1^2}{3} - \frac{A_1^2}{12N^{*2}} - \frac{3(b_1 - b_2)^2(N^{*2} - 1)}{4A_2^2} \right] + (b_2^2 - b_1^2) \\ &= b_2^2 - b_1^2 > 0. \end{aligned}$$

Therefore, D1 delegation yields a higher ex ante payoff. ■

Proposition 3 indicates that decision rights should be delegated to the agent with a smaller bias. Note that this does not depend on which agent's private information has more underlying uncertainty. Intuitively, since equilibrium information transmission only depends on the difference between the biases  $|b_1 - b_2|$ , under either delegation arrangement the agent with the decision rights ends up (after communication) with the same amount of information to utilize (i.e.,  $E(\theta_1^2)E(\bar{m}_2^2)$  under D1 delegation equals  $E(\theta_2^2)E(\bar{m}_1^2)$  under D2 delegation). Thus it is clear that decision rights should be delegated to the agent with a smaller bias to minimize the loss of control.

## 4.2 Comparison between delegation and simultaneous communication

Now without loss of generality, suppose agent 1 has a smaller bias,  $|b_1| < |b_2|$ . Between the two delegation arrangements, D1 delegation is optimal. We are interested in identifying the conditions under which the principal has an incentive to delegate instead of retaining decision rights and engaging in simultaneous cheap talk with both agents.

**Proposition 4** *When two agents have like biases and informative communication is feasible from agent 1 under simultaneous communication, then the DM prefers D1 delegation to simultaneous cheap talk.*

Proposition 4 is related to Dessein (2002), who shows that in a one-sender cheap talk model the principal prefers delegation whenever cheap talk is informative. In our two-sender setting, the comparison becomes more complex and richer. To understand Proposition 4, we list four effects of delegation relative to simultaneous communication and indicate whether these effects make delegation more or less attractive to the DM.

(i) Delegation leads to a loss of control, which is measured by  $b_1^2$ . (-)

(ii) Delegation always leads to more information being utilized in the dimension of  $\theta_1$ , since  $E(\theta_1^2) \geq E(\bar{m}_1^2)$ . (+)

(iii)  $E(\theta_1^2) \geq E(\bar{m}_1^2)$  means  $x_{2D} \leq x_2$ . The strategic complementarity of the agents' information transmission implies that more information will be transmitted by agent 2 under delegation. (+)

(iv) Agent 2's effective bias changes from  $|b_2|$  under simultaneous communication to  $|b_2 - b_1|$  under delegation, which affects agent 2's equilibrium information transmission. (?)

The last three effects measure the potential informational gain under delegation relative to simultaneous communication.<sup>7</sup> While effects (ii) and (iii) always favor delegation, effect (iv) depends on whether the two agents have like or opposing biases. When the two agents have like biases, optimal delegation always leads to a smaller effective bias in communication,  $|b_2 - b_1| < |b_2|$ . Thus both effects (iii) and (iv) work in the same direction: more information is transmitted by agent 2 under delegation. This additional informational gain, which is absent in Dessein's (2002) one-sender cheap talk model, makes the overall informational gain under delegation even larger. Thus delegation is preferred by the DM.

**Example 4** Suppose  $A_1 = 5$ ,  $A_2 = 7$ ,  $b_1 = \frac{1}{8}$  and  $b_2 = \frac{1}{2}$ . Under simultaneous communication,  $N_1^* = 9$ ,  $N_2^* = 4$ ,  $E(\bar{m}_1^2) = 8.28759$ , and  $E(\bar{m}_2^2) = 15.9644$ . Under D1 delegation, the effective bias is  $|b_2 - b_1| = 3/8$ . By the inequality  $N_{2D}(N_{2D} - 1) < \frac{A_1 A_2}{3|b_2 - b_1|}$ ,  $N_{2D}^* = 5$ . The difference between the principal's ex ante payoffs can be expressed as

$$U_{D1}^P - U_{ST}^P = \left[ \left( \frac{25}{3} \right) (16.0688) - \frac{1}{64} \right] - (8.28759)(15.9644) = 1.58446.$$

*Delegation dominates simultaneous communication.*

The condition that two agents have like biases is not necessary for delegation to dominate simultaneous communication. This is because even if two agents have opposing biases such that

---

<sup>7</sup>Dessein (2002) compares the information gain from the second effect to the loss of control in the first effect.

effect (iv) works against delegation, effects (ii) and (iii) can still outweigh effect (iv), meaning that delegation leads to an informational gain. The following example illustrates this.

**Example 5** Suppose  $A_1 = 5$ ,  $A_2 = 7$ ,  $b_1 = \frac{1}{8}$  and  $b_2 = -\frac{1}{4}$ . Under D1 delegation, the effective bias is again  $|b_2 - b_1| = \frac{3}{8}$ . Because  $|b_2 - b_1| = \frac{3}{8}$ ,  $U_{D1}^P$  is the same as in the previous example. Under simultaneous communication,  $N_1^* = 9$ ,  $N_2^* = 4$ ,  $E(\bar{m}_1^2) = 8.28805$ , and  $E(\bar{m}_2^2) = 16.1536$ .

$$U_{D1}^P - U_{ST}^P = \left[ \left( \frac{25}{3} \right) (16.0688) - \frac{1}{64} \right] - (8.28805)(16.1536) = 0.00904693.$$

Again delegation dominates simultaneous communication.

The proof of Proposition 4 indicates that as long as  $E(\bar{m}_{2D}^2) \geq E(\bar{m}_2^2)$  (that is, more information is transmitted by agent 2 under D1 delegation than under simultaneous communication) the principal prefers delegation. Thus we have the following corollary.

**Corollary 3** Suppose two agents have opposing biases. As long as  $|b_1|/|b_2|$  is small enough such that  $E(\bar{m}_{2D}^2) \geq E(\bar{m}_2^2)$  and informative communication is feasible for agent 1 under simultaneous communication, D1 delegation leads to a higher ex ante payoff for the principal.

Of course, when two agents have opposing biases and  $|b_1|$  is big enough, then effect (iv) might outweigh effects (ii) and (iii), leading to a small information gain or even an information loss under D1 delegation relative to simultaneous communication. In this case, simultaneous communication dominates delegation.

**Example 6** Suppose  $A_1 = 5$ ,  $A_2 = 7$ ,  $b_1 = \frac{1}{8}$  and  $b_2 = -\frac{1}{2}$ . By Corollary 2, under simultaneous communication the principal's expected utility is the same as in example 4. Under D1 delegation, it can be verified that  $N_{2D}^* = 2$ . Because  $|b_2 - b_1| = \frac{5}{8}$ ,  $E(\bar{m}_{2D}^2) = 15.7859$ . Thus

$$U_{D1}^P - U_{ST}^P = \left[ \left( \frac{25}{3} \right) (15.7859) - \frac{1}{64} \right] - (8.28759)(15.9644) = -0.772796.$$

Simultaneous communication dominates delegation.

Proposition 4 and Corollary 3 generate some interesting empirical implications. First, between two functionally parallel agents (e.g., division managers), it is possible that decision rights

are delegated to one of the two agents. For example, in the example of a CEO choosing a plant size, it is possible that the CEO delegates decision rights to the production manager if he has a smaller bias than the marketing manager. Second, we are more likely to observe delegation when two agents's preferences are biased in the same direction relative to the principal. In the context of politics, delegation is more likely to be observed when both experts are either both more liberal than the DM or both more conservative than the DM. On the other hand, delegation is less likely to be observed if one expert is more liberal than the DM but the other expert is more conservative than the DM. In the context of firm organization, if both division managers are biased toward choosing a bigger factory size (e.g., empire building) relative to the CEO, then delegation is more likely—again because the agent with the decision rights is able to extract more information from communication with the other agent than the DM would be able to through simultaneous communication.

## 5 Discussions

### 5.1 Sequential Talk

Besides simultaneous talk and delegation, the DM can also let the two agents communicate with him sequentially. Specifically, one agent sends his message first, and this message becomes public. Knowing the first agent's message, the other agent then sends a message. A number of questions regarding sequential talk suggest themselves. First, which agent should talk first in sequential talk to maximize the DM's payoff? In other words, how does the optimal order of cheap talk depend on the parameter values.<sup>8</sup> Second, how does sequential talk compare to simultaneous talk and delegation in terms of the DM's payoff?

Suppose agent 1 sends his message first. Agent 1's communication rule is still  $\mu_1(m_1|\theta_1)$ , but agent 2's communication rule now becomes  $\mu_2(m_2|\theta_2, m_1)$ , which, for simplicity, we denote as  $\mu_2(m_1)$ . We work by backward induction. Given  $m_1$  and  $m_2$ , the DM's optimal decision rule is  $\bar{y} = E[\theta_1|m_1]E[\theta_2|m_2, \mu_2(m_1)]$ . Now given  $m_1$ , consider agent 2's problem. It is straightforward to show that agent 2's equilibrium communication rule takes the form of a partitioning rule. In general, the cutoff  $a_{2,n}$  satisfies the indifference condition:

$$E_{\theta_1} \left[ \left\{ \bar{m}_1 \frac{a_{2,n} + a_{2,n-1}}{2} - (\theta_1 a_{2,n} + b_2) \right\}^2 | m_1 \right] = E_{\theta_1} \left[ \left\{ \bar{m}_1 \frac{a_{2,n} + a_{2,n+1}}{2} - (\theta_1 a_{2,n} + b_2) \right\}^2 | m_1 \right]$$

$$\Leftrightarrow (a_{2,n+1} - a_{2,n}) - (a_{2,n} - a_{2,n-1}) = \frac{4b_2}{\bar{m}_1}. \quad (16)$$

---

<sup>8</sup>Ottaviani and Sorensen (2001) show that in committee debate the order of speech affects information transmission, and thus matters.

From the above equation, we can see that agent 2's communication depends on  $m_1$ . In particular, a larger message sent by agent 1 (i.e.,  $E(\theta_1|m_1)$  is larger) reduces the incremental step size of agent 2's partition, leading to more information being transmitted by agent 2.

Now consider agent 1's problem. We cannot guarantee that agent 1's equilibrium communication rule is of partition form. To see this, consider a modification of Lemma 1 in which

$$E_{\theta_2}[U^{A_1}|v_1, \theta_1] = -E_{\theta_2}[\{v_1 E[\theta_2|\mu_2(v_1)] - \theta_1 \theta_2 - b_1\}^2]. \quad (17)$$

From (17), it can be readily seen that  $\frac{\partial^2}{\partial \theta_1^2}[U^{A_1}|v_1, \theta_1] < 0$ . Again from (17),

$$\begin{aligned} \frac{\partial^2}{\partial \theta_1 \partial v_1}[U^{A_1}|v_1, \theta_1] &= E_{\theta_2}[2\theta_2 E[\theta_2|\mu_2(v_1)]] + v_1 \frac{\partial}{\partial v_1} E_{\theta_2}[2\theta_2 E[\theta_2|\mu_2(v_1)]] \\ &= 2E[\bar{m}_2^2] - 2v_1 \frac{\partial}{\partial v_1} \left[ \frac{b_2^2(N_2^2(v_1) - 1)}{3v_1^2} \right], \end{aligned}$$

where the second term follows (16) and (9). Given that  $N_2$  is increasing in  $v_1$ , we are not sure that the second term in the above equation is greater than 0. Hence, the equilibrium communication rule of agent 1 might not be of partition form.

Now suppose agent 1's equilibrium communication rule is of partition form. To characterize the partition points, define  $\bar{m}_2(a_{1,n}, a_{1,n+1}) = E[\theta_2|m_2, m_1 \in (a_{1,n}, a_{1,n+1})]$ . The indifference condition characterizing  $a_{1,n}$  can be simplified as:

$$\begin{aligned} &E[\bar{m}_2^2(a_{1,n}, a_{1,n+1})][(a_{1,n} + a_{1,n+1})^2 - 4(a_{1,n} + a_{1,n+1})a_{1,n}] - \\ &E[\bar{m}_2^2(a_{1,n-1}, a_{1,n})][(a_{1,n} + a_{1,n-1})^2 - 4(a_{1,n} + a_{1,n-1})a_{1,n}] \\ &= 4E(\theta_2)b_1[(a_{1,n} + a_{1,n+1}) - (a_{1,n} + a_{1,n-1})] \end{aligned} \quad (18)$$

From (16), we see that more information will be transmitted by agent 2 if  $m_1$  increases. Thus we have  $E[\bar{m}_2^2(a_{1,n}, a_{1,n+1})] > E[\bar{m}_2^2(a_{1,n-1}, a_{1,n})]$ . Inspecting (18), we see that this equation is highly nonlinear, so no general properties of the communication equilibrium can be derived. Unless we resort to special numerical examples, we cannot draw conclusions as to the optimal order of communication nor make any meaningful comparisons between sequential talk and simultaneous talk and delegation. We leave these questions for future research.

## 5.2 Robustness

One may wonder how our results change if we allow for more general forms of the optimal decision function  $y^*(\theta_1, \theta_2)$  and for more general distribution functions of  $\theta_1$  and  $\theta_2$ . As is well known, analytical solutions and comparative statics in cheap talk models are very difficult

to derive once we deviate from the uniform-quadratic loss benchmark case. For this reason, we confine our discussion to how the main results regarding simultaneous talk change in more general settings.

Instead of assuming  $y^*(\theta_1, \theta_2) = \theta_1\theta_2$ , suppose we only impose the conditions that  $y^*(\theta_1, \theta_2)$  is twice continuously differentiable,  $y_1^*(\theta_1, \theta_2) > 0$ ,  $y_2^*(\theta_1, \theta_2) > 0$  and  $y_{12}^*(\theta_1, \theta_2) > 0$ . The principal's optimal decision given messages  $m_1$  and  $m_2$  becomes  $y^*(E[\theta_1|m_1], E[\theta_2|m_2])$  or  $y^*(\bar{m}_1, \bar{m}_2)$ .

We first identify the conditions under which the PBE of the simultaneous communication game will be interval equilibria. As in the proof of Lemma 1, suppose that agent 2 employs a communication rule  $\mu_2(\cdot)$  and that the agent 1's communication induces the DM to have a posterior belief  $v_1$  regarding  $\theta_1$ . Agent 1's expected utility becomes

$$E_{\theta_2} [U^{A_1}|v_1, \theta_1] = -E_{\theta_2} \left[ \{y^*(v_1, E[\theta_2|\mu_2(\cdot)]) - y^*(\theta_1, \theta_2) - b_1\}^2 \right]. \quad (19)$$

From Lemma 1, the sufficient conditions for agents 1's equilibrium communication rule to be of the interval form are  $\frac{\partial^2}{\partial \theta_1 \partial v_1} [U^{A_1}|v_1, \theta_1] > 0$  and  $\frac{\partial^2}{\partial \theta_1^2} [U^{A_1}|v_1, \theta_1] < 0$ . By (19),  $y_1^* > 0$  ensures that the first condition is satisfied. The second condition can be written more explicitly as

$$\frac{\partial^2}{\partial \theta_1^2} [U^{A_1}|v_1, \theta_1] = -E_{\theta_2} [2y_{11}^*(\theta_1, \theta_2) \{y^*(\theta_1, \theta_2) - y^*(v_1, \bar{m}_2) + b_1\} + 2(y_1^*(\theta_1, \theta_2))^2] < 0. \quad (20)$$

Note that the conditions we have so far imposed on  $y^*$  do not guarantee that (20) holds. To make sure that (20) holds, we need to impose the additional condition that  $|y_{11}^*/y_1^*$  is "small enough." Similarly, to ensure that agent 2's equilibrium communication is of interval form, we need to impose the condition that  $|y_{22}^*/y_2^*$  is "small enough."

Now suppose that both  $|y_{11}^*/y_1^*$  and  $|y_{22}^*/y_2^*$  are small enough so that both agents' equilibrium communication rules are of interval form. With what functional forms of  $y^*$  will the information transmitted by the two agents exhibit strategic complementarity? The indifference condition that characterizes agent 1's partition points  $a_{1,n}$  can be written as:

$$\begin{aligned} 2b_1 &= E_{\theta_2} \left[ y^* \left( \frac{a_{1,n} + a_{1,n+1}}{2}, \bar{m}_2 \right) + y^* \left( \frac{a_{1,n} + a_{1,n-1}}{2}, \bar{m}_2 \right) - 2y^*(a_{1,n}, \bar{m}_2) \right] \\ &+ \frac{1}{E_{\theta_2} \left[ y^* \left( \frac{a_{1,n} + a_{1,n+1}}{2}, \bar{m}_2 \right) - y^* \left( \frac{a_{1,n} + a_{1,n-1}}{2}, \bar{m}_2 \right) \right]} \times \\ &\{ \text{Var}_{\theta_2} \left[ y^* \left( \frac{a_{1,n} + a_{1,n+1}}{2}, \bar{m}_2 \right) \right] - \text{Var}_{\theta_2} \left[ y^* \left( \frac{a_{1,n} + a_{1,n-1}}{2}, \bar{m}_2 \right) \right] \right. \\ &\left. - 2\text{Cov}_{\theta_2} \left[ y^*(a_{1,n}, \bar{m}_2), y^* \left( \frac{a_{1,n} + a_{1,n+1}}{2}, \bar{m}_2 \right) - y^* \left( \frac{a_{1,n} + a_{1,n-1}}{2}, \bar{m}_2 \right) \right] \right\}. \quad (21) \end{aligned}$$

Inspecting (21), it is hard to tell analytically how the right hand side of (21) will change if  $\bar{m}_2$  becomes more informative.

To get cleaner results, we assume that  $y^*(\theta_1, \theta_2) = s(\theta_1)t(\theta_2)$ , where both  $s'$  and  $t'$  are strictly positive. The indifference condition (21) can be simplified as

$$2b_1 = \frac{[s(\frac{a_{1,n} + a_{1,n+1}}{2}) + s(\frac{a_{1,n} + a_{1,n-1}}{2})]E_{\theta_2}[t(\bar{m}_2)] - 2s(a_{1,n})E_{\theta_2}[t(\theta_2)] + [s(\frac{a_{1,n} + a_{1,n+1}}{2}) + s(\frac{a_{1,n} + a_{1,n-1}}{2})]Var_{\theta_2}(t(\bar{m}_2)) - 2s(a_{1,n})Cov_{\theta_2}(t(\theta_2), t(\bar{m}_2))}{E_{\theta_2}[t(\bar{m}_2)]}. \quad (22)$$

Inspecting (22), it is still difficult to derive analytically comparative static results, the main reason being that we do not know how the ratios among  $Var_{\theta_2}(t(\bar{m}_2))$ ,  $E_{\theta_2}[t(\bar{m}_2)]$ , and  $Cov_{\theta_2}(t(\theta_2), t(\bar{m}_2))$  change as  $\bar{m}_2$  becomes more informative.

We further assume that both  $s(\cdot)$  and  $t(\cdot)$  are linear functions. Then  $E_{\theta_2}[t(\bar{m}_2)] = E_{\theta_2}[t(\theta_2)]$  and  $Cov_{\theta_2}(t(\theta_2), t(\bar{m}_2)) = Var_{\theta_2}(t(\bar{m}_2))$ . Now (22) can be further simplified as

$$s(\frac{a_{1,n} + a_{1,n+1}}{2}) + s(\frac{a_{1,n} + a_{1,n-1}}{2}) - 2s(a_{1,n}) = 2b_1 \frac{E[t(\theta_2)]}{E[(t(\bar{m}_2))^2]}. \quad (23)$$

From (23), we can clearly see that the incremental step size of agent 1 decreases as  $\bar{m}_2$  becomes more informative. Thus the information transmitted by two agents exhibits strategic complementarity.

To summarize, we have shown that, if  $y^*(\theta_1, \theta_2) = s(\theta_1)t(\theta_2)$  with  $s$  and  $t$  being linear functions and  $s' > 0$  and  $t' > 0$ , then under simultaneous talk both agents' equilibrium communication rules are of interval form and their information transmission exhibits strategic complementarity. Of course,  $y^*(\theta_1, \theta_2)$  does not necessarily need to be restricted to the product of two linear functions for the equilibrium properties discussed above to hold. However, no more general conclusions can be drawn without specifying the functional form of  $y^*(\theta_1, \theta_2)$ .

That said, the functional form restrictions we make above—which amount to assuming that  $y^*(\theta_1, \theta_2) = k_1\theta_1 + k_2\theta_2 + k_3\theta_1\theta_2$ , where  $k_1$ ,  $k_2$  and  $k_3$  are some positive constants—are less restrictive than they may appear at first. After all, we always can redefine the underlying states  $\theta_1$  and  $\theta_2$ . For example, suppose  $y^*(\theta_1, \theta_2) = k_1/\theta_1 + k_2\theta_2 + k_3\theta_2/\theta_1$ . By simply redefining or rescaling  $\theta'_1$  as  $1/\theta_1$ ,  $y^*(\theta'_1, \theta_2)$  conforms to the original form.<sup>9</sup>

Now we discuss how our results will change if we generalize the distribution of the underlying states. To keep things simple, we return to the case in which  $y^*(\theta_1, \theta_2) = \theta_1\theta_2$ . Suppose  $\theta_1$  is distributed on  $[0, A_1]$  with density function  $f_1(\theta_1)$  and cumulative distribution function  $F_1(\theta_1)$ , and  $f_1(\theta_1) > 0$  holds everywhere on the support  $[0, A_1]$ . Similarly, suppose  $\theta_2$  is distributed on  $[0, A_2]$  with density function  $f_2(\theta_2)$  and cumulative distribution function  $F_2(\theta_2)$ , and  $f_2(\theta_2) > 0$

<sup>9</sup>Of course, after redefining the underlying states, the distribution of the underlying states will change correspondingly.

holds everywhere on the support  $[0, A_2]$ . Inspecting the proof of Lemma 1, we see that the proof applies to the new setting as well. Therefore, each agent's equilibrium communication rule is of the partition form. Given the sequence of partition points  $a_{i,n}$ , define  $\bar{m}_{i,n}$  as the receiver's posterior regarding  $\theta_i$  after receiving a message  $m_{i,n} \in (a_{i,n-1}, a_{i,n})$ . Now  $\bar{m}_{i,n} = \int_{a_{i,n-1}}^{a_{i,n}} \frac{\theta_i f_i(\theta_i) d\theta_i}{F_i(a_{i,n}) - F_i(a_{i,n-1})}$ . The indifference condition that characterizes  $a_{i,n}$  now is written as:

$$E_{\theta_2} [\{\bar{m}_2 \bar{m}_{1,n} - (\theta_2 a_{1,n} + b_1)\}^2] = E_{\theta_2} [\{\bar{m}_2 \bar{m}_{1,n+1} - (\theta_2 a_{1,n} + b_1)\}^2].$$

After simplification, the above condition becomes:

$$\bar{m}_{1,n+1} + \bar{m}_{1,n} - 2a_{1,n} = \frac{E(\theta_2)}{E(\bar{m}_2^2)} 2b_1. \quad (24)$$

Fixing  $a_{1,n-1}$  and  $a_{1,n}$ , suppose that  $\bar{m}_2$  becomes more informative (i.e., agent 2's incremental step size decreases and he transmits more information). This means that  $E(\bar{m}_2^2)$  increases, and the right hand side of (24) decreases. With  $a_{1,n-1}$  and  $a_{1,n}$  fixed,  $\bar{m}_{1,n+1}$  must decrease, which implies that  $a_{1,n+1}$  must decrease as well. The above argument shows that as agent 2 transmits more information, agent 1's incremental step size decreases as well, leading to more information transmission regarding  $\theta_1$ . Therefore, communication from the two agents also exhibits strategic complementarity under more general distributions.<sup>10</sup>

## 6 Conclusions

We examine a two-sender cheap talk model in which two experts have partial and non-overlapping information regarding the state of the world. In this setting, the marginal impact of one expert's private information on the receiver's ideal action depends on the other expert's private information. Under simultaneous communication, we show that information transmission displays strategic complementarities in that more informative communication from one expert induces more informative communication from the other. Interestingly, the informativeness of communication from both experts in equilibrium does not depend on whether they have like or opposing biases, but only depends on the magnitudes of these biases. When the decision-maker can assign the experts to the different dimensions of the state space, we show that under a broad range of circumstances this assignment will not affect the decision-maker's expected utility.

We then study delegation when the decision rights are delegated to one of the two experts. We show that the decision-maker, if he ever delegates, always prefers to delegate decision rights

---

<sup>10</sup>Under general distributions, the partition points depend on the shape of the density functions.

to the expert with the smaller bias. Comparing delegation to simultaneous communication, we demonstrate that when two experts have like biases, delegation is always superior for the decision-maker whenever informative communication between the decision-maker and the experts is possible. On the other hand, simultaneous communication dominates delegation when the experts have opposing biases and the smaller bias is big enough.

Unlike other models of strategic communication with multiple experts in which two experts observe basically the same realized state of the world, our model highlights how the nature of the relationship between the private information of the two experts influences their communication to the decision-maker. How much information one expert transmits depends on how much information the other experts will transmit. When the experts are uncertain about each other's information, the complementary nature of their information curtails their desires to misrepresent their own information, and the additional incentives for informative communication arising from the relationship between the experts' information widen the scope for the delegation of decision rights to the less-biased expert.

## References

- [1] Aghion, P. and J. Tirole. "Formal and Real Authority in Organizations," *Journal of Political Economy* 105, 1997, 1-29.
- [2] Alonso, R. and N. Matouschek. "Relational Delegation," *Rand Journal of Economics* 38(4), 2007, 1070-1089.
- [3] Alonso, R., W. Dessein and N. Matouschek. "When does Coordination Requires Centralization?" *American Economic Review* 98(1), 2008, 145-179.
- [4] Ambrus, A. and S. Takahashi. "Multi-sender Cheap Talk with Restrictive State Space," *Theoretical Economics* 3(1), 2008, 1-27.
- [5] Austen-Smith, D. "Interested Experts and Policy Advice: Multiple Referrals under Open Rule," *Games and Economic Behavior* 5, 1993, 1-43.
- [6] Battaglini, M. "Multiple Referrals and Multidimensional Cheap Talk," *Econometrica* 70(4), 2002, 1379-1401.
- [7] Chen, Y. "Communication with Two-sided Asymmetric Information," mimeo, 2009.

- [8] Chen, Y., N. Kartik and J. Sobel. "Selecting Cheap-Talk Equilibria," *Econometrica* 76(1), 2008, 117-136.
- [9] Crawford, V. and J. Sobel. "Strategic Information Transmission," *Econometrica* 50(6), 1982, 1431-1451.
- [10] Dessein, W. "Authority and Communication in Organizations," *Review of Economic Studies* 69, 2002, 811-838.
- [11] Epstein, D. "Partisan and Bipartisan Signaling in Congress," *Journal of Law, Economics, and Organization* 14(2), 1998, 183-204.
- [12] Gick, W. "Two-Sender Cheap Talk in One Dimension: A Note of Like Biases," mimeo, 2009.
- [13] Gilligan, T. and K. Krehbiel. "Asymmetric Information and Legislative Rules with a Heterogeneous Committee," *American Journal of Political Science*, 1989, 459-490.
- [14] Goltsman, M. and G. Pavlov. "How to Talk to Multiple Audiences," mimeo, 2009.
- [15] Harris, M. and A. Raviv. "Allocation of Decision-making Authority," *Review of Finance* 9, 2005, 353-383.
- [16] Krishna, V. and J. Morgan. "A Model of Expertise," *Quarterly Journal of Economics* 116, 2001a, 747-775.
- [17] Krishna, V. and J. Morgan. "Asymmetric Information and Legislative Rules," *American Political Science Review* 95, 2001b, 435-452.
- [18] Li, M. "Combining Expert Opinions," working paper, 2007, Concordia University.
- [19] Matthews, S., M. Okuno-Fujiwara and A. Postlewaite. "Refining Cheap Talk Equilibria," *Journal of Economic Theory*, 1991, 247-273.
- [20] McGee, A. "Strategic Communication with an Informed Principal and Contingent Information," working paper, 2008, Ohio State University.
- [21] Ottaviani, M. and P. Sorensen. "Information Aggregation in Debate: Who Should Speak First," *Journal of Public Economics* 81, 2001, 393-421.

# Appendix

## Proof of Lemma 1:

**Proof.** First note that in any PBE of the communication game, the optimal decision given beliefs satisfies (1). We first show that given any communication rule for agent 2,  $\mu_2(\cdot)$ , agent 1's optimal communication rule is of the interval form. Suppose the DM holds a posterior belief  $v_1$  regarding  $\theta_1$ . Then agent 1's expected utility is

$$E_{\theta_2}[U^{A_1}|v_1, \theta_1] = -E_{\theta_2}[\{v_1 E[\theta_2|\mu_2(\cdot)] - \theta_1\theta_2 - b_1\}^2]. \quad (25)$$

It can be readily seen that  $\frac{\partial^2}{\partial\theta_1\partial v_1}[U^{A_1}|v_1, \theta_1] > 0$  and  $\frac{\partial^2}{\partial\theta_1^2}[U^{A_1}|v_1, \theta_1] < 0$ . This implies that for any two different posterior beliefs of the DM, say  $\underline{v}_1 < \bar{v}_1$ , there is at most one type of agent 1 that is indifferent between both. Now suppose that contrary to interval equilibria, there are two states  $\underline{\theta}_1 < \bar{\theta}_1$  such that  $E_{\theta_2}[U^{A_1}|\bar{v}_1, \underline{\theta}_1] \geq E_{\theta_2}[U^{A_1}|\underline{v}_1, \underline{\theta}_1]$  and  $E_{\theta_2}[U^{A_1}|\underline{v}_1, \bar{\theta}_1] > E_{\theta_2}[U^{A_1}|\bar{v}_1, \bar{\theta}_1]$ . Then  $E_{\theta_2}[U^{A_1}|\bar{v}_1, \bar{\theta}_1] - E_{\theta_2}[U^{A_1}|\underline{v}_1, \bar{\theta}_1] < E_{\theta_2}[U^{A_1}|\bar{v}_1, \underline{\theta}_1] - E_{\theta_2}[U^{A_1}|\underline{v}_1, \underline{\theta}_1]$ , which contradicts  $\frac{\partial^2}{\partial\theta_1\partial v_1}[U^{A_1}|v_1, \theta_1] > 0$ .

The same argument can be applied to agent 2 given any communication rule  $\mu_1(\cdot)$  for agent 1. Therefore, all PBE of the communication game must be interval equilibria. ■

**Claim:**  $E[\theta_i \bar{m}_i] = E[\bar{m}_i^2]$ .

**Proof.** Note that  $\bar{m}_i = E[\theta_i|m_i]$  and  $m_i$  is coarser than  $\theta_i$ . Therefore,

$$E[\theta_i \bar{m}_i] = E[\theta_i E[\theta_i|m_i]] = E\{E[\theta_i E[\theta_i|m_i]]|m_i\} = E\{E[\theta_i|m_i]E[\theta_i|m_i]\} = E[\bar{m}_i^2].$$

■

## Proof of Lemma 2.

**Proof.** We prove the results for  $E(\bar{m}_1^2)$ . The results for  $E(\bar{m}_2^2)$  can be proved similarly. From (6), we have

$$\begin{aligned} a_{1,n} - a_{1,n-1} &= \frac{A_1}{N_1} + 2b_1x_2(2n - N - 1), \\ a_{1,n} + a_{1,n-1} &= \frac{A_1}{N_1}(2n - 1) + 2b_1x_2[2n^2 - (2n - 1)(N + 1)]. \end{aligned}$$

By definition,

$$\begin{aligned} E(\bar{m}_1^2) &= \sum_{n=1}^{N_1} \int_{a_{1,n-1}}^{a_{1,n}} \frac{1}{A_1} \frac{(a_{1,n} + a_{1,n-1})^2}{4} = \frac{1}{4A_1} \sum_{n=1}^{N_1} (a_{1,n} - a_{1,n-1})(a_{1,n} + a_{1,n-1})^2 \\ &= \frac{A_1^2}{3} - \frac{A_1^2}{12N_1^2} - \frac{b_1^2x_2^2(N_1^2 - 1)}{3}. \end{aligned}$$

This proves part (i). To show part (ii), consider the change in  $E(\bar{m}_1^2)$  when  $N_1$  decreases to  $N_1 - 1$ :

$$\begin{aligned} E(\bar{m}_1^2)(N_1) - E(\bar{m}_1^2)(N_1 - 1) &= \frac{A_1^2}{12} \left[ \frac{1}{(N_1 - 1)^2} - \frac{1}{N_1^2} \right] - \frac{b_1^2 x_2^2}{3} [N_1^2 - (N_1 - 1)^2] \\ &\propto A_1^2 - 4b_1^2 x_2^2 N_1^2 (N_1 - 1)^2 > 0 \end{aligned}$$

The last inequality follows from  $a_{1,1} > 0$ , which implies that  $A_1 > 2b_1 x_2 N_1 (N_1 - 1)$ . Thus  $E(\bar{m}_1^2)$  is strictly increasing in  $N_1$ .  $x_2$  affects  $E(\bar{m}_1^2)$  in two ways. First, a decrease in  $x_2$  directly increases  $E(\bar{m}_1^2)$ . Second, by (5) a decrease in  $x_2$  leads to a weakly larger  $N_1$ , which increases  $E(\bar{m}_1^2)$  as well. Therefore,  $E(\bar{m}_1^2)$  is strictly decreasing in  $x_2$ . By similar logic,  $E(\bar{m}_1^2)$  is strictly decreasing in  $b_1$ . Since  $x_2$  is decreasing in  $E(\bar{m}_2^2)$ , it follows that  $E(\bar{m}_1^2)$  is strictly increasing in  $E(\bar{m}_2^2)$ .

To prove part (iii), note that  $E(\bar{m}_1^2) = (E(\theta_1))^2 + \text{var}[E(\theta_1|m_1)]$ . Since the conditional variance  $\text{var}[E(\theta_1|m_1)] \in [0, \text{var}(\theta_1)]$ ,  $(E(\theta_1))^2 \leq E(\bar{m}_1^2) \leq E(\theta_1^2)$ . Part (iii) immediately follows. ■

### Proof of Proposition 2:

**Proof.** (i) One sufficient condition for agent 1's communication to be uninformative under both assignments is  $6r|b| \geq A_1 A_2$ . To see this, consider NA assignment.  $N_1^*$  is the largest  $N_1$  such that  $2r|b|x_2 N_1 (N_1 - 1) < A_1$ . Given  $x_2 \geq \frac{3}{2A_2}$ ,  $2r|b|x_2 N_1 (N_1 - 1) \geq 3r|b|N_1 (N_1 - 1)/A_2$ . Now if  $N_1^* \geq 2$ , then  $3r|b|N_1 (N_1 - 1)/A_2 \geq 6r|b|/A_2$ , which by the condition  $6r|b| \geq A_1 A_2$  is bigger than  $A_1$ . This contradicts  $2r|b|x_2 N_1^* (N_1^* - 1) < A_1$ . Therefore,  $N_1^* = 1$ . By a similar argument, one can show that under PA assignment  $N_2'^* = 1$ .

Now consider NA assignment. Given  $N_1^* = 1$ ,  $E(\bar{m}_1^2) = \frac{A_1^2}{4}$  and  $x_1 = \frac{2}{A_1}$ . Then  $N_2^*$  is the largest integer such that  $4|b|N_2(N_2 - 1) < A_1 A_2$ . Under PA assignment,  $N_2'^* = 1$ ,  $E(\bar{m}_2'^2) = \frac{A_2^2}{4}$ ,  $x_2' = \frac{2}{A_2}$ . Then  $N_1'^*$  is the largest integer such that  $4|b|N_1'(N_1' - 1) < A_1 A_2$ . It can be readily seen that  $N_1'^* = N_2^* \equiv N^*$ . The difference between the ex ante payoffs from NA assignment and PA assignment is

$$\begin{aligned} E(\bar{m}_1^2)E(\bar{m}_2^2) - E(\bar{m}_1'^2)E(\bar{m}_2'^2) &= \frac{A_1^2}{4} \left[ \frac{A_2^2}{3} - \frac{A_2^2}{12N^{*2}} - \frac{4b^2(N^{*2} - 1)}{3A_1^2} \right] \\ - \frac{A_2^2}{4} \left[ \frac{A_1^2}{3} - \frac{A_1^2}{12N^{*2}} - \frac{4b^2(N^{*2} - 1)}{3A_2^2} \right] &= 0. \end{aligned}$$

Therefore, both assignments lead to the same ex ante payoff.

(ii) When  $b \rightarrow 0$ , under NA assignment  $E(\bar{m}_2^2) = \frac{A_2^2}{3}$  and  $x_1 = \frac{3}{2A_2}$ . Moreover,  $N_1^*$  is the largest integer such that  $4r|b|N_1(N_1 - 1) < A_1 A_2$ . Similarly, under PA assignment  $E(\bar{m}_1'^2) = \frac{A_1^2}{3}$

and  $x'_2 = \frac{3}{2A_1}$ . Moreover,  $N_2^*$  is the largest integer such that  $4r|b|N_2'(N_2' - 1) < A_1A_2$ . It can be readily seen that  $N_2^* = N_1^* \equiv N^*$ . The difference between the ex ante payoffs from NA assignment and PA assignment is

$$\begin{aligned} E(\bar{m}_1^2)E(\bar{m}_2^2) - E(\bar{m}_1'^2)E(\bar{m}_2'^2) &= \frac{A_2^2}{3} \left[ \frac{A_1^2}{3} - \frac{A_1^2}{12N^{*2}} - \frac{3r^2b^2(N^{*2} - 1)}{4A_2^2} \right] \\ - \frac{A_1^2}{3} \left[ \frac{A_2^2}{3} - \frac{A_2^2}{12N^{*2}} - \frac{3r^2b^2(N^{*2} - 1)}{4A_1^2} \right] &= 0. \end{aligned}$$

Thus both assignments lead to the same ex ante payoffs.

(iii) Define  $s \equiv x_1x_2$  and  $t = x_1/x_2$ . Now  $x_1 = \sqrt{st}$  and  $x_2 = \sqrt{s/t}$ . Using  $s$  and  $t$ , under NA assignment equations (8) and (9) can be rewritten as

$$\frac{A_2}{3} - \frac{A_2}{12N_2^{*2}} - \frac{b_2^2st(N_2^{*2} - 1)}{3A_2} = \frac{A_1}{3}t - \frac{A_1}{12N_1^{*2}}t - \frac{b_1^2s(N_1^{*2} - 1)}{3A_1} = \sqrt{t/s}/2. \quad (26)$$

From (26), we can solve for  $t$  as a function of  $s$ :

$$t = \frac{\frac{A_2}{3} - \frac{A_2}{12N_2^{*2}} + \frac{b_1^2s(N_1^{*2}-1)}{3A_1}}{\frac{A_1}{3} - \frac{A_1}{12N_1^{*2}} + \frac{b_2^2s(N_2^{*2}-1)}{3A_2}}.$$

Substituting the above expression for  $t$  into (26) and rearranging, we get an equation in  $s$ :

$$C - D = \frac{1}{2\sqrt{s}} \times \sqrt{C + D + \left(\frac{1}{3} - \frac{1}{N_1^{*2}}\right) \frac{b_1^2s(N_1^{*2} - 1)}{3} + \left(\frac{1}{3} - \frac{1}{N_2^{*2}}\right) \frac{b_2^2s(N_2^{*2} - 1)}{3}}, \quad (27)$$

where  $C = A_1A_2\left(\frac{1}{3} - \frac{1}{N_1^{*2}}\right)\left(\frac{1}{3} - \frac{1}{N_2^{*2}}\right)$  and  $D = \frac{b_1^2b_2^2s^2(N_1^{*2} - 1)(N_2^{*2} - 1)}{9A_1A_2}$ .

Under PA assignment, define  $s' \equiv x'_1x'_2$  and  $t' = x'_1/x'_2$ . Following the same procedure, we can get an equation in  $s'$  similar to (27). Given that  $N_1^* = N_2^{*'}$  and  $N_2^* = N_1^{*'}$ , the equation in  $s'$  is the same as that in  $s$  with the positions of  $b_1$  and  $b_2$  switched and the positions of  $N_1^*$  and  $N_2^*$  switched. Inspecting (27), we see that the equation remains the same if we simply switch  $b_1$  and  $b_2$  and switch  $N_1^*$  and  $N_2^*$ . Therefore, both  $s$  and  $s'$  are defined by the same equation (27), and we must have  $s = s'$ . Thus  $x_1x_2 = x'_1x'_2$ , which implies  $E(\bar{m}_1^2)E(\bar{m}_2^2) = E(\bar{m}_1'^2)E(\bar{m}_2'^2)$ . ■

#### Proof of Proposition 4:

**Proof.** We first show that  $E(\bar{m}_2^2)$  under simultaneous communication is smaller than  $E(\bar{m}_{2D}^2)$  under D1 delegation. From previous results, under simultaneous communication  $N_2^*$  is the largest integer such that  $2|b_2|\frac{A_1}{2E(\bar{m}_1^2)}N_2(N_2 - 1) < A_2$ . Under D1 delegation,  $N_{2D}^*$  is the largest integer such that  $2|b_2 - b_1|\frac{A_1}{2E(\theta_1^2)}N_2(N_2 - 1) < A_2$ . Since  $b_1$  and  $b_2$  have the same sign and

$|b_1| < |b_2|$ ,  $|b_2 - b_1| < |b_2|$ . Moreover,  $E(\theta_1^2) \geq E(\bar{m}_1^2)$ . Therefore,  $N_{2D}^* \geq N_2^*$ . Comparing  $E(\bar{m}_2^2)$  and  $E(\bar{m}_{2D}^2)$ ,

$$\begin{aligned} E(\bar{m}_2^2) &= \frac{A_2^2}{3} - \frac{A_2^2}{12N_2^{*2}} - \frac{b_2^2 A_1^2 (N_2^{*2} - 1)}{12[E(\bar{m}_1^2)]^2}, \\ E(\bar{m}_{2D}^2) &= \frac{A_2^2}{3} - \frac{A_2^2}{12N_{2D}^{*2}} - \frac{(b_2 - b_1)^2 A_1^2 (N_{2D}^{*2} - 1)}{12[E(\theta_1^2)]^2}, \\ E(\bar{m}_{2D}^2) - E(\bar{m}_2^2) &\geq \frac{b_2^2 A_1^2 (N_2^{*2} - 1)}{12[E(\bar{m}_1^2)]^2} - \frac{(b_2 - b_1)^2 A_1^2 (N_{2D}^{*2} - 1)}{12[E(\bar{m}_1^2)]^2} \geq 0; \end{aligned}$$

where the first inequality holds because  $E(\bar{m}_{2D}^2)$  is increasing in  $N_{2D}^*$  and  $N_{2D}^* \geq N_2^*$ , and the second inequality follows from the fact that  $|b_2 - b_1| < |b_2|$  and  $E(\theta_1^2) \geq E(\bar{m}_1^2)$ .

The difference between the DM's ex ante payoffs can be expressed as

$$\begin{aligned} U_{D1}^P - U_{ST}^P &= E(\theta_1^2)E(\bar{m}_{2D}^2) - E(\bar{m}_1^2)E(\bar{m}_2^2) - b_1^2 \\ &\geq E(\bar{m}_2^2)[E(\theta_1^2) - \frac{b_1^2}{E(\bar{m}_2^2)} - E(\bar{m}_1^2)], \end{aligned}$$

where the inequality is due to  $E(\bar{m}_{2D}^2) - E(\bar{m}_2^2) \geq 0$ . Now  $U_{D1}^P - U_{ST}^P > 0$  is equivalent to the term in the bracket being strictly greater than 0. More explicitly,

$$\begin{aligned} E(\theta_1^2) - \frac{b_1^2}{E(\bar{m}_2^2)} - E(\bar{m}_1^2) &= \frac{A_1^2}{12N_1^{*2}} + \frac{b_1^2 A_2^2 (N_1^{*2} - 1)}{12[E(\bar{m}_2^2)]^2} - \frac{b_1^2}{E(\bar{m}_2^2)} \\ &> \frac{b_1^2 A_2^2 (N_1^{*2} - 1)}{12[E(\bar{m}_2^2)]^2} - \frac{b_1^2}{E(\bar{m}_2^2)} \geq \frac{b_1^2}{E(\bar{m}_2^2)} \left( \frac{N_1^{*2} - 1}{4} - 1 \right), \end{aligned}$$

where the second inequality follows from the fact that  $E(\bar{m}_2^2) \leq \frac{A_2^2}{3}$ . From the above expression, it is evident that  $U_{D1}^P - U_{ST}^P > 0$  if  $N_1^* \geq 3$ .

Now consider the case in which  $N_1^* = 2$ . Note that under simultaneous communication, the partition  $a_{1,1} > 0$  satisfies

$$a_{1,1} + a_{1,1} + \frac{2b_1 A_2}{E(\bar{m}_2^2)} = A_1.$$

Given that  $a_{1,1} > 0$ , we have  $\frac{2|b_1|A_2}{E(\bar{m}_2^2)} < A_1$  and

$$\begin{aligned} E(\theta_1^2) - \frac{b_1^2}{E(\bar{m}_2^2)} - E(\bar{m}_1^2) &= \frac{A_1^2}{48} + \frac{b_1^2 A_2^2}{4[E(\bar{m}_2^2)]^2} - \frac{b_1^2}{E(\bar{m}_2^2)} \\ &> \frac{A_2^2 b_1^2}{12E(\bar{m}_2^2)} + \frac{b_1^2 A_2^2}{4[E(\bar{m}_2^2)]^2} - \frac{b_1^2}{E(\bar{m}_2^2)} = \frac{b_1^2 A_2^2}{3[E(\bar{m}_2^2)]^2} - \frac{b_1^2}{E(\bar{m}_2^2)} \geq \frac{b_1^2}{E(\bar{m}_2^2)}(1 - 1) = 0, \end{aligned}$$

where the first inequality follows from  $\frac{2|b_1|A_2}{E(\bar{m}_2^2)} < A_1$  and the second follows from the fact that  $E(\bar{m}_2^2) \leq \frac{A_2^2}{3}$ . Thus  $U_{D1}^P - U_{ST}^P > 0$  if  $N_1^* = 2$ . ■