# Investment Cycles, Strategic Delay, and Self-Reversing Cascades*

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**Abstract**

We study investment cycles and information flows in a model of social learning in which investment returns fluctuate according to a Markov process. In our Waiting Game, agents observe the investment history and a private signal correlated with the current period's investment return. Agents then decide whether to invest in round 1 or to delay their decision to round 2 of the current period. Cascades in which everyone invests and no one invests eventually correct themselves. As compared to the No-Waiting Game with no opportunity for delay, the Waiting Game has shorter investment cascades, longer recessions, and shorter booms. The Waiting Game also has more underinvestment and less overinvestment.

## 1 Introduction

The beliefs of economic agents about investment prospects sometimes receives more media attention than the prospects themselves. The importance of business confidence is particularly salient during the current "Great Recession." Some government officials and some economists speak about the importance of boosting business confidence, to encourage businessmen who are on the fence to undertake projects that would increase employment and pull us out of recession.[1] Not only are beliefs a crucial element, but they differ substantially across agents, due to private information. Beliefs and private information influence economic activity, but at the same time, beliefs endogenously respond to economic activity. Because agents may have an incentive to delay investment opportunities to learn others' information, strategic delay has the potential to affect dramatically the flow of information and the course of economic fluctuations.

This paper studies how the possibility of strategic delay affects the extent to which markets aggregate private information, and investigates the implications for investment cycles. We are the first to study cycles in the context of social learning with strategic delay and a fluctuating investment return. Information cascades, in which either all agents invest or no agents invest,

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[1] A good example is the speech by Lawrence H. Summers (Director of the National Economic Council) at the Brookings Institution, March 13, 2009, in which he argues that "entrepreneurship and the search for opportunity is what we need today."

occur but eventually reverse themselves. Moreover, we find that the option to delay investment leads to shorter investment booms and longer recessions, as compared to the situation without the option to delay.

Our "Waiting Game" (henceforth, WG) extends the seminal work of Chamley and Gale (1994). Specifically, the investment return can be either high or low, and agents privately observe a binary signal that is correlated with the investment return. After observing their signal and the history of transactions, remaining agents simultaneously decide whether to invest right away or wait.[2] The previous papers on herding with endogenous timing assume that the investment return remains constant over time.[3] Chamley and Gale (1994) and Levin and Peck (2008) motivate their analysis with a discussion of how recessions can be prolonged, when many firms receive signals that the investment climate has improved to the point of being profitable, but they delay investment in an attempt to improve their information by observing whether other firms invest. While this motivation is legitimate, there are no cycles in those models. Our innovation is to introduce fluctuations in the investment return, which now evolves according to a Markov process. In each period, a new generation of agents is born, and each agent observes the history of previous investment decisions, receives a signal correlated with the current investment return, and decides whether to invest.

In our WG, there are two rounds, and an agent that does not invest in round 1 observes investment activity in round 1 before deciding whether to invest in round 2. The payoff of investment in the second round is discounted, but the ability to observe market activity gives rise to an option value of waiting. We find that, depending on the beginning-of-period expected investment return, the economy can be in one of four possible regimes in equilibrium: Regime 0 in which no agent invests, Regime 2 in which every agent invests, Regime 1 in which only the agents who receive the high signal invest in round 1, and Regime M, in which agents receiving the low signal wait and agents receiving the high signal mix, investing in round 1 with probability between zero and one. The amount of information aggregation depends crucially on which regime we are in. No private information is revealed in Regime 0 and Regime 2, all the signals are revealed in Regime 1, and partial information is revealed in Regime M. We characterize the equilibrium behavior within a period, based on the agents' beliefs about the investment return, and we characterize the evolution of beliefs as a function of history.

Moscarini, Ottaviani, and Smith (1998) model self-reversing cascades in the literature on herding with exogenous timing.[4] As a benchmark to compare to our WG, we analyze the No-Waiting Game (henceforth, NWG), which is the obvious extension of Moscarini, Ottaviani, and Smith to the case of $n$ agents per period. We see our contributions beyond Moscarini, Ottaviani, and Smith (1998) and Chamley and Gale (1994) as: (i) successfully embedding

[2] See also the extension by Levin and Peck (2008) to two-dimensional signals, in which agents receive a common-value signal, either high or low, and a private-value signal (interpreted as the cost of investment), drawn from a continuous distribution.

[3] See Caplin and Leahy (1994), Gul and Lundholm (1995) and Chari and Kehoe (2004). In Caplin and Leahy (1994), the economy eventually leaves the "business as usual" regime, not because the state could have changed, but because agents accumulate more private information over time. In our model, on the other hand, the economy leaves Regime 0 or Regime 2 because agents become *less* sure of the investment state over time, to the point that different types might choose different actions.

[4] This literature also includes Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992), and Smith and Sorensen (2000).

a changing investment return into a tractable endogenous timing model, (ii) deriving results about long run dynamics, and (iii) comparing the equilibria of the No-Waiting and Waiting Games, to understand the effect of the ability to delay investment on cycles. We show that the range of beliefs giving rise to Regime 2 is narrower in the WG than in the NWG, which implies that a Regime 2 cascade is shorter. We show for the case of the large, persistent economy that the average length of a boom is shorter, and the average length of a recession is longer, in the WG than in the NWG. The WG has less overinvestment (investment when the return is low) and more underinvestment (lack of investment when the return is high) than the NWG. When we depart from the large, persistent economy, our simulations indicate that these long-run properties of the dynamics hold more generally.

There is a huge literature, which we see as complementary, on cycles in models with payoff externalities. Gale (1996) develops a dynamic model that incorporates both delay and cycles generated by payoff externalities. See also Jovanovic (2006). Chamley (1999) and Yang (2006) consider dynamic coordination games, and show that equilibrium cycles and regime switches can occur. Information is asymmetric, but the payoff externality greatly affects the analysis.

The papers by Zeira (1994), Van Nieuwerburgh and Veldkamp (2006), and Veldkamp (2005) look at informational cycles in a setting of symmetric information. In Veldkamp (2005), the knowledge that an agent has invested provides no information, but the outcome of each investment is observed. Thus, each investment is a small experiment that provides information about the common success probability. As a result, more investment activity is more informative, which leads to cycles exhibiting a slow boom and a sudden crash. This is in contrast to our model, in which agents learn nothing when everyone invests. Van Nieuwerburgh and Veldkamp (2006) provide evidence that analysts' forecasts of real GDP are both less accurate and more dispersed near business cycle troughs. Their model explains well why forecasts are less accurate, but has difficulty explaining dispersion because all agents share the same beliefs in their model. Our model can explain both findings. On the other hand, our model cannot explain why cycles exhibit a slow boom and sudden crash.

In our model, business confidence can be interpreted as the expected investment return, which is the key variable that endogenously both responds to and influences activity. We see cycles of euphoria (Regime 2) and fear (Regime 0), in which agents ignore their private information about fundamentals. Most importantly, as the recession continues and the economy transitions from Regime 0 to Regime M, agents with favorable signals believe that investment would be profitable, but with high probability everyone delays, remaining on the fence hoping that someone else will take the lead. In the model, a single agent with the urge to invest can set in motion an avalanche of information flow that ends the recession.[5] We believe that the presence of asymmetric information, and the tendency of agents with favorable information to wait for confirmation, plays an important role in actual business cycles. Thus, our model offers a formal explanation for why recessions are extended due to delay, and why policy makers acting in the

[5] The notion of an urge to invest is reminiscent of the animal spirits literature. See Keynes (1936), Shleifer (1986), Howitt and McAfee (1992), and Francois and Lloyd-Ellis (2003). In the recent literature, animal spirits are equated to an aggregate state of expectation corresponding to one of several equilibria. In our setting, for a given state of expectation and a given equilibrium, an individual agent would have animal spirits if she would receive the same payoff from investing or not, but she invests because of what Keynes termed "a spontaneous urge to action." See also Akerlof and Shiller (2009).

social interest would seek to encourage investment in those circumstances. Furthermore, since agents with favorable signals are indifferent in Regime M, it only takes the smallest nudge to push such an agent off the fence; this could explain why speeches to encourage entrepreneurship could yield beneficial results.

The rest of the paper is laid out as follows. Section 2 contains the benchmark of the NWG, including the equilibrium characterization. In Section 3, the WG is presented and equilibrium is characterized. Section 4 contains analytical results about long-run dynamics of both the NWG and the WG, for the case of the large, persistent economy. Section 5 contains some brief concluding remarks. All of the technical or long proofs are contained in the Appendix.

## 2 The No-Waiting Game

In our benchmark game, each agent has only one opportunity to invest. Time is discrete, $t = 1, 2, ...$, and in each period, there are $n$ agents or potential investors, who live for a single period. The investment return in period $t$ is common to all investors, and is normalized to be either zero or one. We denote the investment return in period $t$ by $S^t$, and assume that it follows a Markov process with persistence parameter $\rho > \frac{1}{2}$. That is, we have

$$\Pr(S^{t+1} = 0|S^t = 0) = \Pr(S^{t+1} = 1|S^t = 1) = \rho,$$

and $\Pr(S^1 = 0) = \Pr(S^1 = 1) = \frac{1}{2}$. There is also a deterministic investment cost, $c$, which is strictly between zero and one and common to all investors. Thus, the realized payoff to an investor in period $t$ is $S^t - c$, and the payoff to an agent that does not invest is 0.

At the beginning of each period, each agent receives a binary private signal correlated with the investment return. These private signals are independent across agents, conditional on the investment return. Denoting the signal of an agent in period $t$ as $s$, we have

$$\Pr(s = 0|S^t = 0) = \Pr(s = 1|S^t = 1) = \alpha,$$

where the parameter $\alpha \in (1/2, 1)$ captures the accuracy of signals. We will refer to agents as either type-0 or type-1, depending on whether they receive the low signal or the high signal. Also, we denote the number of agents who invest in period $t$ as $I^t$, and the history of investments as $h^{t-1} = (I^1, ..., I^{t-1})$ for $t > 1$ and $h^0 = \varnothing$.

The timing of the No-Waiting Game (NWG) is as follows. At the beginning of period $t$, the investment return is realized according the Markov process described above. Each agent observes her signal and the history of past investments, $h^{t-1}$. Then the agents alive in period $t$ simultaneously decide whether to invest or not. Then period $t + 1$ begins with a new generation of agents, and so on.

Since all agents alive in period $t$ observe the same investment history, they share the same initial belief about the investment return (before observing a private signal). Denote the probability that the investment return is high, conditional on the history $h^{t-1}$, as $\mu(h^{t-1})$. We will sometimes suppress the history, and simply refer to the beginning-of-period belief as $\mu$. We denote the probability of the high investment return, conditional on an agent being type-1 or type-0, and conditional on the beginning-of-period belief $\mu$, as

$$\mu_1 \equiv \Pr(S^t = 1|s = 1, \mu) = \frac{1}{1 + (\frac{1-\alpha}{\alpha})(\frac{1-\mu}{\mu})} \tag{1}$$

$$\mu_0 \equiv \Pr(S^t = 1|s = 0, \mu) = \frac{1}{1 + (\frac{\alpha}{1-\alpha})(\frac{1-\mu}{\mu})}. \tag{2}$$

Clearly, $\mu_1 > \mu_0$ since $\alpha > 1/2$, and both $\mu_0$ and $\mu_1$ are increasing in the initial belief $\mu$.

The following proposition shows that the NWG has a Bayesian Nash equilibrium, characterized by cutoffs of initial beliefs,

$$\underline{\mu}^{NW} \equiv \frac{1}{1 + (\frac{\alpha}{1-\alpha})(\frac{1-c}{c})} \text{ and } \overline{\mu}^{NW} \equiv \frac{1}{1 + (\frac{1-\alpha}{\alpha})(\frac{1-c}{c})}, \tag{3}$$

determining whether no one invests, just type-1 agents invest, or all agents invest. To simplify notation, we define the following belief transition function: $f(\mu) \equiv \rho\mu + (1-\rho)(1-\mu)$.

**Proposition 2.1:** *The NWG has a Bayesian Nash equilibrium, characterized as follows.*

*(i) Within-period behavior: If $\mu(h^{t-1}) < \underline{\mu}^{NW}$ holds, then no one invests in period $t$ (Regime 0). If $\underline{\mu}^{NW} \leq \mu(h^{t-1}) < \overline{\mu}^{NW}$ holds, then all of the type-1 agents and none of the type-0 agents invest in period $t$ (Regime 1). If $\overline{\mu}^{NW} \leq \mu(h^{t-1})$ holds, then all agents invest in period $t$ (Regime 2).*

*(ii) Updating of beliefs: If the economy is in Regime 0 or Regime 2 in period $t$, beliefs in period $t+1$ are given by $\mu(h^t) = f(\mu(h^{t-1}))$. If the economy is in Regime 1 in period $t$, beliefs at the end of period $t$ are given by*

$$\mu^{I^t} = \frac{1}{1 + \frac{1-\mu(h^{t-1})}{\mu(h^{t-1})}(\frac{1-\alpha}{\alpha})^{2I^t-n}}, \tag{4}$$

*and beliefs in period $t+1$ are given by $\mu(h^t) = f(\mu^{I^t})$.*

**Proof.** Given any history, a type-1 agent strictly prefers to invest if and only if $\mu_1 - c > 0$, which is equivalent to $\mu > \underline{\mu}^{NW}$. Similarly, a type-0 agent strictly prefers to invest if and only if $\mu_0 - c > 0$, which is equivalent to $\mu > \overline{\mu}^{NW}$. It follows that no agent has an incentive to deviate from their behavior as specified in part (i). When the economy is in Regime 0 or Regime 2, then nothing is learned from activity in period $t$, so beliefs are updated by considering the probability of a change in the investment return, using to Bayes' rule, according to $f(\mu(h^{t-1}))$. When the economy is in Regime 1, the number of type-1 agents is revealed to be $I^t$. By Bayes' rule, the posterior after observing $I^t$ investment can be computed as (4), and the initial belief in period $t+1$ is updated according to $f(\mu^{I^t})$. □

The proof makes clear that the equilibrium characterized in Proposition (2.1) is essentially the unique Bayesian Nash equilibrium of the NWG.[6] It is worth emphasizing that different

[6] The only possibility for multiplicity occurs when beliefs happen to lie exactly on the boundary between regimes. The characterization of behavior and beliefs off the equilibrium path does not affect the incentive to deviate and is therefore unimportant.

regimes give rise to different degrees of information aggregation. Specifically, no information about the current underlying state is revealed in either Regime 0 or Regime 2, where either no one invests or everyone invests. However, in Regime 1, all available information about the investment return is aggregated.

If not for the possibility of a shift in the investment return from period to period, Regime 0 and Regime 2 would lead to traditional information cascades, where subsequent generations do not pay attention to their signals. However, the possiblity of a shift in the investment return causes subsequent generations to update their beliefs, so that the expected investment return rises towards 1/2 in Regime 0, and falls towards 1/2 in Regime 2. Thus, for reasonable parameter values, the equilibrium results in self-reversing information cascades. To make regime switches interesting, we assume $c < \alpha$ throughout the paper.[7]

**Proposition 2.2:** *For the NWG, if $c < 1-\alpha$ holds, the economy starts and remains in Regime 2. If* $1-\alpha < c < \alpha$ holds, the economy starts in Regime 1, and *all Regime 0 and Regime 2 cascades are self-reversing. That is, whenever the economy reaches either Regime 0 or Regime 2, it will leave that regime with probability one.*

Proposition (2.2) allows us to describe the equilibrium regime switches. Consider the case in which all three regimes are possible ($1-\alpha < c < \alpha$).[8] The economy starts in Regime 1, in which all private information is revealed by investment activity. In Regime 1, the number of realized investments, $I^t$, determines the Regime for period $t+1$. In general, there are two cutoffs, such that the economy switches to Regime 2 if $I^t$ exceeds the larger cutoff, and the economy switches to Regime 0 if $I^t$ is below the smaller cutoff. For intermediate values of $I^t$, the economy will remain in Regime 1. Also, because $\underline{\mu}^{NW} < 1/2 < \overline{\mu}^{NW}$ holds, all transitions out of Regime 0 or Regime 2 must go through Regime 1.

# 3 The Waiting Game

In the Waiting Game (WG), agents have an opportunity to postpone investment. For tractability, we assume that every period is divided into two rounds. An agent can either invest in the first round (and remain invested in second round) or postpone the decision until the second round. By waiting until round 2, the payoff will be discounted by a factor, $\delta < 1$. However, the realized investment return remains constant during the two rounds and agents only receive a signal once at the beginning of the first round. Thus, the realized payoff to an investor in period $t$, round 1, is $S^t - c$, and the realized payoff to an investor in period $t$, round 2, is $\delta(S^t - c)$. All the other elements of the model are the same as those in the NWG.

Let $k_1^t$ and $k_2^t$ denote the number of investments in round 1 and round 2 of period $t$, respectively. Thus, $I^t = k_1^t + k_2^t$. Now the history at the beginning of period $t$ is given by $h^{t-1} = (k_1^1, k_2^1, ..., k_1^{t-1}, k_2^{t-1})$. We assume that $k_1^t$ is observed before round 2 begins. By observing $k_1^t$, an agent is able to make more informative decisions.

[7]If $\alpha < c$ holds, the economy starts and remains in Regime 0.

[8]Regime 0 and Regime 2 will be reached in equilibrium, for sufficiently large $\rho$. A necessary condition is $\rho > \max[\overline{\mu}^{NW}, 1-\underline{\mu}^{NW}]$, which is also sufficient as $n \to \infty$.

As in Chamley and Gale (1994) and Levin and Peck (2008), the opportunity to delay investment in round 1 leads to a tradeoff between waiting and investing in round 1: waiting incurs the cost of delay but enables a more informative decision in round 2. As a result, the equilibrium may involve mixing by the type-1 agents. Let $q$ denote the probability that other type-1 agents invest in round 1, and let $\mu_i^{k,q}$ ($\mu^{k,q}$) denote a type-$i$ agent's (an outside observer's) posterior that the investment return is high after observing $k$ investments in round 1. More explicitly, we have

$$\mu_i^{k,q} = \frac{1}{1 + \frac{1-\mu_i}{\mu_i}(\frac{1-\alpha}{\alpha})^k[\frac{1-(1-\alpha)q}{(1-\alpha q)}]^{n-1-k}}. \tag{5}$$

Let $V(\mu_i, q)$ be the expected payoff of a type-$i$ agent, who plans to delay the investment decision until round 2. In particular,

$$V(\mu_i, q) = \delta \sum_{k=0}^{n-1} \max\{\mu_i^{k,q} - c, 0\} \Pr(k_1^t = k | s = i, q, \mu_i).$$

Define $T(k, \mu_i, q)$, which represents the option value of investing in round 2 after observing $k$ investors in round 1, as

$$\begin{aligned} T(k, \mu_i, q) \quad = \quad & \mu_i(1-c)\binom{n-1}{k}(\alpha q)^k(1-\alpha q)^{n-1-k} \\ & -(1-\mu_i)c\binom{n-1}{k}[(1-\alpha)q]^k[1-(1-\alpha)q]^{n-1-k}. \end{aligned} \tag{6}$$

Then $V(\mu_i, q)$ can be rewritten as

$$V(\mu_i, q) = \delta \sum_{k=0}^{n-1} \max\left[0, T(k, \mu_i, q)\right]. \tag{7}$$

**Lemma 3.1:** (i) *The function, $V(\mu_i, q)$, is continuous in $\mu_i$ and $q$. (ii) $V(\mu_i, q)$ is weakly increasing in $\mu_i$ and strictly increasing if $T(n-1, \mu_i, q) > 0$. $V(\mu_i, q)$ is weakly increasing in $q$, and strictly increasing if $T(n-1, \mu_i, q) > 0$ and $T(0, \mu_i, q) < 0$. (iii) The difference between the payoff from investing in round 1 and the payoff from waiting, $\mu_i - c - V(\mu_i, q)$, is strictly increasing in $\mu_i$.*

Lemma 3.1 establishes that the expected payoff from waiting, $V(\mu_i, q)$, is weakly increasing in initial belief, and strictly increasing unless an agent will never invest in round 2. The expected payoff from waiting is weakly increasing in the probability with which type-1 agents invest in round 1, and strictly increasing unless the agent would either never invest or always invest in round 2. This is because more accurate information will be learned in round 2 if type-1 agents invest with a higher probability in round 1. Finally, the advantage, of investing in round 1 over waiting until round 2, is strictly increasing in an agent's belief. The intuition for this last result is that, since the agent is trading off the cost of delay with the option not to invest if unfavorable information is revealed, the option not to invest is more valuable for an agent who has more pessimistic beliefs. This implies that agents have a stronger incentive to wait when the initial belief is low and a weaker incentive to wait when the initial belief is high.

Similar to the NWG, the WG has a Bayesian Nash equilibrium, characterized by cutoffs in initial beliefs, $\underline{\mu}^W$, $\widehat{\mu}^W$, and $\overline{\mu}^W$, determining whether no one invests, type-0 agents do not invest and type-1 agents mix, type-0 agents do not invest and all type-1 agents invest, or all agents invest. To characterize these cutoffs, define $\mu^*$ be the unique value of $\mu_i$ solving

$$\mu_i - c - V(\mu_i, 1) = 0. \tag{8}$$

That is, $\mu^*$ is the belief (after learning one's type) such that an agent is indifferent between investing in round 1 and making a decision in round 2 after learning the types of all agents. A solution to (8) exists, because the expression is continuous, takes the value $-c$ at $\mu_i = 0$, and takes the value $(1-c)(1-\delta) > 0$ at $\mu_i = 1$. Uniqueness follows from the strict monotonicity of the left hand side of (8), according to Lemma 3.1. The cutoffs are defined as follows

$$\underline{\mu}^W = \frac{1}{1+(\frac{\alpha}{1-\alpha})(\frac{1-c}{c})}; \quad \widehat{\mu}^W = \frac{1}{1+(\frac{\alpha}{1-\alpha})(\frac{1-\mu^*}{\mu^*})}; \quad \overline{\mu}^W = \frac{1}{1+(\frac{1-\alpha}{\alpha})(\frac{1-\mu^*}{\mu^*})}. \tag{9}$$

**Proposition 3.2:** *The WG has a Bayesian Nash equilibrium, characterized as follows.*

*(i) Regime 0: If $\mu(h^{t-1}) < \underline{\mu}^W$ holds, then no one invests in period $t$. Beliefs in period $t+1$ are given by $f(\mu(h^{t-1}))$.*

*(ii) Regime M: If $\underline{\mu}^W \leq \mu(h^{t-1}) < \widehat{\mu}^W$ holds, then the type-0 agents wait and the type-1 agents invest in round 1 with probability $q(h^{t-1})$, where $q(h^{t-1})$ is the unique solution to $\mu_1 - c - V(\mu_1, q) = 0$. Based on round-1 investment, if $\mu_1^{k_1^t, q(h^{t-1})} < c$ holds, then no one invests in round 2, and beliefs in period $t+1$ are given by $f(\mu^{k_1^t, q(h^{t-1})})$. If $\mu_0^{k_1^t, q(h^{t-1})} \leq c < \mu_1^{k_1^t, q(h^{t-1})}$ holds, then the remaining type-1 agents (but no type-0 agents) invest in round 2, and beliefs in period $t+1$ are given by $f(\mu^{I^t})$. If $c \leq \mu_0^{k_1^t, q(h^{t-1})}$ holds, then all remaining agents invest in round 2, and beliefs in period $t+1$ are given by $f(\mu^{k_1^t, q(h^{t-1})})$.*

*(iii) Regime 1: If $\widehat{\mu}^W \leq \mu(h^{t-1}) < \overline{\mu}^W$ holds, then the type-0 agents wait and the type-1 agents invest in round 1. Based on round-1 investment, if $\mu_0^{k_1^t, 1} < c$ holds, then no one invests in round 2. If $c \leq \mu_0^{k_1^t, 1}$ holds, then all type-0 agents invest in round 2. Beliefs in period $t+1$ are given by $f(\mu^{k_1^t, 1})$.*

*(iv) Regime 2: If $\overline{\mu}^W \leq \mu(h^{t-1})$ holds, then all agents invest in round 1. Beliefs in period $t+1$ are given by $f(\mu(h^{t-1}))$.*

**Proof.** (i). If $\mu(h^{t-1}) < \underline{\mu}^W$ holds, then investment in round 1 is not profitable for either type of agent, and nothing is learned from observing no investment in round 1.

(ii). If $\underline{\mu}^W \leq \mu(h^{t-1}) < \widehat{\mu}^W$ holds, then investment in round 1 is profitable for a type-1 agent, so that $\mu_1 - c - V(\mu_1, 0) \geq 0$ holds. Also, from the definition of $\widehat{\mu}^W$ in (9), a type-1 agent would prefer to wait if waiting would allow her to learn the signals of all agents, so $\mu_1 - c - V(\mu_1, 1) < 0$ holds. From Lemma (3.1) there must be a unique $q$ such that $\mu_1 - c - V(\mu_1, q) = 0$ holds, which we denote by $q(h^{t-1})$. For agents that do not invest in round 1, the specified behavior is to invest in round 2 if and only if investment is profitable given their beliefs at that point. Clearly, no beneficial deviation in round 2 is possible. Type-1 agents have no incentive to deviate in round 1 since they are indifferent between investing and waiting. Type-0 agents strictly prefer to wait in round 1, since they have more pessimistic beliefs than type-1 agents and Lemma (3.1) implies $\mu_0 - c - V(\mu_0, q(h^{t-1})) < 0$. The end-of-period beliefs are determined from Bayes' rule. If round-1 investment is either small or large, then either no one invests or everyone invests in round 2, so nothing additional is learned. If round-1 investment is intermediate, then only type-1 agents invest in round 2, and the signals of all agents are revealed.

(iii). If $\widehat{\mu}^W \leq \mu(h^{t-1}) < \overline{\mu}^W$ holds, then a type-1 agent would prefer to invest in round 1 rather than wait, and a type-0 agent would rather wait than invest, even if waiting would allow her to learn the signals of all agents. As a result, round-1 investments reveal all the signals.

(iv). If $\overline{\mu}^W \leq \mu(h^{t-1})$ holds, then each agent would prefer to invest in round 1 rather than wait, even if waiting would allow her to learn the signals of all agents, so nothing is learned from behavior in period $t$. Given the end-of-period-$t$ belief $\mu$, the initial belief in period $t+1$ is given by $f(\mu)$. □

As in the NWG, in Regimes 0 and 2 nothing is revealed, while in Regime 1, all of the private information is revealed. In Regime M, information is revealed by round-1 investments and potentially by round-2 investments. In round 1, an increase in mixing probablility $q$ leads to more information being revealed, as the behavior of type-1 agents becomes more different from that of type-0 agents. Since the mixing probability $q$ is increasing in $\mu(h^{t-1})$, the informativeness of market activity is increasing in initial beliefs. The additional information revealed in round 2 depends on the realized investment in round 1, $k_1^t$. If $k_1^t$ is too small or too big, then either no agents or all remaining agents invest in round 2, thus no additional information is revealed. If $k_1^t$ is intermediate, then all remaining type-1 agents invest and no type-0 agents invest in round 2. In this case, all information is revealed by investment activity in two rounds.

**Remark on Multiple Equilibrium.** In general, the WG will have multiple equilibria, as in Chamley (2004). Besides the equilibrium characterized in Proposition (3.2), all of the other type-symmetric equilibria involve an "informational coordination failure." For example, after some histories in which a type-0 agent finds investment profitable but not as profitable as waiting and learning all of the signals, it is consistent with equilibrium for all agents to invest, supported by the self-fulfilling expectation that no information will be revealed.[9] It is worth emphasizing that all of these equilibria are qualitatively similar to the equilibrium of Proposition (3.2), in the following sense. The range of beliefs corresponding to Regime 0 is $[0, \underline{\mu}^W]$ in all equilibria, the same range as in the NWG. For sufficiently high persistence, when beliefs are slightly higher than $\underline{\mu}^W$, then we are in Regime M in all equilibria of the WG, while we are in Regime 1 in the NWG; thus, the option to wait reduces information flow when we are leaving Regime 0, inducing type-1 firms with profitable investment prospects to sit on the fence with high probability, thereby lengthening recessions. The range of beliefs corresponding to Regime 2 is weakly larger in the NWG than in all of the equilibria of the WG; thus, the option to wait

[9] It is also possible to have an equilibrium in which the type-1 agents invest in round 1 and the type-0 agents mix. There is a self-fulfilling expectation that mixing will dilute the information content of observing market activity, to the point that type-0 agents are indifferent between investing and waiting.

weakly speeds the exit of Regime 2, which tends to shorten booms.[10] While it is interesting to note that informational coordination failures are possible here, coordination failures based on payoff externalities have already been featured in the neo-Keynesian literature (see Shleifer (1986)). Given space limitations, we focus on the most informationally efficient equilibrium, the one without coordination failures. In so doing, we make clear that informational coordination failures are in no way necessary to generate the result that strategic delay can lengthen recessions.

**Proposition 3.3:** *The range of beliefs corresponding to Regime 0 is identical for the two games, $\underline{\mu}^W = \underline{\mu}^{NW}$. The range of beliefs corresponding to Regime 2 is strictly smaller for the WG than for the NWG, $\overline{\mu}^W > \overline{\mu}^{NW}$. For the WG, if the economy starts in Regime M or Regime 1 (a sufficient condition is $1-\alpha < c < \alpha$), then all Regime 0 and Regime 2 cascades are self-reversing.*

**Proof.** That $\underline{\mu}^W = \underline{\mu}^{NW}$ holds follows immediately from (3) and (9). From (3) and (9), showing $\overline{\mu}^W > \overline{\mu}^{NW}$ is equivalent to showing $c < \mu^*$. We know that $\mu^*$ solves (8). Also, $\mu_i - c - V(\mu_i, 1)$, evaluated at $\mu_i = c$, is strictly negative, because the initial beliefs yield an expected payoff of zero. From the monotonicity of $\mu_i - c - V(\mu_i, 1)$, $c < \mu^*$ follows.

Clearly, $1 - \alpha < c < \alpha$ is sufficient for the economy to start in Regime M or Regime 1, because investment in round 1 of period 1 is profitable for a type-1 agent (rule out Regime 0), but it is unprofitable for a type-0 agent (rule out Regime 2). The argument that the economy deterministically moves out of Regime 0 or Regime 2 in a finite number of periods, assuming that we start in Regime M or Regime 1, is the same as in the proof of Proposition (2.2). In particular, $\overline{\mu}^W > \overline{\mu}^{NW} > 1/2$ holds, so the economy cannot remain in Regime 2 forever. □

By Proposition (3.3), the set of beliefs for which the economy is in Regime 0 is identical for two games. This is because in both games the cutoff is determined by whether it is profitable for type-1 agents to invest in round 1. On the other hand, the set of beliefs for which the economy is in Regime 2 is strictly smaller when waiting is allowed. Intuitively, the threshold to fall into Regime 1 occurs when investment is unprofitable for type-0 agents in the NWG. However, in the WG it occurs when investment is *less profitable* than waiting and learning all agents' signals. Thus, beliefs do not have to fall all the way to the zero-profit point for type-0 agents.

**Proposition 3.4:** *Suppose $\delta' > \delta''$ holds. (i) In the WG, $\overline{\mu}^W(\delta') > \overline{\mu}^W(\delta'')$, so the range of beliefs corresponding to Regime 2 is strictly larger for a lower discount factor. (ii) In Regime M of the WG, holding $\mu$ constant, $q(\delta') < q(\delta'')$. That is, type-1 agents' investment probability is strictly decreasing in the discount factor.*

**Proof.** Let $\mu^*(\delta')$ and $\mu^*(\delta'')$ be the solutions to equation (8) for $\delta = \delta'$ and $\delta = \delta''$ respectively. Since $\delta' > \delta''$, we have

$$0 = \mu^*(\delta') - c - V(\mu^*(\delta'), 1, \delta') < \mu^*(\delta') - c - \frac{\delta''}{\delta'} V(\mu^*(\delta'), 1, \delta').$$

[10]The lengthening of recessions and increased long-run probability of being in a recession, as compared to the NWG, can be proven for the large, persistent economy when agents are not too patient, and remains a strong conjecture otherwise.

That is, for $\mu_i = \mu^*(\delta')$, the left hand side of (8) is strictly positive for $\delta = \delta''$. Given that the left hand side of (8) is strictly increasing in $\mu_i$, we must have $\mu^*(\delta') > \mu^*(\delta'')$. Following the monotonicity of $\overline{\mu}^W$ in $\mu^*$ by (9), we have $\overline{\mu}^W(\delta') > \overline{\mu}^W(\delta'')$. This proves part (i).

For part (ii), by $\delta' > \delta''$ in Regime M we have

$$0 = \mu_1 - c - V(\mu_1, q(\delta'), \delta') < \mu_1 - c - \frac{\delta''}{\delta'} V(\mu_1, q(\delta'), \delta') = \mu_1 - c - V(\mu_1, q(\delta'), \delta'').$$

By the fact that $V(\cdot, q, \cdot)$ is strictly increasing in $q$, we must have $q(\delta') < q(\delta'')$. $\square$

Proposition (3.4) states the comparative statics regarding the discount factor. Intuitively, as the cost of delay increases when $\delta$ decreases in the WG, agents have less incentive to delay investment. As a result, in Regime M, type-1 agents will invest with a higher probability, given the initial belief. In addition, the range of beliefs for which type-0 agents invest in round 1 (Regime 2) becomes larger.

**Comparing the long-run dynamics of the NWG and the WG.** The long-run dynamics of the WG differ from that of the NWG in two major aspects. First, a smaller region of Regime 2 means that the length of a Regime 2 cascade is shorter in the WG. A second difference is that the economy typically enters Regime M when it emerges from Regime 0 in the WG, but it enters Regime 1 in the NWG. Since information aggregation is less efficient in Regime M than in Regime 1, it will take agents longer, in the WG than in the NWG, to learn that the investment return has switched from low to high.

It is desirable to study the long-run cyclical patterns of boom and recession. We define a boom to be a period in which investment is the predominant activity, i.e., a period in which more than half of the agents invest. We define a recession to be a period in which we are not in a boom. We also define underinvestment as lack of investment in the high return state and overinvestment as investment in the low return state.

Unfortunately, in general it is not possible to derive analytical expressions for the long-run probability of recession and boom. The reason is that the dynamical systems characterized by Propositions (2.1) and (3.2) cannot be represented as finite-state Markov processes, because beliefs depend on the entire history (the transition matrix is not time-homogenous). However, equilibrium trajectories can be computed numerically. Specifically, equilibrium can be computed, history by history, by updating beliefs, determining the regime, computing the investment probability if necessary, and so on. We conduct simulations with the following parameter values: $\alpha = 0.65$, $c = 0.5$, $n = 100$, $\rho = 0.95$, and a time horizon of 1000 periods. The statistics are reported in Table 1.[11]

## Table 1: Simulation Statistics

[11] The average overinvestment (underinvestment) is the probability that the state is low (high) and a random agent invests (does not invest). The welfare is the average total payoff that all agents get in a period. Programming was done using SAS version 9.1. Source code and graphs are available upon request. We are grateful to Hammad Qureshi for doing an excellent job programming the algorithms.

| | NWG | WG ($\delta = 0.9$) | WG ($\delta = 0.7$) | WG ($\delta = 0.5$) |
|---|---|---|---|---|
| Mean of Investment | 54.941 | 33.255 | 38.255 | 43.167 |
| Overinvestment Index | 0.13446 | 0.01584 | 0.04652 | 0.06419 |
| Underinvestment Index | 0.09405 | 0.19229 | 0.17297 | 0.14152 |
| Welfare | 14.0245 | 14.75895 | 13.9886 | 14.5535 |

Several features emerge from Table 1. First, as compared to the NWG, in the WG the economy spends more time in recession and less time in boom. Second, there is less overinvestment and more underinvestment in the WG than in the NWG. Third, as $\delta$ decreases, in the WG the economy spends less time in recession and more time in boom, and there is more overinvestment and less underinvestment. These features generalize across all parameter values for which we have performed simulations. Table 1 also indicates that either the NWG or the WG can provide higher welfare, and that the welfare comparison is non-monotonic in $\delta$.

In the next section, we will analytically study the long-run dynamics of the large and persistent economy. To proceed, we first prove a usefully lemma when $n \to \infty$.[12]

**Lemma 3.5:** *As $n \to \infty$, the regime cutoffs for WG are as follows*

$$\widehat{\mu}^W = \frac{1}{1 + (\frac{\alpha}{1-\alpha})(\frac{1-c}{c})(1-\delta)}, \; \overline{\mu}^W = \frac{1}{1 + (\frac{1-\alpha}{\alpha})(\frac{1-c}{c})(1-\delta)}.$$

**Proof.** As $n \to \infty$, when all type-1 agents invest in round 1, the investment return is revealed, so an agent who waits can invest if and only if the investment return is high. Therefore, we have

$$V(\mu_i, 1) = \delta \mu_i (1-c). \tag{10}$$

From (8) and (10), we have

$$\lim_{n \to \infty} \mu^* = \frac{c}{1 - \delta(1-c)},$$

which, when substituted into (9), allows us to derive the expressions for $\widehat{\mu}^W$ and $\overline{\mu}^W$. □

# 4 The Large, Persistent Economy

All of our simulations indicate that investment cycles have longer recessions and shorter booms in the WG than in the NWG. While we conjecture that this is a general result, the nonstationary nature of the dynamics makes proving this result impossible (at least, for us). However, in this section we derive analytical results for the important case of the *large, persistent economy.* We consider the limiting case in which $n \to \infty$, $\rho \to 1$, and the order of limits is such that $n \to \infty$ for any value of $\rho$. We will argue that this allows us to characterize the equilibria of the NWG and

[12] For large economies in Regime M, it is easy to show that an individual agent's investment probability must converge to zero as $n \to \infty$. However, the probability that no agent invests in the good and bad investment states, $(1 - \alpha q(h^{t-1}))^n$ and $(1 - (1-\alpha) q(h^{t-1}))^n$, converge to a positive limit.

the WG as finite-state Markov processes, from which we can study the long run probabilities of boom and recession. Intuitively, large $n$ allows us to use the law of large numbers, thus getting rid of the randomness of beliefs in Regime 1 and Regime M. Meanwhile, large n also leads to nice cutoffs in the WG as shown by Lemma (3.5). Large $\rho$ makes belief transitions across periods (in either Regime 0 or Regime 2) smoothly and guarantees that Regime 1 and Regime M occur when beliefs first cross the boundaries.

## 4.1 The Large, Persistent NWG

The economy can be represented by $r^{NW} + 2 + b^{NW}$ Markov states. Regime 0 has $r^{NW}$ states (state 1 to state $r^{NW}$). Regime 1 has two states: state $r^{NW} + 1$ (corresponding to a low investment return) and state $r^{NW} + 2$ (corresponding to a high investment return). Regime 2 has $b^{NW}$ states (state $r^{NW} + 3$ to state $r^{NW} + 2 + b^{NW}$).

Since $n \to \infty$, by the law of large numbers, in Regime 1 the investment return is always fully revealed. Therefore, if the economy is in Markov state $r^{NW} + 1$, in the following period the economy will be in state 1 (corresponding to the beginning of Regime 0) for sure, with initial belief $1-\rho$. Similarly, state $r^{NW}+2$ transitions to state $r^{NW}+3$ (corresponding to the beginning of Regime 2) for sure, with inital belief $\rho$. From state 1, the economy transitions through $r^{NW}$ states of Regime 0 deterministically, until state $r^{NW}$ is reached. State $r^{NW}$ transitions to Regime 1 with probability one. Specifically, it transitions to state $r^{NW} + 1$ with probability $1 - \underline{p}^{NW}$, which is the probability of the investment return being low after $r^{NW}$ periods, given that it was low initially; and it transitions to state $r^{NW} + 2$ with probability $\underline{p}^{NW}$. Similarly, once the economy reaches state $r^{NW}+3$, it transitions through $b^{NW}$ states of Regime 2 deterministically, until state $r^{NW}+2+b^{NW}$ is reached. State $r^{NW}+2+b^{NW}$ transitions to either state $r^{NW}+1$ or state $r^{NW} + 2$. The probability of transitioning to state $r^{NW} + 2$, denoted by $\overline{p}^{NW}$, is the probability of the investment return being high after $b^{NW}$ periods, given that it was high initially.

Note that the stationarity of the processes is guaranteed by the law of large numbers, as the investment return is fully revealed in regime 1. We summarize the state transitions in the NWG in the following transition probability matrix $P^{NW}$ (from top to bottom rows and from left to right columns are state 1 to state $r^{NW} + 2 + b^{NW}$).

$$P^{NW} = \begin{bmatrix} & 1 & & & & & & & & \\ & & 1... & & & & & & & \\ & & & 1 & & & & & & \\ & & & & (1-\underline{p}^{NW}) & \underline{p}^{NW} & & & & \\ 1 & & & & & & & & & \\ & & & & & & 1 & & & \\ & & & & & & & 1 & & \\ & & & & & & & & 1... & \\ & & & & & & & & & 1 \\ & & & & (1-\overline{p}^{NW}) & \overline{p}^{NW} & & & & \end{bmatrix}$$

Since all of the Markov states are ergodic, we can calculate the stationary distribution of states $\pi^{NW}$ (a row vector), which is defined as $\pi^{NW} P^{NW} = \pi^{NW}$. Using standard techniques, the fraction of time the economy spends in booms, $\pi_B^{NW}$, can be calculated as

$$\pi_B^{NW} = \frac{(b^{NW}+1)\underline{p}^{NW}}{(r^{NW}+1)(1-\overline{p}^{NW}) + (b^{NW}+1)\underline{p}^{NW}}, \tag{11}$$

with the probability of recession equal to $1-\pi_B^{NW}$.

We can determine $r^{NW}$ ($b^{NW}$) by calculating the number of periods that must pass, in order for the probability of the high investment return to first exceed $\underline{\mu}^{NW}$ (fall below $\overline{\mu}^{NW}$), given that the investment return was low (high) initially. Thus, we have[13]

$$r^{NW} = \frac{\log(1-2\underline{\mu}^{NW})}{\log(2\rho-1)}; \quad b^{NW} = \frac{\log(2\overline{\mu}^{NW}-1)}{\log(2\rho-1)}. \tag{12}$$

Equations (12) indicate that the number of periods in Regime 0 and Regime 2 grow without bound as $\rho \to 1$, but the ratio converges to a well defined limit. We can compute the limiting boom probability by noting that $\lim_{\rho\to 1}(\underline{p}^{NW}) = \underline{\mu}^{NW}$ and $\lim_{\rho\to 1}(\overline{p}^{NW}) = \overline{\mu}^{NW}$, yielding[14]

$$\pi_B^{NW} = \frac{1}{1 + \frac{\log(1-2\underline{\mu}^{NW})}{\log(2\overline{\mu}^{NW}-1)}\left[\frac{1-\overline{\mu}^{NW}}{\underline{\mu}^{NW}}\right]}. \tag{13}$$

## 4.2 The Large, Persistent WG

For the WG of the large, persistent economy, we want to show that the dynamics are approximated by a first-order Markov process. Specifically, there are $r^W+4+b^W$ Markov states. States 1 to $r^W$ correspond to Regime 0. Regime M has two states: state $r^W+1$ (corresponding to a low investment return) and state $r^W+2$ (corresponding to a high investment return). Regime 1 has two states: state $r^W+3$ (corresponding to a low investment return) and state $r^W+4$ (corresponding to a high investment return). States $r^W+5$ to $r^W+4+b^W$ correspond to Regime 2.

The state transitions are as follows. By the law of large numbers, state $r^W+3$ transitions to state 1 for sure, and state $r^W+4$ transitions to state $r^W+5$ for sure. From state 1, the economy goes through the $r^W$ states of Regime 0. State $r^W$ transitions to Regime M: to state $r^W+1$ with probability $(1-\underline{p}^W)$ and to state $r^W+2$ with probability $\underline{p}^W$. From state $r^W+5$, the economy goes through the $b^W$ states of Regime 2. State $r^W+4+b^W$ transitions to Regime 1: to state $r^W+3$ with probability $(1-\overline{p}^W)$ and to state $r^W+4$ with probability $\overline{p}^W$.

Now we specify transitions from Regime M. From state $r^W+1$, with probability $\lambda_0$ the investment return is revealed to be low and the economy transitions to state 1; with probability $(1-\lambda_0)\rho$ it remains in state $r^W+1$ next period, and with probability $(1-\lambda_0)(1-\rho)$ it switches

[13] Of course, the expressions (12) are not generally integers, so $r^{NW}$ and $b^{NW}$ are actually the smallest integers greater than or equal to the corresponding expressions.

[14] Clearly, if $c=1/2$, then $\underline{\mu}^{NW} = 1-\overline{\mu}^{NW}$ and $b^{NW} = r^{NW}$ holds. By (13), we have $\pi_B^{NW} = \pi_R^{NW} = 1/2$. As we will see, this symmetry of investment cycles for the symmetric model with $c=1/2$ does not carry over to the WG.

to state $r^W + 2$. Similarly, from state $r^W + 2$, with probability $\lambda_1$ the investment return is revealed to be high and the economy transitions to state $r^W + 5$; with probability $(1 - \lambda_1)(1 - \rho)$ it switches to state $r^W + 1$ next period, and with probability $(1 - \lambda_1)\rho$ it remains in state $r^W + 2$. The Markov transition matrix $P^W$ is summarized below (from top to bottom rows and from left to right columns are state 1 to state $r^W + 4 + b^W$):

$$P^W = \begin{bmatrix} & 1 & & & & & & & & & & \\ & & 1... & & & & & & & & & \\ & & & 1 & & & & & & & & \\ & & & & (1-\underline{p}^W) & \underline{p}^W & & & & & & \\ \lambda_0 & & & & (1-\lambda_0)\rho & (1-\lambda_0)(1-\rho) & & & & & & \\ & & & & (1-\lambda_1)(1-\rho) & (1-\lambda_1)\rho & & & \lambda_1 & & & \\ 1 & & & & & & & & & & & \\ & & & & & & & & 1 & & & \\ & & & & & & & & & 1 & & \\ & & & & & & & & & & 1... & \\ & & & & & & & & & & & 1 \\ & & & & & & (1-\overline{p}^W) & \overline{p}^W & & & & \end{bmatrix}$$

The law of large numbers guarantees that the investment return is revealed in Regime 1, or if the economy is in Regime M and $k_1^t$ induces all remaining type-1 agents and no type-0 agents to invest in round 2. For $\rho$ sufficiently close to one, it is clear that: when the investment return is revealed to be high (low), then the following period is in Regime 2 (0); during the first period that the economy leaves Regime 2, it is in Regime 1;[15] and during the first period that the economy leaves Regime 0, it is in Regime M. To finish our justification that $P^W$ is an accurate approximation to the transition dynamics, we will show that for $\rho$ sufficiently close to one, (i) in Regime M, if one agent invests in round 1, $k_1^t = 1$, then the remaining type-1 agents and none of the type-0 agents will invest in round 2, thereby revealing the investment return, (ii) in Regime M, the probability of more than one agent investing in round 1, $k_1^t > 1$, is negligible, even as compared to the already small probability that exactly one agent invests, and (iii) in Regime M, the beginning of period beliefs are approximately constant, independent of the history.

It will be useful to define the following notation for the probabilities of aggregate round-1 investment in Regime M, given beginning of period beliefs, $\mu$, and the investment return:

$$prk0(\mu) \equiv pr(k_1^t = k | S^t = 0) \quad \text{and} \quad prk1(\mu) \equiv pr(k_1^t = k | S^t = 1).$$

Also, based on the investment probability $q$ (which implicitly depends on $\mu$), we define $Q(\mu)$ to be the ratio of the probability that no one invests when the investment return is low, to the probability that no one invests when the investment return is high:

$$Q(\mu) \equiv (\frac{1 - (1 - \alpha)q}{1 - \alpha q})^n. \tag{14}$$

[15] Regime 2 and Regime 1 always exists (not bypassed) for fixed $\delta < 1$ and $\rho \to 1$. If $\delta \to 1$ as well, then to ensure Regime 2 and Regime 1 exist the limiting ratio $(1 - \delta)/(1 - \rho)$ must be sufficiently large.

**Lemma 4.1:** *For the large, persistent WG in Regime M, let the beginning of period belief be given by $\mu(h^{t-1}) = \mu < 1$. Then in the limit, as $n \to \infty$, round-1 investment probabilities are characterized as follows.*

$$
\begin{aligned}
pr00(\mu) &= Q(\mu)^{-(1-\alpha)/(2\alpha-1)} \\
pr10(\mu) &= \left(\frac{1-\alpha}{2\alpha-1}\right) Q(\mu)^{-(1-\alpha)/(2\alpha-1)} \log(Q(\mu)) \\
pr01(\mu) &= Q(\mu)^{-\alpha/(2\alpha-1)} \\
pr11(\mu) &= \left(\frac{\alpha}{2\alpha-1}\right) Q(\mu)^{-\alpha/(2\alpha-1)} \log(Q(\mu))
\end{aligned}
$$

*Also, if $n \to \infty$ and $\mu \to \underline{\mu}^W$, then $Q(\mu)$, $pr00(\mu)$, and $pr01(\mu)$ converge to one. Although the probability of $k_1^t = 1$ is converging to zero, the probability of $k_1^t > 1$, relative to the probability of $k_1^t = 1$, also converges to zero.*

If beliefs in Regime M are close to $\underline{\mu}^W$, Lemma (4.1) allows us to ignore the possibility of more than one agent investing in round 1. Also, if $k_1^t = 1$ holds, then from (5) and Lemma (4.1), a type-1 agent finds it profitable to invest and a type-0 agent does not; thus, only the remaining type-1 agents invest in round 2, thereby revealing the investment return. Lemma (4.2) below completes our justification that $P^W$ is an accurate approximation to the transition dynamics.

**Lemma 4.2:** *Consider the limiting large, persistent WG as $n \to \infty$, for fixed $\rho$ close to one. Also assume that $\frac{1}{2} < \delta < 1$ holds. Then for all histories such that $k_1^t = 0$ in Regime M, in the next period $\mu(h^t)$ will be in Regime M, in the interval, $[\underline{\mu}^W, \rho\underline{\mu}^W + (1-\rho)(1-\underline{\mu}^W)]$. Furthermore, the beginning of period beliefs converge to a constant, $\mu^{fix}$, following a sequence of periods of $k_1^t = 0$ in Regime M.*

Standard techniques allow us to compute the steady state distribution of Markov states, $\pi^W$, which is defined as $\pi^W P^W = \pi^W$. To simplify notation, we denote the probability of one of the Regime 0 (2) states as $\pi_0^W$ ($\pi_2^W$), the probability of Regime M, low (high) investment return as $\pi_{M0}^W$ ($\pi_{M1}^W$), and the probability of Regime 1, low (high) investment return as $\pi_{10}^W$ ($\pi_{11}^W$). After much manipulation, we can solve the following equations for $\pi_{M1}^W$ and $\pi_{M0}^W$:

$$1 = \pi_{M1}^W\left(\frac{b^W+1}{1-\overline{p}^W}\lambda_1 + 1 + \lambda_1 r^W\right) + \pi_{M0}^W(1+\lambda_0 r^W), \tag{15}$$

$$\pi_{M1}^W = \frac{1-\rho+\lambda_0[\rho-(1-\underline{p}^W)]}{1-\rho+\lambda_1(\rho-\underline{p}^W)}\pi_{M0}^W. \tag{16}$$

Next, the steady-state probability of being in one of the Markov states corresponding to a boom can be written as

$$\pi_B^W = \frac{b^W+1}{1-\overline{p}^W}\lambda_1 \pi_{M1}^W. \tag{17}$$

Finally, we take limit as $\rho \to 1$, yielding

$$\underline{p}^W = \underline{\mu}^W = \frac{1}{1+(\frac{\alpha}{1-\alpha})(\frac{1-c}{c})}, \quad \overline{p}^W = \overline{\mu}^W = \frac{1}{1+(\frac{1-\alpha}{\alpha})(\frac{1-c}{c})(1-\delta)}, \quad \frac{b^W}{r^W} = \frac{\log(2\,\overline{\mu}^W - 1)}{\log(1-2\underline{\mu}^W)},$$

and $\frac{\lambda_0}{\lambda_1}$, $\frac{1-\rho}{\lambda_1}$, and $\lambda_0 r^W$ are computed in the proof of Proposition (4.3) below. This allows us to compute the limiting boom probability, and the recession probability $1-\pi_B^W$.[16]

In Chamley and Gale (1994), when the number of agents is large and agents are extremely patient, underinvestment is the only source of inefficiency; there is no overinvestment, in the sense that when the investment return is low, the probability that an agent ever invests is zero. This finding seems to contradict our finding for the large, persistent economy, that the economy spends a positive fraction of periods in Regime 2 with all agents investing. In fact, our analysis shows that the Chamley-Gale no-overinvestment result is robust to our setting. In the large, persistent economy as $\delta \to 1$, the length of a single Regime 0 cascade approaches infinity, but the length of a single Regime 2 cascade is bounded.[17] Thus, booms are characterized by a sequence of many Regime 2 cascades, which repeat themselves as long as the investment return remains high. Overinvestment during the entire cycle only occurs during Regime M periods with a single round 1 investor, or (almost certainly) during at most one Regime 2 cascade, which together comprise a negligible fraction of the length of that cycle.

### 4.3 Comparing the Long Run Dynamics Across Games

In this subsection, based on the Markov matrices $P^{NW}$ and $P^W$, we compare the long-run dynamics of the NWG and the WG, for the large, persistent economy. We demonstrate that the expected length of a boom is shorter and the expected length of a recession is longer in the WG. The probability of being in a recession is greater in the WG than in the NWG. We also show that overinvestment (investing when the return is low) is more prevalent in the NWG, and underinvestment (not investing when the return is high) is more prevalent in the WG.

Let $L_B^{NW}$ be the expected length of a boom for the NWG. The actual length of a boom is a random variable that can take one of the values, $b^{NW}+1$, $2(b^{NW}+1)$, $3(b^{NW}+1)$, and so on. The probability that a boom lasts for $k(b^{NW}+1)$ periods is $(1-\overline{p}^{NW})(\overline{p}^{NW})^{k-1}$. Thus, we have

$$L_B^{NW} = (b^{NW}+1)\sum_{k=1}^{\infty} k(1-\overline{p}^{NW})(\overline{p}^{NW})^{k-1} = (b^{NW}+1)\frac{1}{(1-\overline{p}^{NW})} = (b^{NW}+1)\frac{2}{1-(2\rho-1)^{b^{NW}+1}} \tag{18}$$

Similarly, the expected length of recessions $L_R^{NW}$ is

$$L_R^{NW} = (r^{NW}+1)\frac{1}{\underline{p}^{NW}} = (r^{NW}+1)\frac{2}{1-(2\rho-1)^{r^{NW}+1}}. \tag{19}$$

[16] For the large, patient, persistent economy ($\delta \to 1$) the limiting probability of boom is $\pi_B^W = \frac{2c(\alpha-c)}{6c\alpha-2c-4c^2\alpha-(\alpha-c)\log(\frac{\alpha-c}{c+\alpha-2c\alpha})}$.

[17] The limiting ratio $(1-\delta)/(1-\rho)$ must be sufficiently large to ensure that there is a Regime 2.

Although the expected lengths of booms and recessions grow without bound as $\rho \to 1$, the ratios have well defined limits.

Similarly, the expected length of booms in the WG, denoted by $L_B^W$, is given by

$$L_B^W = (b^W + 1) \sum_{k=1}^{\infty} k(1 - \overline{p}^W)(\overline{p}^W)^{k-1} = \frac{b^W + 1}{1 - \overline{p}^W}. \tag{20}$$

Denote the expected length of recessions in the WG by $L_R^W$. It will be convenient to keep track of the expected length of recessions starting from Regime M when the investment return is high, which we denote by $\ell_1$, and starting from Regime M when the investment return is low, which we denote by $\ell_0$. From $P^W$, we have the following equations:

$$L_R^W = (r^W + 1) + \underline{p}^W \ell_1 + (1 - \underline{p}^W)\ell_0 \tag{21}$$
$$\ell_1 = (1 - \lambda_1)[1 + \rho\ell_1 + (1 - \rho)\ell_0] \tag{22}$$
$$\ell_0 = (1 - \lambda_0)[1 + \rho\ell_0 + (1 - \rho)\ell_1] + \lambda_0 L_R^W \tag{23}$$

Solving the above equations simultaneously, we can compute $L_R^W$.

**Proposition 4.3:** *In the large, persistent economy, the expected length of a boom is shorter, and the expected length of a recession is longer, in the WG than in the NWG. That is,* $L_B^W < L_B^{NW}$ *and* $L_R^W > L_R^{NW}$.

The average length of a boom is shorter in the WG than in the NWG, because Regime 2 cascades are shorter with the possibility of waiting. The shorter Regime 2 cascades reduce the chance that the investment return switches from high to low without being detected. The average length of a recession is longer in the WG, because of the presence of Regime M. Suppose the investment return switches to high during a Regime 0 cascade. In the NWG, the economy moves to Regime 1, and the high investment return is revealed, ending the recession. However, in the WG, the economy moves to Regime M, and is likely to stay there for many periods. This directly prolongs the recession, and also allows for the possibility that the investment return switches back to low before the high return is detected.

Next we consider long-run probabilities of boom and recession.

**Proposition 4.4:** *In the large, persistent economy, the long-run probability of being in a recession in the WG is greater than in the NWG,* $\pi_R^W > \pi_R^{NW}$ *and* $\pi_B^W < \pi_B^{NW}$.

The intuition for Proposition (4.4) is the following. The economy is oscillating between boom and recession. Given that the average length of a boom is shorter and the average length of a recession is longer in the WG than in the NWG (Proposition (4.3)), the economy must spend relatively more time in a recession in the WG.

Finally, we show that the possibility of waiting reduces expected overinvestment and increases expected underinvestment. Let $O$ ($U$) be the overinvestment (underinvestment) index, measuring the average investment (lack of investment) when the return is low (high). More specifically,

$$O = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}[\frac{I^t}{n}|S^t = 0]; \;\; U = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}[\frac{n-I^t}{n}|S^t = 1].$$

For the large, persistent NWG, $O^{NW}$ and $U^{NW}$ can be expressed as

$$O^{NW} \;\; = \;\; \{\frac{b^{NW}}{2} - \frac{2\rho-1}{4(1-\rho)}[1-(2\rho-1)^{b^{NW}}]\}\pi_2 + (1-\alpha)\pi_0, \qquad (24)$$

$$U^{NW} \;\; = \;\; \{\frac{r^{NW}}{2} - \frac{2\rho-1}{4(1-\rho)}[1-(2\rho-1)^{r^{NW}}]\}\pi_0 + (1-\alpha)\pi_{11}. \qquad (25)$$

To understand (24), note that the term in braces is the expected number of periods that the investment return is low during $b^{NW}$ consecutive periods of Regime 2. The second term is the probability that the investment return is low during Regime 1, multiplied by the fraction of agents that invest.

For the large and persistent WG, $O^W$ and $U^W$ can be computed as:

$$O^W \;\; = \;\; (1-\alpha)(\lambda_1\pi^W_{M1} + \lambda_0\pi^W_{M0}) + \frac{\lambda_1\pi^W_{M1}}{1-\overline{p}^W}\{\frac{b^W}{2} - \frac{2\rho-1}{4(1-\rho)}[1-(2\rho-1)^{b^W}]\} \qquad (26)$$

$$U^W \;\; = \;\; \pi^W_{M1}[(1-\lambda_1) + \lambda_1(1-\alpha)] + \pi^W_0\{\frac{r^W}{2} - \frac{2\rho-1}{4(1-\rho)}[1-(2\rho-1)^{r^W}]\} \qquad (27)$$

**Proposition 4.5:** *In the large, persistent economy, (i) the overinvestment index is higher for the NWG than for the WG, $O^{NW} > O^W$, and (ii) the underinvestment index is higher for the WG than for the NWG, $U^{NW} < U^W$.*

The reason for part (i) of Proposition (4.5) is that Regime 2 cascades are longer in the NWG than in the WG. Longer Regime 2 cascades on average lead to higher probabilities of overinvestment, since it becomes more likely that the investment return has switched from high to low during a Regime 2 cascade. The result of part (ii) is due to the presence of Regime M in the WG. The presence of Regime M increases the probability that no agent invests, even though the investment return has switched from low to high.

**Proposition 4.6:** *Consider the large, persistent WG with $\delta' > \delta''$. Then we have (i) $L^W_B(\delta') < L^W_B(\delta'')$ and $L^W_R(\delta') = L^W_R(\delta'')$; (ii) $\pi^W_R(\delta') > \pi^W_R(\delta'')$; (iii) $O^W(\delta') < O^W(\delta'')$ and $U^W(\delta') > U^W(\delta'')$.*

Propsoition (4.6) shows that as $\delta$ decreases, the long-run dynamics of the WG become closer to those of the NWG. The underlying intuition is that, as $\delta$ decreases, Regime 2 cascades become longer, because $\overline{\mu}^W$ decreases, which tends to increase the average length of booms. On the other hand, since $\rho$ is very close to one, the transition probability from Regime M to Regime 2 does not depend on $\delta$. This means that the average length of recessions remains the same as $\delta$ changes. Combining these two effects, as $\delta$ decreases the economy spends more and more time in booms rather than in recessions. As a result, the overinvestment probability increases and underinvestment probability decreases. If $\rho$ is strictly less than 1, then a decrease in $\delta$ would lead to a higher investing probability for type-1 agents in Regime M, which in general shortens the average length of recessions.

Proposition (4.6) has some potentially testable implications. We can interpret a larger $\delta$ as a smaller cost of waiting. Then our model would predict that, as the cost of delaying investment decreases, booms will tend to be shorter and recessions will tend to be longer; that the economy will tend to spend less time in booms; and that the averge investment (or output) will decrease.

Our results are consistent with the empirical evidence provided by Van Nieuwerburgh and Veldkamp (2006), that analysts' forecasts of real GDP are both less accurate and more dispersed near business cycle troughs. Suppose that forecasters are the agents (or outsiders who observe a signal) of our large, persistent economy, and forecasts are the conditional beliefs of the investment return. Then measuring the inaccuracy of forecasts as the squared deviation between the forecast and the true investment return, the average inaccuracy is a function of $\alpha$ and the beginning of period belief $\mu$, given by

$$\mu[\alpha(1-\mu_1)^2 + (1-\alpha)(1-\mu_0)^2] + (1-\mu)[\alpha{\mu_0}^2 + (1-\alpha){\mu_1}^2],$$

which is symmetric in $\mu$ with a peak at $\mu = 0.5$. Assume that Regime 0 cascades are longer than Regime 2 cascades, which occurs if the model parameters are symmetric (i.e., $c = 0.5$) or if agents are reasonably patient. Then forecasts are most accurate during Regime 2 (boom), somewhat less accurate during Regime 0 (recession), and far less accurate during Regime M (also recession). In terms of dispersion, type-0 and type-1 forecasters will have almost the same beliefs during Regime 2, somewhat more dispersed beliefs during Regime 0, and far more dispersed beliefs during Regime M.

The WG can also shed some light on the timing of goverment policy to pull the economy out of recession. Suppose the government only observes the history of aggregate investments. Our model predicts that an investment subsidy will be most effective when the economy transitions to Regime M, i.e., after the recession has lasted for some time. At that point, beliefs are reasonably optimistic, and the subsidy needed to induce type-1 agents to invest (with a high probability) is small. Moreover, it is reasonably likely that market activity will reveal the investment return to be high (in a large economy) and that the recession can be ended. On the other hand, if there is a subsidy at the beginning of Regime 0, agents are very pessimistic, which means that a large subsidy rate is required to induce type-1 agents to invest, and the investment return is very likely to be low. As a result, even if type-1 agents are induced to invest, the revealed information is very likely to be bad news, and thus the recession will continue.

## 5 Concluding Remarks

This paper studies how the option to delay investment in order to learn from others' activity affects the process of information aggregation and the course of information cascades. We also make the following contributions to the macro literature on business cycles. First, we have formally demonstrated that the option to delay (or a decrease in the cost of delay) investment will tend to lengthen recessions and shorten booms. The macro literature cites Chamley and Gale (1994) on this point, but their model does not actually contain cycles. Second, we show that asymmetric information gets aggregated by market activity, but imperfectly and in chunks. Only periodically does the economy find itself in a regime in which different types choose different actions, where a lot of information gets revealed. Third, even large economies can find themselves in a situation where investment by a single agent sets in motion a process whereby others invest and information is revealed that either prolongs the recession further or else starts a boom.

Many of the key features of real world economies relevant to cycles are left out of the model. For example, our model does not contain liquidity constraints, collateral, or asset prices. Instead, we isolate the role of information flows and the incentive to delay to learn from market activity, in order to demonstrate as cleanly as possible the effect of delay on lengthening recessions. We believe that the presence of asymmetric information, and the tendency of agents with favorable information to wait for confirmation, plays an important role in actual business cycles. This feature is particularly salient in the current recession, as government officials and economists speak about the importance of boosting business confidence. However, it is useful to discuss how our results would change if we were to expand the model to include these additional features. A simple way to introduce financing constraints would be to assume that investors must borrow the full cost, $c$, in order to undertake their investment. We would now reinterpret the agents of our model as the lenders, who receive signals and decide whether to lend immediately or delay their decision to learn from the decisions of other lenders.[18] If we were to model a procyclical investment cost, $c$, this would reduce both the average length of recessions and the average length of booms. A procyclical cost of waiting tends to increase both the average length of recessions and the average length of booms. This is because a stronger incentive to wait in recession prolongs recessions and a weaker incentive to wait in booms prolongs booms.[19]

We could match any desired ratio of boom length to recession length by adjusting the investment cost. Empirical patterns of investment or GDP for actual economies are usually characterized by steep downturns and short recessions, followed by gradual recoveries and long booms. While our model as currently formulated cannot match this pattern, introducing more than two investment returns could lead to smoother and more realistic looking fluctuations. Also, it would be interesting to extend the model, by allowing some information to flow directly from investment outcomes, perhaps delayed several periods.[20]

[18] With limited liability, potential investors will always want to invest. Introducing collateral would require a new model and not just a reinterpretation. Also, the investment returns, 0 and 1, should be interpreted as the returns to the lenders.

[19] Avery and Zemsky (1998) model a financial market with asset prices in a herding model with exogenous timing. Unfortunately, including asset markets with endogenous timing is far more complicated, and remains an unsolved problem even in simpler models with a constant investment return. For asymmetric movement of asset prices in a model with symmetric information, see Veldkamp (2005).

[20] Veldkamp (2005) has this feature, without the asymmetric information, which generates slow booms and sudden crashes.

# 6 Appendix

**Proof of Proposition 2.2.** Note that in period 1 the initial belief is 1/2, hence the expected investment return is $1-\alpha$ for a type-0 agent and $\alpha$ for a type-1 agent. If $c < 1-\alpha$ holds, both type-0 and type-1 agents will invest in period 1. Moreover, expected returns in period 2 are the same as they were in period 1, and thus everyone invests, and so on.

Now suppose that $1-\alpha < c < \alpha$ holds, and suppose that the economy reaches Regime 0 after some history with initial belief $\mu(h^{t-1}) < \underline{\mu}^{NW}$. We will show that the economy moves out of Regime 0 deterministically, in a finite number of periods. Given $\alpha > c$, we have $\mu(h^{t-1}) < \underline{\mu}^{NW} < 1/2$. Since no information is revealed as long as we are in Regime 0, beliefs evolve according to $f(\mu(h^{t-1}))$, which defines a sequence of increasing beliefs that converges to 1/2. After a finite number of periods, the initial belief must exceed $\underline{\mu}^{NW}$, at which point the economy moves out of Regime 0. The same reasoning implies that the economy moves out of Regime 2 deterministically in a finite number of periods. □

**Proof of Lemma 3.1.** The continuity of $V(\mu_i, q)$ follows immediately from (7) and (6).

From (6), it is easy to verify that $T(k, \mu_i, q)$ is strictly increasing in $k$. Therefore, we have the following three mutually exclusive cases: $T(k, \mu_i, q) \geq 0$ for all $k$, $T(k, \mu_i, q) \leq 0$ for all $k$, or there is a cutoff $k^*$, $0 < k^* < n-1$, such that $T(k, \mu_i, q) \geq 0$ for $k \geq k^*$ and $T(k, \mu_i, q) < 0$ for $k < k^*$. From (6) it is easy to see that $T(k, \mu_i, q)$ is strictly increasing in $\mu_i$. Thus, whenever $T(n-1, \mu_i, q) > 0$ holds, then $V(\mu_i, q)$ is a sum containing at least one positive term, each of which are strictly increasing in $\mu_i$. Hence $V(\mu_i, q)$ is strictly increasing in $\mu_i$.

The fact that $V(\mu_i, q)$ is weakly increasing in $q$, and strictly increasing when $T(0, \mu_i, q) < 0$ holds, is a special case of a result in Chamley and Gale (1994). Consider two investment probabilities $q > q'$. We can think of $q'$ as being generated by type-1 agents first mixing with probability $q$ and then having the "successful" agents mix again with probability $q'/q$. Then the second, garbled information structure is less informative than the first information structure in the sense of Blackwell, and Blackwell's theorem implies that the expected payoff must be weakly higher. When $T(n-1, \mu_i, q) > 0$ and $T(0, \mu_i, q) < 0$ holds, then reducing the mixing probability leads to a strictly lower $V(\mu_i, q)$.

For any $q$, investment in round 1 must yield the same payoff as always investing in round 2, but without discounting. We can therefore express $\mu_i - c$ as follows: $\mu_i - c = \sum_{k=0}^{n-1} T(k, \mu_i, q)$. Let us consider the possibilities. First, suppose that $T(k, \mu_i, q) \geq 0$ holds for all $k$. Then we have

$$\mu_i - c - V(\mu_i, q) = (1-\delta)V(\mu_i, q),$$

which is strictly increasing in $\mu_i$, because $T(n-1, \mu_i, q) > 0$ must hold. Second, suppose that $T(k, \mu_i, q) \leq 0$ holds for all $k$. Then we have $\mu_i - c - V(\mu_i, q) = \mu_i - c$, which is obviously strictly increasing in $\mu_i$. Finally, suppose that there is an interior cutoff, $k^*$. Then we have

$$\begin{aligned}\mu_i - c - V(\mu_i, q) &= \sum_{k=0}^{n-1} T(k, \mu_i, q) - \delta \sum_{k=0}^{n-1} \max[0, T(k, \mu_i, q)] = \\ & \sum_{k=0}^{k^*-1} T(k, \mu_i, q) + (1-\delta) \sum_{k=k^*}^{n-1} T(k, \mu_i, q). \end{aligned} \tag{28}$$

We have already shown that $T(k, \mu_i, q)$ is strictly increasing in $\mu_i$. Thus, $k^*$ remains constant as a function of $\mu_i$, except for a finite number of values at which increasing $\mu_i$ causes $k^*$ to decrease by one. Away from the jump points, (28) is strictly increasing in $\mu_i$, because each term is strictly increasing. At one of the jump points, as $k^*$ decreases from, say, $\kappa + 1$ to $\kappa$, the term $T(\kappa, \mu_i, q)$ moves from the left summation to the right summation in (28). However, since this movement occurs exactly when we have $T(\kappa, \mu_i, q) = 0$, changing the weight on $T(\kappa, \mu_i, q)$ in (28) from 1 to $1 - \delta$ has no effect on the overall expression. □

**Proof of Lemma 4.1.** For a type-1 agent that does not invest in round 1 and observes $k_1^t = 0$, the probability of the high investment return, $\mu_1^{0,q}$, is

$$\mu_1^{0,q} = \frac{1}{1 + \frac{1-\mu}{\mu}(\frac{1-\alpha}{\alpha})(\frac{1-(1-\alpha)q}{1-\alpha q})^{n-1}} = \frac{1}{1 + \frac{1-\mu}{\mu}(\frac{1-\alpha}{\alpha})Q}. \tag{29}$$

The last equality holds because $q$ must be near zero as $n$ approaches infinity.[21] From (14), the investment probability $q$ can be written as $q = \frac{Q^{\frac{1}{n}}-1}{\alpha Q^{\frac{1}{n}}-(1-\alpha)}$. The probability of $k_1^t = 0$ in the high investment return is given by

$$pr01 = (1 - \alpha q)^n = [\frac{\alpha Q^{\frac{1}{n}} - (1-\alpha)}{2\alpha - 1}]^{-n}.$$

Taking the limit of $\log(pr01)$, as $n$ approaches infinity, yields

$$\lim_{n\to\infty} \log(pr01) = -\lim_{n\to\infty} \frac{\log[\frac{\alpha Q^{\frac{1}{n}}-(1-\alpha)}{2\alpha-1}]}{1/n} = -\frac{\alpha}{2\alpha-1}\log Q.$$

Thus, we have

$$\lim_{n\to\infty} pr01 = Q^{-\alpha/(2\alpha-1)}.$$

By a similar computation, one can show that the probability of $k_1^t = 0$ in the low investment return is given by

$$\lim_{n\to\infty} pr00 = Q^{-(1-\alpha)/(2\alpha-1)}.$$

[21] To economize on clutter, we suppress the dependence of $Q$, $pr00$, $pr01$, etc. on beginning of period beliefs, $\mu$.

The probability of $k_1^t = 1$ in the high investment return is given by $n\alpha q(1-\alpha q)^{n-1}$. Therefore, we have

$$\begin{aligned}\lim_{n\to\infty} pr11 &= \lim_{n\to\infty}(n\alpha q(pr01)) = \lim_{n\to\infty}(n\alpha\left[\frac{Q^{\frac{1}{n}}-1}{\alpha Q^{\frac{1}{n}}-(1-\alpha)}\right]pr01) \\ &= \left(\frac{\alpha}{2\alpha-1}\right)Q^{-\alpha/(2\alpha-1)}\log(Q).\end{aligned}$$

By a similar computation, one can show that the probability of $k_1^t = 1$ in the low investment return is given by

$$\lim_{n\to\infty} pr10 = \left(\frac{1-\alpha}{2\alpha-1}\right)Q^{-(1-\alpha)/(2\alpha-1)}\log(Q).$$

If $\mu$ is close to $\underline{\mu}^W$, it follows that round 1 investment is only slightly profitable for a type-1 agent. For a type-1 agent to be indifferent between investing in round 1 and waiting, the option value of not having to invest if $k_1^t = 0$ must be small. It follows that profits from investing in round 2 are only slightly negative if $k_1^t = 0$, and are positive if $k_1^t > 0$. The indifference equation can therefore be written as

$$(1-\delta)(\mu_1 - c)/\delta = Pr(k_1^t = 0|s=1,q,\mu_1)(\mu_1^{0,q} - c),$$

which can be written as

$$\frac{(1-\delta)}{\delta}\left[1-c-c(\frac{1-\alpha}{\alpha})(\frac{1-\mu}{\mu})\right]Q^{\alpha/(2\alpha-1)} = \left[c-1+cQ(\frac{1-\alpha}{\alpha})(\frac{1-\mu}{\mu})\right]. \qquad (30)$$

As $n\to\infty$ and $\mu\to\underline{\mu}^W$, the left side of (30) converges to zero, which implies $Q\to 1$. Therefore, we have $\lim_{\mu\to\underline{\mu}^W}\lim_{n\to\infty} pr01 = 1$ and $\lim_{\mu\to\underline{\mu}^W}\lim_{n\to\infty} pr00 = 1$ hold. Although we have $\lim_{\mu\to\underline{\mu}^W}\lim_{n\to\infty} pr11 = 0$ and $\lim_{\mu\to\underline{\mu}^W}\lim_{n\to\infty} pr10 = 0$, having one agent invest is infinitely more likely than having more than one agent invest. To see this, note that

$$\lim_{\mu\to\underline{\mu}^W}\lim_{n\to\infty}\left(\sum_{k=2}^{n} prk1\right) = 1 - \lim_{\mu\to\underline{\mu}^W}\lim_{n\to\infty} pr11 - \lim_{\mu\to\underline{\mu}^W}\lim_{n\to\infty} pr01 = 1 - \lim_{\mu\to\underline{\mu}^W}\lim_{n\to\infty} pr11 - Q^{-\frac{\alpha}{2\alpha-1}}.$$

Thus, we have

$$\begin{aligned}\lim_{\mu\to\underline{\mu}^W}\lim_{n\to\infty}\left(\frac{\sum_{k=2}^{n} prk1}{pr11}\right) &= \lim_{Q\to 1}[\frac{1-Q^{-\frac{\alpha}{2\alpha-1}}}{\frac{\alpha}{2\alpha-1}Q^{-\frac{\alpha}{2\alpha-1}}\log Q}] - 1 = \lim_{Q\to 1}[\frac{Q^{\frac{\alpha}{2\alpha-1}}-1}{\frac{\alpha}{2\alpha-1}\log Q}] - 1 \\ &= \lim_{Q\to 1}\frac{\frac{\alpha}{2\alpha-1}Q^{\frac{\alpha}{2\alpha-1}-1}}{\frac{\alpha}{2\alpha-1}\frac{1}{Q}} - 1 = 0.\end{aligned}$$

A similar calculation yields

$$\lim_{n\to\infty}\left(\frac{\sum_{k=2}^{n} prk0}{pr10}\right) = 0,$$

which completes the proof. □

**Proof of Lemma 4.2.** During the first period that the economy emerges from Regime 0 into Regime M, clearly beliefs must satisfy $\mu \in [\underline{\mu}^W, \rho\underline{\mu}^W + (1-\rho)(1-\underline{\mu}^W)]$. For beginning of period $t$ beliefs in this interval, consider the mapping to beginning of period $t+1$ beliefs, $\Psi(\mu)$, based on the equilibrium mixing probability and outcome $k_1^t = 0$. For $\rho$ sufficiently close to one, from the proof of Lemma (4.1) the equilibrium mixing condition is (30). Denoting the left hand side of (30) as LHS, we have

$$Q = \frac{LHS + 1 - c}{c(\frac{1-\alpha}{\alpha})(\frac{1-\mu}{\mu})}. \tag{31}$$

Using (5) and (31), $\mu^{0,q}$ can be simplified to

$$\mu^{0,q} = \frac{1}{1 + (1+D)(\frac{1-c}{c})(\frac{\alpha}{1-\alpha}) - D(\frac{1-\mu}{\mu})}, \tag{32}$$

where $D = \frac{(1-\delta)}{\delta} Q^{\alpha/(2\alpha-1)}$ is governed by the discount factor, since $Q^{\alpha/(2\alpha-1)}$ is nearly one for all $\mu \in [\underline{\mu}^W, \rho\underline{\mu}^W + (1-\rho)(1-\underline{\mu}^W)]$. Since no one invests in round 2 when $k_1^t = 0$, beliefs at the beginning of period $t+1$ are given by

$$\Psi(\mu) = \frac{2\rho - 1}{1 + (1+D)(\frac{1-c}{c})(\frac{\alpha}{1-\alpha}) - D(\frac{1-\mu}{\mu})} + 1 - \rho. \tag{33}$$

It is straightforward to check that $\Psi(\underline{\mu}^W) > \underline{\mu}^W$ and $\Psi(\rho\underline{\mu}^W + (1-\rho)(1-\underline{\mu}^W)) < \rho\underline{\mu}^W + (1-\rho)(1-\underline{\mu}^W)$ hold, so $\Psi$ must have a fixed point within the interval, which we denote by $\mu^{fix}$. From (33), the slope of the mapping is $-D(2\rho-1)$. Therefore, for $\delta > \frac{1}{2}$ and $\rho$ close to one, it can be shown that beliefs converge to $\mu^{fix}$ over time (in an oscillatory fashion) and remain within the interval, $[\underline{\mu}^W, \rho\underline{\mu}^W + (1-\rho)(1-\underline{\mu}^W)]$, as long as no one invests in round 1.[22] $\square$

**Proof of Proposition 4.3.** Since $r^W = r^{NW}$ and $\underline{p}^W = \underline{p}^{NW}$, we will drop the superscripts without causing confusion. It will also be convenient to adopt the shorthand notation,

$$z = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1-c}{c}\right), \tag{34}$$

$z > 1$ by $\alpha > c$. From (12), we have

$$\lim_{\rho\to 1}(r+1)(1-\rho) = \lim_{\rho\to 1} r(1-\rho) = \lim_{\rho\to 1} \frac{(1-\rho)\log[\frac{z-1}{z+1}]}{\log(2\rho-1)} = \frac{1}{2}\log\frac{1+z}{z-1}. \tag{35}$$

From Lemma (4.1), we have[23]

$$\lim_{\rho\to 1}\frac{\lambda_0}{\lambda_1} = \lim_{\rho\to 1}\frac{pr10(\mu^{fix})}{pr11(\mu^{fix})} = \frac{1-\alpha}{\alpha}, \tag{36}$$

$$\lim_{\rho\to 1}\frac{(1-\rho)}{\lambda_1} = \lim_{\rho\to 1}\frac{1-\rho}{1-Q(\mu^{fix})^{\frac{-\alpha}{2\alpha-1}}} = \lim_{\rho\to 1}\frac{-1}{\frac{\alpha}{2\alpha-1}Q'(\mu^{fix})} = \frac{2\alpha-1}{\alpha}\frac{z}{z^2-1}. \tag{37}$$

[22] To demonstrate that we remain within the interval (and therefore do not drop out of Regime M), solve the quadratic equation based on (33) for $\mu^{fix}$, then show that $(\mu^{fix} - \underline{\mu}^W)/(\rho\underline{\mu}^W + (1-\rho)(1-\underline{\mu}^W) - \mu^{fix})$ is greater in absolute value than the slope of $\Psi$. Computations were performed using Maple 10.

[23] In (37), we use l'Hopital's rule and analytically evaluate $\lim_{\rho\to 1}[Q'(\mu^{fix})]$ using Maple 10 software.

By using the fact that $\lambda_1$ goes to 0 as $\rho$ goes to 1 and (35)-(37) , we have

$$\lim_{\rho\to 1} r\lambda_1 = \lim_{\rho\to 1}(r+1)\lambda_1 = \lim_{\rho\to 1}(r+1)(1-\rho)\frac{\lambda_1}{1-\rho} = \frac{1}{2}\frac{\alpha}{2\alpha-1}\frac{z^2-1}{z}\log\frac{z+1}{z-1}, \quad (38)$$

$$\lim_{\rho\to 1} r\lambda_0 = \lim_{\rho\to 1} r\lambda_1\frac{\lambda_0}{\lambda_1} = \frac{1}{2}\frac{1-\alpha}{2\alpha-1}\frac{z^2-1}{z}\log\frac{z+1}{z-1}, \quad (39)$$

$$\lim_{\rho\to 1}\frac{\pi^W_{M0}}{\pi^W_{M1}} = \lim_{\rho\to 1}\frac{\frac{(1-\rho)}{\lambda_1}+(1-\frac{1}{1+z})}{\frac{(1-\rho)}{\lambda_1}+\frac{\lambda_0}{\lambda_1}\frac{1}{1+z}} = \frac{1+\frac{\alpha}{2\alpha-1}(z-1)}{1+\frac{1-\alpha}{2\alpha-1}\frac{z-1}{z}}. \quad (40)$$

By (18) and (20), we have

$$L^{NW}_B - L^W_B = \frac{b^{NW}+1}{1-\overline{p}^{NW}} - \frac{b^W+1}{1-\overline{p}^W}. \quad (41)$$

To show that (41) is positive, we define the function, $p^{10}(b) \equiv \frac{1}{2} - \frac{1}{2}(2\rho-1)^{b+1}$, which is the probability of the investment return switching from high to low after $b$ periods. Note that $p^{10}(b^{NW}) = 1-\overline{p}^{NW}$ and $p^{10}(b^W) = 1-\overline{p}^W$ hold. We will now show that $\frac{b+1}{p^{10}(b)}$ is increasing in $b$. We have

$$\begin{aligned} & \frac{b+1}{p^{10}(b)} - \frac{b}{p^{10}(b-1)} \propto [1-(b+1)(2\rho-1)^b + b(2\rho-1)^{b+1}] \\ = & \ [1-(2\rho-1)]\{[1+(2\rho-1)+...+(2\rho-1)^b] - (b+1)(2\rho-1)^b]\} \\ > & \ [1-(2\rho-1)]\{(b+1)(2\rho-1)^b - (b+1)(2\rho-1)^b\} = 0. \end{aligned} \quad (42)$$

Thus, $\frac{b+1}{p^{10}(b)}$ is increasing in $b$, so we have $L^{NW}_B > L^W_B$.

To show $L^W_R > L^{NW}_R$, first recall that $L^{NW}_R = \frac{r+1}{\underline{p}}$ holds. By (21), (22), and (23), $L^W_R - L^{NW}_R$ can be simplified to

$$\begin{aligned} L^W_R - L^{NW}_R \propto \ & \underline{p}^2(1-\lambda_1)[\lambda_0 + (1-\lambda_0)\frac{(1-\rho)\lambda_1}{1-\rho+\rho\lambda_1}] + (1-\lambda_0)\underline{p}[1-\rho+\rho\lambda_1-\underline{p}\lambda_1](2-\frac{\lambda_1}{1-\rho+\rho\lambda_1}) \\ & -(r+1)(1-\rho)[\lambda_1(1-\lambda_0)(1-\underline{p}) - \lambda_0(1-\lambda_1)\underline{p}]. \end{aligned}$$

Since $\frac{\lambda_1}{1-\rho+\rho\lambda_1} < 1$ holds, to show $L^{NW}_R < L^W_R$, it is sufficient to show that the third term is smaller than the first term in the above expression, which is implied by the following condition:

$$(r+1)(1-\rho)[1-\frac{\underline{p}}{1-\underline{p}}\frac{1-\lambda_1}{1-\lambda_0}\frac{\lambda_0}{\lambda_1}] \le \underline{p} + \frac{\underline{p}}{1-\underline{p}}\frac{1-\lambda_1}{\lambda_1}(1-\rho) + \frac{\underline{p}^2}{1-\underline{p}}\frac{1-\lambda_1}{1-\lambda_0}\frac{\lambda_0}{\lambda_1}. \quad (43)$$

Using the limits (36)-(40), when $\rho$ converges to 1, (43) becomes

$$\begin{aligned} \frac{1}{2}(1-\frac{1-\alpha}{\alpha}\frac{1}{z})\log\frac{1+z}{z-1} \ & \le \ \frac{1}{1+z} + \frac{1}{z^2-1}\frac{2\alpha-1}{\alpha} + \frac{1}{z(z+1)}\frac{1-\alpha}{\alpha} \\ & \Leftrightarrow \ \frac{1}{2}\log\frac{1+z}{z-1} \le \frac{1}{z^2-1}\frac{z^2-\frac{1-\alpha}{\alpha}}{z-\frac{1-\alpha}{\alpha}}. \end{aligned}$$

since $z > 1$ holds, the following inequality is sufficient to show $L_R^{NW} < L_R^W$:

$$\frac{2z}{z^2-1} - \log\frac{1+z}{z-1} \geq 0. \tag{44}$$

Given $z$ is bounded, (44) holds. To verify the condition, the derivative of the expression with respect to $z$ is $\frac{-4}{(z^2-1)^2}$, which is negative, so the left side of (44) is decreasing in $z$. Moreover, we have $\lim_{z\to\infty}[\frac{2z}{z^2-1} - \log\frac{1+z}{z-1}] = 0$. Therefore, inequality (44) holds. □

**Proof of Proposition 4.4.** To establish part (i), it is sufficient to show

$$\lim_{\rho\to 1}\frac{\pi_R^{NW}}{\pi_B^{NW}} < \lim_{\rho\to 1}\frac{\pi_R^W}{\pi_B^W}, \tag{45}$$

because $\pi_R^{NW} + \pi_B^{NW} = 1$ and $\pi_R^W + \pi_B^W = 1$ hold.

By (11) and (17) we have

$$\begin{aligned}
\lim_{\rho\to 1}\frac{\pi_R^{NW}}{\pi_B^{NW}} &= \lim_{\rho\to 1}\frac{\frac{r+1}{\underline{p}}}{\frac{b^{NW}+1}{1-\overline{p}^{NW}}} = \lim_{\rho\to 1}\frac{\frac{r+1}{\underline{p}}\lambda_1}{\frac{b^{NW}+1}{1-\overline{p}^{NW}}\lambda_1} \quad \text{and} \\
\lim_{\rho\to 1}\frac{\pi_R^W}{\pi_B^W} &= \lim_{\rho\to 1}\frac{(r\lambda_1+1)+(1+r\lambda_0)\frac{\pi_{M0}^W}{\pi_{M1}^W}}{\frac{b^W+1}{1-\overline{p}^W}\lambda_1} > \lim_{\rho\to 1}\frac{(r\lambda_1+1)+(1+r\lambda_0)\frac{\pi_{M0}^W}{\pi_{M1}^W}}{\frac{b^{NW}+1}{1-\overline{p}^{NW}}\lambda_1}.
\end{aligned} \tag{46}$$

Inequality (46) comes from the fact that $\frac{b^W+1}{1-\overline{p}^W} < \frac{b^{NW}+1}{1-\overline{p}^{NW}}$ holds. To show (45), it is sufficient to show

$$\lim_{\rho\to 1}(r\lambda_1+1)+(1+r\lambda_0)\frac{\pi_{M0}^W}{\pi_{M1}^W} \geq \lim_{\rho\to 1}\frac{r+1}{\underline{p}}\lambda_1. \tag{47}$$

Using the previous limiting results (36)-(40), inequality (47) is equivalent to

$$4+2\frac{1-\alpha}{2\alpha-1}\frac{z-1}{z}+2\frac{\alpha}{2\alpha-1}(z-1)+\frac{1-\alpha}{2\alpha-1}\frac{z^2-1}{z}\log\frac{z+1}{z-1}-\frac{\alpha}{2\alpha-1}(z^2-1)\log\frac{z+1}{z-1} \geq 0. \tag{48}$$

It is easy to verify that inequality (48) holds if the following condition holds

$$\frac{1}{2}\log\frac{z+1}{z-1} - \frac{1}{z-1} < 0. \tag{49}$$

To see (49) holds, note that the derivative with respect to $z$ is positive, which implies that the left side of (49) is increasing in $z$. Moreover, we have $\lim_{z\to\infty}[\frac{1}{2}\log\frac{z+1}{z-1} - \frac{1}{z-1}] = 0$. Therefore, (49) holds. □

**Proof of Proposition 4.5.** First, define the function, $A(b)$, by

$$A(b) \equiv \frac{1}{b}\{\frac{b}{2} - \frac{2\rho-1}{4(1-\rho)}[1-(2\rho-1)^b]\} = \frac{1}{2} - \frac{2\rho-1}{4(1-\rho)}\frac{[1-(2\rho-1)^b]}{b}.$$

That is, $A(b)$ is the probability that the investment return is low during one of the $b$ periods of Regime 2, chosen at random, and $A(r)$ is the probability that the investment return is high during one of the $r$ periods of Regime 0, chosen at random. We show that $A(\cdot)$ is an increasing function. To see this, it is sufficient to show that $\frac{[1-(2\rho-1)^b]}{b}$ is decreasing in $b$. This condition is satisfied, since we have

$$\frac{1-(2\rho-1)^b}{b} - \frac{1-(2\rho-1)^{b+1}}{b+1} \propto [1-(2\rho-1)^{b+1} - (b+1)(2\rho-1)^b(1-(2\rho-1))] > 0,$$

where the last inequality follows from (42).

By (26) and the fact that both $\lambda_1$ and $\lambda_0$ go to 0 as $\rho$ goes to 1, we have

$$\lim_{\rho\to 1} O^W = \lim_{\rho\to 1} \frac{b^w+1}{1-\overline{p}^W}\lambda_1\pi^W_{M1}A(b^W) = \lim_{\rho\to 1}\pi^W_B A(b^W). \tag{50}$$

By (24) and the fact that $r$ and $b^{NW}$ go to infinity as $\rho$ goes to 1, we have

$$\lim_{\rho\to 1} O^{NW} = \lim_{\rho\to 1} A(b^{NW})\frac{(b^{NW}+1)\underline{p}}{(r+1)\overline{p}^{NW}+(b^{NW}+1)\underline{p}} = \lim_{\rho\to 1}\pi^{NW}_B A(b^{NW}). \tag{51}$$

Now we compare (50) and (51). From Proposition (4.4), we have $\lim_{\rho\to 1}\pi^{NW}_B > \lim_{\rho\to 1}\pi^W_B$. And by the fact $b^{NW} > b^W$, we have $A(b^{NW}) > A(b^W)$. Therefore, $\lim_{\rho\to 1} O^W < \lim_{\rho\to 1} O^{NW}$. This proves part (i).

Now we show part (ii). By (25), we have

$$\lim_{\rho\to 1} U^{NW} = \lim_{\rho\to 1}\frac{r\overline{p}^{NW}}{(r+1)\overline{p}^{NW}+(b^{NW}+1)\underline{p}}A(r) = \lim_{\rho\to 1}\pi^{NW}_R A(r). \tag{52}$$

On the other hand, by (27), we have

$$\begin{aligned}
\lim_{\rho\to 1} U^W &= \lim_{\rho\to 1}\pi^W_{M1} + (r\lambda_1\pi^W_{M1} + r\lambda_0\pi^W_{M0})A(r) \\
&= \lim_{\rho\to 1}[(r\lambda_1+1)\pi^W_{M1} + (r\lambda_0+1)\pi^W_{M0}]A(r) + [1-A(r)]\pi^W_{M1} - A(r)\pi^W_{M0} \\
&= \lim_{\rho\to 1}\pi^W_R A(r) + \lim_{\rho\to 1}[1-A(r)]\pi^W_{M1} - A(r)\pi^W_{M0}, \qquad (53)
\end{aligned}$$

where the last equality follows from (17). Now we compare (52) and (53). Since by Proposition (4.3), $\lim_{\rho\to 1}\pi^W_R > \lim_{\rho\to 1}\pi^{NW}_R$ holds, the following condition is sufficient to show $\lim_{\rho\to 1}(U^W - U^{NW}) > 0$:

$$\lim_{\rho\to 1}[1-A(r)]\pi_{M1} - A(r)\pi_{M0} \geq 0, \tag{54}$$

After using (40) and simplifying, we can rewrite (54) as

$$\frac{1}{2}\left(\frac{1-\alpha}{2\alpha-1}\right)\frac{z-1}{z} + \frac{1}{(z+1)\log\frac{z+1}{z-1}}[2 + \frac{1-\alpha}{2\alpha-1}\frac{z-1}{z} + \frac{\alpha}{2\alpha-1}(z-1)] - \frac{1}{2}\frac{\alpha}{2\alpha-1}(z-1) \geq 0.$$

The above inequality holds since $\frac{1}{2}\log\frac{z+1}{z-1} < \frac{1}{z-1}$, by (49). Therefore, inequality (54) holds. □

**Proof of Proposition 4.6.** By Proposition (3.5), we have $\overline{\mu}^W(\delta') > \overline{\mu}^W(\delta'')$. This implies that $b^W(\delta') < b^W(\delta'')$. On the other hand, $r^W$ does not depend on $\delta$. The limit $\lim_{\rho\to 1}\pi^W_{M1}/\pi^W_{M0}$ does not depend on $\delta$ either (see (40)). Following the proof of Proposition 4.3, We have

$$L^W_B(\delta') - L^W_B(\delta'') = \frac{b^W(\delta')+1}{1-\overline{p}^W(\delta')} - \frac{b^W(\delta'')+1}{1-\overline{p}^W(\delta'')} < 0.$$

Inspecting the proof of Proposition (4.3), one can see that $L^W_R$ does not depend on $\delta$, thus $L^W_R(\delta') = L^W_R(\delta'')$. This proves part (i).

To show part (ii), it is sufficient to show that

$$\lim_{\rho\to 1}\frac{\pi^W_R(\delta')}{\pi^W_B(\delta')} > \lim_{\rho\to 1}\frac{\pi^W_R(\delta'')}{\pi^W_B(\delta'')},$$

because $\pi^W_R + \pi^W_B = 1$ hold. By the proof in Proposition (4.4),

$$\lim_{\rho\to 1}\frac{\pi^W_R(\delta')}{\pi^W_B(\delta')} - \lim_{\rho\to 1}\frac{\pi^W_R(\delta'')}{\pi^W_B(\delta'')} = \lim_{\rho\to 1}\frac{(r\lambda_1+1)+(1+r\lambda_0)\frac{\pi^W_{M0}}{\pi^W_{M1}}}{\lambda_1}\left[\frac{1}{\frac{b^W(\delta')+1}{1-\overline{p}^W(\delta')}} - \frac{1}{\frac{b^W(\delta'')+1}{1-\overline{p}^W(\delta'')}}\right] > 0.$$

The last inequality follows part (i) and $\lim_{\rho\to 1}\pi^W_{M1}/\pi^W_{M0}$ does not depend on $\delta$.

By the proof in Proposition (4.5), $\lim_{\rho\to 1} O^W(\delta) = \lim_{\rho\to 1}\pi^W_B(\delta)A(b^W(\delta))$. Moreover, $A(b^W)$ is increasing in $b^W$. Given that $b^W(\delta') < b^W(\delta'')$, we have $A(b^W(\delta')) < A(b^W(\delta''))$. In addition, from part (ii) we have $\pi^W_B(\delta') < \pi^W_B(\delta'')$. Thereforefore, $\lim_{\rho\to 1} O^W(\delta') < \lim_{\rho\to 1} O^W(\delta'')$.

Similarly,

$$\lim_{\rho\to 1} U^W(\delta) = \lim_{\rho\to 1}\pi^W_{M1}(\delta) + [r\lambda_1\pi^W_{M1}(\delta) + r\lambda_0 k\pi^W_{M1}(\delta)]A(r),$$

where $k \equiv \frac{\pi^W_{M0}}{\pi^W_{M1}}$. Note that when $\rho$ goes to 1, by (37)-(40) the limits of $r\lambda_1$, $r\lambda_0$ and $k$ do not depend on $\delta$. Therefore, to show $U^W(\delta') > U^W(\delta'')$ it is sufficient to show that $\lim_{\rho\to 1}\pi^W_{M1}(\delta') > \lim_{\rho\to 1}\pi^W_{M1}(\delta'')$. By (15) and (16),

$$\lim_{\rho\to 1}\pi^W_{M1}(\delta) = \frac{1}{\lim_{\rho\to 1}[\frac{b^W(\delta)+1}{1-\overline{p}^W(\delta)}\lambda_1 + 1 + k + r\lambda_1 + rk\lambda_0]}.$$

Since $\frac{b^W(\delta)+1}{1-\overline{p}^W(\delta)}$ is decreasing in $\delta$, we have $\lim_{\rho\to 1}\pi^W_{M1}(\delta') > \lim_{\rho\to 1}\pi^W_{M1}(\delta'')$. This proves part (iii).

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