# Short-lived Teams with Mutual Monitoring*

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**Abstract**

We show that the free-riding problem in short-lived teams is not as severe as previously thought. Two critical conditions are: team members can observe each other's effort periodically, which makes mutual monitoring possible; technology is convex (increasing marginal returns) or has a "completion benefit." In principal-agent settings, mutual monitoring reduces the necessary wage payment to induce efforts, thus making team production relatively more attractive. In partnership settings, mutual monitoring can reduce the range of inefficiency. The more convex is the technology, and the more frequently team members interact with each other, the more effective mutual monitoring is in discouraging free-riding.

Key words: Teams, Free-riding, Mutual Monitoring, Increasing marginal return
JEL: D23, J41, L23

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# 1 Introduction

Classical works on team production (Alchian and Demsetz, 1972; Holmstrom, 1982) concluded that team production is usually inefficient because of the well-known free-riding problem. But teams have been increasingly popular in the business world. According to Osterman's (1994) survey, work teams are present in 54.5 percent of American establishments. Similarly, Dumain (1994) estimated that about two-thirds of U.S. firms use work teams.

Several recent papers tried to reconcile the mismatch between theory and empirical evidence. Essentially being static models, the classical works ignored two important features of team production. First, the same team may last longer than one period; in other words, team members may interact repeatedly. Second, team members have a great chance to observe each other's effort or contribution to the team. Combining these two features, the possibility of mutual monitoring or peer sanction arises: if one team member shirks in one period, other team members can punish him by shirking in later periods. In an infinitely repeated game setting, Che and Yoo (2001) (CY henceforth) showed that mutual monitoring can provide implicit incentive to team members, thus alleviating the classical free-riding problem in team production.[1]

However, the model of CY has two problems. Theoretically, the required punishment, which is a grim trigger strategy, to support "work" equilibrium is not so credible. Suppose one team member shirks in some period. According to the grim trigger strategy, all the members of the team are going to shirk in all later periods. But this will affect the output realizations of the team. Since the principal can observe the output realization in each period, he will soon infer that the "work" equilibrium has broken down and the team is in punishment phase. Because the punishment (shirking) is also costly to the principal, he would have incentive to dissolve the team. But this possibility would make the required punishment (grim trigger strategy) non-credible, thus destroying the "work" equilibrium.[2]

In terms of practical application, the results of CY only apply to teams that have the potential to last forever. For finitely repeated team production, backward induction would destroy the "work equilibrium."[3] In real business world, a lot of teams are short-lived. Typical examples are problem-solving teams: a team is usually formed when a specific problem arises, and it is dissolved right after the problem is solved. These kinds of short-lived teams are not only popular

[1] Their model is in a principal-agent setting and the same team (at least has the potential to) lasts for infinite periods. At the end of each period, the principal only observes the outcome of the team production but not each team member's effort. On the other hand, at the end of each period each team memebr observes other members' effort in that period.

[2] Actually, all the repeated team production (with or without a principal, finitely or infinitely repeated) models have this problem. Off the equilibrium path, the team either dissolves voluntarily (if outside option is more attractive than staying in the punishment phase) or is dissolved by the principal. And this in turn makes trigger strategy non-credible and limits the effectiveness of mutual monitoring.

[3] One can change the model to a finite-horizon repeated team production with imcomplete information, but this still requires the horizon to be "longer enough" to induce "work" equilibrium in early periods.

among law firms, consulting firms and medical group partnerships, but also are increasingly popular among manufacturing or service firms (Osterman (1994); Boning, Ichniowski, and Shaw (2001)). Apparently, CY cannot explain the popularity of short-lived teams. Moreover, team regrouping is a common practice in business, mainly due to the ever-changing business environment.

This motivates us to study the role of mutual monitoring in short-lived teams. Actually, even for a team which is formed to implement a specific project (dissolves right after the project is implemented), the free-riding problem is not as severe as previously thought. Due to the very nature of a team (team members work together and interact frequently), team members might observe each other's effort made during the implementation of the project. For instance, team members usually need to communicate intermittently to carry out the project. To make the idea clear, consider the following example. A team of two members is formed to implement a project in two weeks. In every weekend, they get together to discuss the development of the project. During the discussion, they can at least infer whether the other member has exerted some effort in the first period. A more concrete example is coauthorship: two authors need to interact frequently to write a paper, and by interaction they can know each other's effort level. Therefore, mutual monitoring is still possible for short-lived teams.

To fix the idea, consider a team of two members to implement a project in two weeks. At the end of the first week, each member observes the other member's effort level in the first week.[4] Then, both members might make their effort level in the second week conditional on the observed effort levels of the first week. In particular, both of them may adopt a trigger strategy: make effort in the second week if and only if both members have made effort in the first week. This threat of mutual sanction has the potential to deter free-riding in the first week. How about the second week? Since it is the last week, mutual monitoring does not alleviate the free-riding problem because there is no future punishment. If the cost of effort is linear and the return of effort is linear as well, then mutual monitoring does not help at all: in the last week, teams members' incentive to free-ride is as strong as their incentive to free-ride in both weeks when effort is not observable.

However, if the marginal return of the project is increasing in total effort ( the value of the project is an increasing and convex function of the degree of completeness of the project), or the project has a "completion benefit," then mutual monitoring can effectively alleviate the free-riding problem. To see this, if both members exert effort in the first week, the marginal return of effort in the second week is higher; as a result, both members have less incentive to free-ride in the second week. In short, mutual monitoring discourages free-riding in the first week by possible punishment, while increasing marginal return alleviates free-riding problem in the last week as total effort accumulates on the equilibrium path.

Increasing marginal return or "completion benefit" is usually an essential feature of some complex projects. Consider the previous example of coauthorship. The value of a paper is usually

[4] You can interpret that they get toghater at the end of the first week to discuss the development of the project.

an increasing and convex function of the completeness of the paper: when a paper is just half finished, it almost has no value; when it is eighty percent finished, it can be presented in some conference or workshop; when it is completed, it can be sent for publication. Next consider a team developing a new drug. This project at least exhibits "completion benefit": compared to the situation where a new drug is not developed, the development of the new drug creates additional values. In general, for more complex projects the values of the projects are usually more sensitive to the completeness of the projects. Therefore, it is natural to think that the more complex is the project, the bigger the "completion benefit" or the more convex is the total return function.

Thus, we reach the main conclusion of the paper: even for short-lived teams, free-riding problem is not as severe as people previously thought. Mutual monitoring still can alleviate the free-riding problem, if the production technology exhibits increasing marginal return or has a "completion benefit." This applies to both principal-agent settings and partnerships. In principal-agent settings, mutual monitoring reduces the required wage payment to motivate team members to exert effort, thus increasing the attractiveness of teams relative to individual production. In partnerships, by discouraging free-riding, mutual monitoring makes the implementation of the "efficient" action easier, thus making the "range of inefficiency" smaller.

The paper also yields some interesting comparative statics results. First, other things being equal, as the marginal return of effort increases faster or the "completion benefit" becomes bigger, mutual monitoring becomes more effective in alleviating the free-riding problem. Since we argued before that usually complex projects have a bigger "completion benefit" or their marginal returns increase faster, this model predicts that team production should be more common for the implementation of complex projects. Some empirical evidence (Osterman, 1994; Boning, Ichniowski and Shaw, 2001) supports this prediction: they found that technologically more complex projects are more likely to adopt teams.

Second, as team members interact more often, the free-riding problem becomes less severe. The intuition is fairly simple. More frequent interaction is equivalent to the decrease of period length and the increase of the number of periods. With the presence of mutual monitoring, a team member can at most free-ride for one period. The gain from doing so is decreasing as the number of periods increases, because each period's effort is smaller. In the limit (as team members interact infinitely often or period length shrinks to zero), free-riding problem can be completely solved by mutual monitoring.

This paper complements and extends CY. Note that the mutual monitoring equilibrium in our model is free of the criticism mentioned before. Since a short-lived team just implements one project once, (inefficient) perpetual punishment is neither feasible nor necessary. Arya, Fellingham and Glover (1997) studied the role of mutual monitoring in a team which repeats a task twice. They show that implicit incentive can be provided by mutual monitoring in the first period.[5] But

[5] Their model is also subject to the criticisim mentioned earlier, since in the second period the principal has

our model studies teams which just implement a task once. Miller (1997) examined the efficiency in partnerships with joint monitoring. He demonstrated that when one partner can observe at least one of other partners' effort, the efficient effort vector can be sustained in an equilibrium by sharing rule that exhibits budget balance and limited liability. But the equilibrium he derived is based on the observing partner's report conditional on his observation. Strausz (1999) studied efficiency in sequential partnership, in which partners choose effort sequentially and later movers observe all the effort choices of previous partners. Under this setting, he proved that efficient production can be induced by a sharing rule which is budget-balanced and does not rely on unlimited liability. This paper is also related with the literature of optimal task design and compensation scheme in multi-agents setting (Itoh, 1991; Rob and Zemsky, 2002).

Another related literature is dynamic contributions to a public good (Admati and Perry, 1991; Marx and Matthews, 2000, MM henceforth). The basic logic of this literature is that if the contribution is divided into consecutive small contributions, each member can try the other's good faith for a small price, and the free-riding problem can be alleviated. Admati and Perry (1991) studied a sequential model, in which two players move alternately. Our model is more closely related with MM, in which the stage contribution game is a simultaneous move game. Except for different applications, the main differences between our model and MM are the following. First, in our model, the number of periods to implement a project is fixed, while in MM the completion date is endogenous. This makes their setting more flexible in terms of the completion of the project: the project can always be completed as long as the contribution horizon is long enough; the only inefficiency is the delay of the project. In our model (partnership setting), the fixed completion date might cause the non-completion of an efficient project. Second, in MM the contribution is divisible, which may cause a dynamic free-riding problem: current player can free-ride on future players. In our model, the contribution of effort in each period is indivisible, thus dynamic free-riding is not an issue. Our model is also related to the class of monotone games with positive spillovers studied by Gale (2001).

The rest of the paper is organized as follows. Section 2 studies a two-period model in a principal-agent setting. Also using a two-period model, Section 3 shows how mutual monitoring can reduce the inefficiency of partnership. A generalization to $n$-period model is provided in Section 4. The last section concludes.

## 2 Principal-agent setting

A manager (principal) hired two workers (agents) (1 and 2) to implement some projects. The projects can be either two independent ones ($a$ and $b$) or a combined one. If the manager chooses

incentive to dissolve the team off the equilibrium path. But the authors did discuss about the principal's ability to commit to a two-period contract.

two individual projects, each project is assigned to one worker. Instead, if the manager chooses the combined project, two workers are assigned and they work as a team. The outcome of each project can be either a success $s$, or a failure $f$. If succeeds, each individual project yields a revenue of $S > 0$, and the combined project yields a revenue of $2S$. Each project yields revenue 0 if it fails. Each project needs two periods to complete. One way to interpret the time structure is the following: each project has a deadline, after which no further effort can affect the outcome; from the inception to the deadline, if the combined project is chosen, two players might meet once to discuss the ongoing of the project. Denote worker $i$'s effort in period $j$ as $e_i^j$. $e_i^j \in \{0, 1\}$; in other words, he can either work or shirk.[6] The probability of success of each project depends on relevant agents' effort. In particular, we assume that the probability of success of individual project $i$ is

$$q(s_i|e_i) \equiv q_{e_i^1+e_i^2}$$

$e_i$ is the effort vector of agent $i$. Depending on effort choices, there are three possibilities: $q_0$, $q_1$ and $q_2$. Note that the probability of success only depends on the sum of the efforts. We assume that

$$q_2 > q_1 > q_0 \geq 0 \text{ and } q_2 - q_1 > q_1 - q_0 \tag{1}$$

That is, the probability of success is increasing in total effort; moreover, the marginal return of exerting effort is increasing in total effort.

For the combined project, the probability of success is

$$p(s|e) = p_{\sum_{i,j} e_i^j}$$

Depending on effort choices, there are five possibilities: $p_0$, $p_1$, $p_2$, $p_3$, $p_4$. Note that the probability success only depends on the total efforts,[7] hence the efforts of agent 1 and agent 2 are perfect substitutes in some sense. We further assume that

$$\begin{aligned} p_4 &> p_3 > p_2 > p_1 > p_0 \geq 0 \text{ and} \\ p_4 - p_3 &> p_3 - p_2 > p_2 - p_1 > p_1 - p_0 \end{aligned} \tag{2}$$

In words, the probability of success is increasing in total effort; moreover, the marginal return of effort is increasing in total effort. That is, the marginal increase in the probability of success by exerting an extra unit of effort is increasing in the total effort accumulated. Put it another way, the probability of success is an increasing and convex function of total effort. One can interpret

[6] The assumption that effort choices are discrete is for the sake of simplicity in exposition. This assumption rules out the possibility that one worker may overwork in one period to make up the shirking happened in previous periods. An alternative way of modelling is that effort choice is continuous but the cost of efforts is convex, this can also rule out workers' incentive to overwork.

[7] The probability of success does not depend on the distribution of efforts. This assumption captures the bite of free-riding problem. If the probability of success does depend on the distribution of efforts between two workers, workers incentive to free-ride would decrease, as illustrated by Legos and Matthews (1993).

assumption (2) in the following way. Total effort determines the degree of completeness of the project in a linear way, with full effort producing a fully completed project. But the value of the project is increasing and convex in the degree of completeness.[8]

The manager does not observe workers' effort choice in any case; the only verifiable signal is the outcome of the projects. So the compensation contract or wage contract can only be conditional on the outcome of the projects. The form of contracts is $w = (w^s, w^f)$, with $w^f$ denotes the wage payment paid by the manager to the worker if the project fails, and $w^s$ is the wage payment when the project succeeds. Workers are subject to limited liability, which requires requires $w^f \geq 0$ and $w^s \geq 0$. The workers' outside options are normalized to be 0. All the parties are risk neutral. Given contract $w$, worker $i$'s utility is,[9]

$$u_i(w, e) = Ew - \frac{e_i^1 + e_i^2}{2}$$

And the manager's payoff is,

$$\pi(w, e) = (q_a + q_b)S - E(w_1 + w_2)$$

if two individual projects are chosen; and it is

$$\pi(w, e) = 2pS - E(w_1 + w_2)$$

if the combined project is chosen.

We assume that $S$ is big enough such that the efficient actions are full effort level: exerting efforts in both periods for both workers. Specifically, we assume that

$$(q_2 - q_0)S - 1 > 0 \quad (3)$$
$$2(p_4 - p_0)S - 2 > 0 \quad (4)$$

Given these two assumptions, the efficient action for each individual project is exerting effort in both periods, because

$$(q_2 S - 1) - (q_1 S - \frac{1}{2}) = (q_2 - q_1)S - \frac{1}{2} > \frac{1}{2}(q_2 - q_0) - \frac{1}{2} > 0$$

The first inequality comes from assumption (1) and the second inequality comes from (3). Similarly, the efficient action for the combined project is exerting effort in both periods for both

[8] The results of the paper hold under a more general technology. Define the expected return of the combined project as $V(e)$, where $e$ is the total effort. What we require is that $V(e)$ exhibits increasing returns: $V_4 - V_3 > V_3 - V_2 > V_2 - V_1 > V_1 - V_0$. In the paper, we fix $S$ and only $p$ is a function of $e$, which is a special case of the general technology.

[9] Here we assume that there is no discounting. The main reason is that we focus primarily on short-term teams. Adding discounting would not change the qualitative results of the paper.

workers, since for $k = 1, 2, 3$

$$(2p_4S - 2) - (2p_kS - \frac{k}{2}) = 2(p_4 - p_k)S - \frac{1}{2}(4-k) > 2\frac{4-k}{4}(p_4 - p_0)S - \frac{1}{2}(4-k) > 0$$

The first inequality comes from assumption (2) and the second one follows (4).

There are two decisions to be made for the manager. The first one is which projects to choose. We call this task design decision. The second one is to choose contracts to induce effort. If the combined project exhibits no synergy, that is $p_4 \leq q_2$, then choosing the combined project is always dominated by choosing the individual projects. This is because of the free-riding problem associated with team production. Therefore, a necessary condition for the manager to choose the combined project is $p_4 > q_2$. In other words, team production exhibits synergy. We make this assumption in this section. It also follows that the overall efficient action is for the manager to choose the combined project and for both workers to exert effort in both periods.

An immediate observation is that $w^f = 0$ in the optimal contracts. Given that workers are risk neutral, the difference between $w^s - w^f$ should be as big as possible to motivate workers to exert effort. By the assumption of limited liability, $w^f = 0$ in optimal contracts. To ease exposition, we drop $w^f$ from now on.

## 2.1 The optimal contract for individual projects

If two individual projects are chosen, we assume that workers cannot observe each other's effort choice.[10] Thus two projects are completely independent. Suppose that the manager intends to implement the full effort for a project, the optimal contract is characterized by

$$\begin{aligned} & \min_{w^s} q_2 w^s \\ \text{subject to} \quad : \quad & q_2 w^s - 1 \geq q_1 w^s - \frac{1}{2} \Leftrightarrow w^s \geq \frac{1}{2(q_2 - q_1)} \\ q_2 w^s - 1 \quad \geq \quad & q_0 w^s \Leftrightarrow w^s \geq \frac{1}{(q_2 - q_0)} \\ q_2 w^s - 1 \quad \geq \quad & 0 \end{aligned}$$

The first two inequalities are incentive compatibility constraints and the third one is participation constraint. Note that only the second constraint is binding in the optimal contract, because $2(q_2 - q_1) > (q_2 - q_0)$ by assumption (1). Therefore, the optimal contract is $w^{s*} = \frac{1}{q_2 - q_0}$. Under this contract, the manager's payoff is $q_2(S - \frac{1}{q_2 - q_0})$. And his total payoff from two projects are $2q_2(S - \frac{1}{q_2 - q_0})$. Notice that it is positive by assumption (3).

[10] If two workers work in the same office, they may occasionally observe whether the other worker shirked. But since two projects are independent, this kind of observation might not be perfect. In Che and Yoo (2001), they assume that workers may observe each other's effort even if they are assigned two independent jobs.

If the manager wants to implement effort $(0,0)$, it is obvious that the optimal contract is $w=0$, and the manager's payoff is $q_0 S$. On the other hand, it can be shown that $(0,1)$ or $(1,0)$ cannot be implemented. The reason is that exerting partial effort is always dominated either by exerting full effort or by exerting zero effort, given that marginal return of exerting effort is increasing. The relative magnitudes of $q_2(S-\frac{1}{q_2-q_0})$ and $q_0 S$ are ambiguous. As a result, the efficient action may not be implemented. The efficient action can be implemented if and only if

$$S \geq \frac{q_2}{(q_2-q_0)^2}$$

which is obviously satisfied if $q_0 = 0$.

### 2.2 Team production without mutual monitoring

Now suppose that the combined project is chosen but workers do not observe each other's effort. We are only interested in the implementation of the efficient action, i.e., both workers exert effort in both periods. The optimal contract is characterized by

$$\begin{aligned} & \min_{w^s} p_4 w^s \\ \text{subject to} \quad : \quad & p_4 w^s - 1 \geq p_3 w^s - \frac{1}{2} \Leftrightarrow w^s \geq \frac{1}{2(p_4-p_3)} \\ p_4 w^s - 1 \quad \geq \quad & p_2 w^s \Leftrightarrow w^s \geq \frac{1}{(p_4-p_2)} \\ p_4 w^s - 1 \quad \geq \quad & 0 \end{aligned}$$

The first two inequalities are incentive compatibility constraints and the third one is participation constraint. The first constraint makes sure that each worker has no incentive to deviate in single period given the other worker exerts effort in both periods. The second condition ensures that each worker has no incentive to deviate in both periods. Note that only the second constraint is binding in the optimal contract, since $2(p_4-p_3) > (p_4-p_2)$ by assumption (2). Therefore, the optimal contract is $w^{s*} = \frac{1}{p_4-p_2}$. Under this contract, the manager's payoff is $2p_4(S-\frac{1}{p_4-p_2})$. Notice that it may be negative.

### 2.3 Team production with mutual monitoring

In this subsection, we study team production when workers can observe each other's effort choice at the end of each period. We are interested in how to implement the efficient action, that is, both workers exert efforts in both periods. Since each worker's effort in the first period is observable to the other worker at the beginning of the second period, they might be conditional their effort choices in the second period on the observed first period's efforts. In particular, we are interested in the following strategy profile: each worker exerts effort in the first period, and he exerts effort in the second period if and only if both workers exerted effort in the first period. That is, workers may “sanction” each other in second period if some worker shirks in the first period. Note that this is not a repeated game: worker’s incentive to exert effort in the second period depending on the effort accumulated in the first period. But this game belongs to multi-stage game, so one-stage deviation principle still applies (Fudenberg and Tirole, 1991). The strategy profile specified before constitutes a subgame perfect equilibrium under a contract $w$ if and only if the following conditions are met.

**Condition 1** *Each worker should have no incentive to deviate on the equilibrium path.*

This requires

$$p_4 w^s - 1 \geq p_1 w^s \tag{5}$$

$$p_4 w^s - 1 \geq p_3 w^s - \frac{1}{2} \tag{6}$$

The first condition says following equilibrium strategy is better than shirking in the first period. The second condition makes sure that following equilibrium strategy is better than shirking in the second period.

**Condition 2** *Each worker should have no incentive to deviate off the equilibrium path.*

This requires

$$\begin{aligned} p_2 w^s - 1 &\leq p_1 w^s - \frac{1}{2} & (7) \\ p_1 w^s - \frac{1}{2} &\leq p_0 w^s & (8) \end{aligned}$$

The first constraint assures that each worker will not exert effort in the second period if one worker deviates in the first period, while the second one makes sure that each worker will not exert effort in the second period if both workers deviate in the first period.[11]

The optimal contract for the manager is characterized by

$$\begin{aligned} & \min_{w^s} p_4 w^s \\ \text{subject to} \quad &: \quad (5), (6), (7), (8) \text{ and} \\ p_4 w^s - 1 &\geq 0 \end{aligned}$$

[11] Actually, we ignored one possibility: if one worker shirks in the first period (the other worker works in the first period), he should have no incenitve to work in the second period. Mathematically, $p^2 w^s - \frac{1}{2} \leq p^1 w^s$. But this constraint is the same as (10).

**Proposition 1** *When mutual monitoring is feasible, the optimal contract that induces full effort level is* $w^{s*} = \max\{\frac{1}{(p_4-p_1)}, \frac{1}{2(p_4-p_3)}\}$.

**Proof.** The first observation is that (8) is redundant if (7) is satisfied, since $p_2 - p_1 > p_1 - p_0$ by assumption (2). The participation constraint is also redundant given (5) is satisfied. Then the programming problem can be simplified as

$$\min_{w^s} p_4 w^s$$

$$(p_4 - p_1)w^s \geq 1 \tag{9}$$

$$(p_4 - p_3)w^s \geq \frac{1}{2} \tag{10}$$

$$(p_2 - p_1)w^s \leq \frac{1}{2} \tag{11}$$

From assumption (2), we have $(p_4 - p_3) > (p_2 - p_1)$ and $(p_4 - p_1) > (p_4 - p_2) > 2(p_2 - p_1)$. Therefore, there exists a $w^s$ such that (9), (10) and (11) are all satisfied. The optimal contract depends on the magnitudes of $(p_4 - p_1)$ and $(p_4 - p_3)$. If $(p_4 - p_1) \geq 2(p_4 - p_3)$, then the optimal wage $w^{s*} = \frac{1}{2(p_4-p_3)}$. On the other hand, if $(p_4 - p_1) < 2(p_4 - p_3)$, then $w^{s*} = \frac{1}{(p_4-p_1)}$. ■

Note that under the optimal contract, the manager's payoff is $2p_4(S - \max\{\frac{1}{(p_4-p_1)}, \frac{1}{2(p_4-p_3)}\})$.

## 2.4 Comparisons

Relative to the case without mutual monitoring, team production with mutual monitoring yields a higher payoff for the manager. To see this, note that both $(p_4 - p_1)$ and $2(p_4 - p_3)$ are greater than $(p_4 - p_2)$ by assumption (2). Therefore,

$$\max\{\frac{1}{(p_4 - p_1)}, \frac{1}{2(p_4 - p_3)}\} < \frac{1}{p_4 - p_2}$$

That is, the expected wage payment with mutual monitoring is strictly less than those without mutual monitoring. The expected revenue is the same since both induce the full effort level. So the manager is strictly better off with the presence of mutual monitoring. Thus, we have the following proposition.

**Proposition 2** *Mutual monitoring strictly improves the Principal's payoff, thus making team production relatively more attractive.*

The main intuition for this result is that, with mutual monitoring workers may sanction each other if one worker shirks in the first period. As a result, a worker can at most free-ride for one period with mutual monitoring. On the other hand, a worker can free-ride for two periods without mutual monitoring. This difference is reflected in the difference between the term $\frac{1}{(p_4-p_1)}$ and the term $\frac{1}{p_4-p_2}$. In the second period, workers' incentive to free-ride on the equilibrium path decreases, since the marginal return of exerting effort increases as total effort accumulated. This is reflected in the difference between $\frac{1}{2(p_4-p_3)}$ and $\frac{1}{p_4-p_2}$. On the other hand, in the second period worker's incentive to shirk increases off the equilibrium path, which in turn makes the trigger strategy credible. Overall, mutual monitoring reduces the gain from free-riding, thus the manager needs to pay less to induce the "work" equilibrium.

Note that the condition of increasing marginal returns plays an important role. Suppose that the marginal return of effort is constant, that is, $p_k = p_0 + \frac{k}{4}(p_4 - p_0)$ for $k = 1, 2, 3$. In other words, the probability of success is linear in total effort. Then $2(p_4 - p_3) = (p_4 - p_2) < (p_4 - p_1)$. Therefore, $\max\{\frac{1}{(p_4-p_1)}, \frac{1}{2(p_4-p_3)}\} = \frac{1}{p_4-p_2}$. Thus, the presence of mutual monitoring does not reduce the necessary wage payment to motivate workers. The main reason for this is that without increasing marginal returns, worker's incentive to free-ride in the second (last) period is as strong as his incentive to free-ride in both periods without mutual monitoring. Mutual monitoring can discourage free-riding in the first period, but it cannot discourage free-riding in the last period since there is no future punishment anymore. Increasing marginal return ensures that worker's incentive to free-ride decreases on the equilibrium path as early effort accumulates.

It is natural to think that as the technology becomes more convex (marginal return increases faster), the expected wage payment in the optimal contract decreases. To prove that formally, we first define the criterion to compare two technologies. Fix $p_4$ and $p_0$, for two set of $p_{i'}$and $p_{i''}$ $(i = 1, 2, 3)$ satisfying assumption (2), if

$$(p_4 - p_{i'}) > (p_4 - p_{i''}) \text{ for all } i = 1, 2, 3$$

then we say that technology $p_{i'}$ is more convex than technology $p_{i''}$. It follows immediately that

$$\max\{\frac{1}{(p_4 - p_{1'})}, \frac{1}{2(p_4 - p_{3'})}\} < \max\{\frac{1}{(p_4 - p_{1''})}, \frac{1}{2(p_4 - p_{3''})}\}$$

That is, the necessary wage payment to motivate workers decreases as the technology becomes more convex. This result is pretty intuitive. As the technology becomes more convex, shirking in the first period becomes less attractive, which is reflected in the decrease of the term $\frac{1}{(p_4-p_1)}$. Moreover, shirking in the second period on the equilibrium path is more costly, since doing so means forego a bigger increase in return; this effect is embodied by the decrease in term $\frac{1}{2(p_4-p_3)}$. Overall, mutual monitoring becomes more effective in deterring shirking as the technology becomes more convex. The following corollary summarizes the comparative statics result:

**Corollary 1** : *As the technology becomes more convex, the expected wage that is necessary to motivate workers in team production decreases.*

Since complex projects usually have more convex technology, the corollary implies that more complex projects should be more likely to adopt teams. The results of two empirical studies support this prediction. Based on the survey data from 875 American establishments, Osterman (1994) found that as an enterprise's technology becomes more complex, it is more likely that teams are used. Boning, Ichniowski and Shaw (2001) studied the use of teams in the operations of U.S. steel minimills. They reached the conclusion that "problem-solving teams are adopted only in the presence of incentive plans, and that more technologically complex production lines are much more likely to adopt teams."

One may doubt that why two workers choose to sanction each other, which actually reduces their payoff compared to the case without mutual monitoring. Standard repeated games usually have multiple equilibria. Likewise, under the optimal contract there are two subgame perfect Nash equilibrium in this game: mutual monitoring equilibrium and shirking equilibrium (shirking in each period regardless of history). Shirking equilibrium definitely hurts the principal. But we argue that the mutual monitoring equilibrium is a more plausible one. Given the optimal wage contract, if workers accept the contract, ex post workers have incentive to carry out mutual monitoring. To see this, in mutual monitoring equilibrium each worker gets $p_4 \max\{\frac{1}{(p_4-p_1)}, \frac{1}{2(p_4-p_3)}\} - 1$. On the other hand, the shirking equilibrium yields payoff $p_0 \max\{\frac{1}{(p_4-p_1)}, \frac{1}{2(p_4-p_3)}\}$ for each worker. But

$$\begin{aligned} & p_4 \max\{\frac{1}{(p_4-p_1)}, \frac{1}{2(p_4-p_3)}\} - 1 - p_0 \max\{\frac{1}{(p_4-p_1)}, \frac{1}{2(p_4-p_3)}\} \\ = \quad & (p_4 - p_0) \max\{\frac{1}{(p_4-p_1)}, \frac{1}{2(p_4-p_3)}\} - 1 > 0 \end{aligned}$$

Therefore, workers will have incentive to carry out mutual sanction if the optimal contract is offered.

To make the comparison of project selection simple, we further assume that $q_0 = p_0 = 0$. Then for two individual projects, the manager's highest total payoff is $2q_2(S - \frac{1}{q_2})$, which is achieved when the "work" equilibrium is implemented. Without mutual monitoring, choosing the combined project is optimal if and only if

$$p_4(S - \frac{1}{p_4 - p_2}) \geq q_2(S - \frac{1}{q_2}) \tag{12}$$

Note that two necessary conditions for (12) to hold are: $p_4 > q_2$ and $S > \frac{1}{p_4-p_2}$.

On the other hand, with mutual monitoring choosing the combined project is optimal if and only if

$$p_4(S - \max\{\frac{1}{(p_4-p_1)}, \frac{1}{2(p_4-p_3)}\}) \geq q_2(S - \frac{1}{q_2}) \tag{13}$$

Since $\max\{\frac{1}{(p_4-p_1)}, \frac{1}{2(p_4-p_3)}\} < \frac{1}{p_4-p_2}$, (13) is less stringent than (12). Therefore, mutual monitoring can make team production relatively more attractive: there is a set of parameter values such that without mutual monitoring two individual projects will be chosen, but with mutual monitoring team production will be chosen. Moreover, as the technology becomes more convex., choosing the combined project is more likely to be optimal.

Actually, mutual monitoring is also effective under one special technology. For total effort level $i$, suppose that $p_i = p_0 + ki$ for $i = 1, 2, 3$, where $k$ is some constant; and $p_4 = p_0 + 3k + b$, where $b$ is another constant. That is, the marginal return of exerting effort is constant for partial effort, but full effort has an additional bonus $b$, probably due to the completion of the project. We call this $b$ "completion benefit." Under this technology,

$$2(p_4 - p_3) = 2b + 2k > b + 2k = (p_4 - p_2)$$

Therefore,

$$\max\{\frac{1}{(p_4 - p_1)}, \frac{1}{2(p_4 - p_3)}\} < \frac{1}{p_4 - p_2}$$

The presence of mutual monitoring still decreases the necessary wage payment thus increases the relative attractiveness of team production. Fix $p_4$ and $p_0$, an increase in $b$ means a decrease in $k$, the marginal return of partial effort. One can easily show that $\max\{\frac{1}{(p_4-p_1)}, \frac{1}{2(p_4-p_3)}\}$ decreases as $b$ increases. Therefore, corollary 1 still holds under this technology.

## 2.5 Examples

In this subsection, we provide two concrete examples. The technologies of example 1 is the following:

$p_0 = 0 \quad p_1 = 0.1 \quad p_2 = 0.25 \quad p_3 = 0.5 \quad p_4 = 0.8$
$q_0 = 0 \quad q_1 = 0.2 \quad q_2 = 0.6$

Obviously, technology $p$ satisfies assumption (2) and technology $q$ satisfies assumption (1). $S = 2$. The table below reports the optimal contracts and the associated profits of the principal.

| | Optimal contract | Profits |
|---|---|---|
| Individual projects | $w^{s*} = \frac{5}{3}$ | $\frac{1}{5}$ |
| Combined project without mutual monitoring | $w^{s*} = \frac{20}{11}$ | $\frac{8}{55}$ |
| Combined project with mutual monitoring | $w^{s*} = \frac{5}{3}$ | $\frac{4}{15}$ |

From the table, we see that the presence of mutual monitoring decreases the expected wage payment from $\frac{20}{11}$ to $\frac{5}{3}$. Without mutual monitoring, choosing individual projects is optimal for the principal ($\frac{1}{5} > \frac{8}{55}$). But with mutual monitoring, choosing the combined project is optimal ($\frac{4}{15} > \frac{1}{5}$).

In the second example, the technology exhibits constant marginal returns of partial effort, but with a positive "completion benefit." In particular,

$$p_0 = 0 \quad p_1 = 0.15 \quad p_2 = 0.3 \quad p_4 = 0.45 \quad p_4 = 0.8$$

and the "completion benefit" $b = 0.2$. Without mutual monitoring, to induce full effort $w^{s*} = 2$ , which yields a zero expected profit for the principal. With mutual monitoring, $w^{s*} = \frac{20}{13} < 2$, and the expected profit for the principal is $\frac{24}{65} > 0$.

# 3 Partnership

In this section we study how mutual monitoring can reduce the inefficiency of partnership. We assume that only the combined project is available, and two entrepreneurs form a partnership (there is no longer a principal to break the budget). All the other features of the model are the same as the one presented in Section 2. Assumption (2) is satisfied, so effort exhibits increasing marginal returns. Assumption (4) is also satisfied, so the efficient action is exerting effort in both periods for both workers. We are interested in when the efficient action can be implemented by simple sharing rules, that is, sharing rules exhibits balanced budget and limited liability. Let the sharing rule be $s(y) = (s_1(y), s_2(y))$, $s_i(y)$ is the share of player $i$ if the output is $y$. Balanced budget requires $s_1(y) + s_2(y) = 1$, and limited liability requires $s_i(y) \geq 0$. Because $y = 0$ if the project fails, the only relevant sharing rule is $s_i(y = S)$. To simplify notation, we simply drop the argument and denote the relevant sharing rule as $s_i$.

## 3.1 Partnership without mutual monitoring

To implement the efficient action without mutual monitoring, the following conditions have to be met. First, no player should have incentive to deviate in one period only. That is, $s_iSp_4 - 1 \geq s_iSp_3 - \frac{1}{2}$, $i = 1, 2$. Second, no player should have incentive to deviate in both periods. Mathematically, $s_iSp_4 - 1 \geq s_iSp_2 - 0$, $i = 1, 2$. Given assumption (2), $2(p_4 - p_3) > (p_4 - p_2)$, the first condition is redundant. Moreover, to make the second condition easier to be satisfied for both players, the best sharing rule is $s_i = 0.5$ for $i = 1, 2$. Therefore, the efficient action, or the working equilibrium can be implemented if and only if

$$(p_4 - p_2)S \geq 2$$

Recall that for working in both periods to be the efficient action, the condition is $(p_4 - p_0)S \geq 2$. Therefore, if $\frac{2}{p_4 - p_0} < S < \frac{2}{p_4 - p_2}$, the efficient action cannot be implemented without mutual monitoring. The bite comes from the classical free-riding problem; to discourage free-riding, the return of the project should be high enough.

## 3.2 Partnership with mutual monitoring

With mutual monitoring, players can use trigger strategies to "sanction" or "punish" non-cooperative behavior. Specifically, we consider the following strategy: work in the first period, and work in the second period if and only if both players work in the first period. We call this mutual monitoring equilibrium if this strategy profile can be supported as a subgame perfect equilibrium. To support this strategy profile as an equilibrium, the following conditions have to be met:

$$s_i S p_4 - 1 \geq s_i S p_1 \tag{14}$$

$$s_i S p_4 - 1 \geq s_i S p_3 - \frac{1}{2} \tag{15}$$

$$s_i S p_2 - 1 \leq s_i S p_1 - \frac{1}{2} \tag{16}$$

$$s_i S p_1 - \frac{1}{2} \leq s_i S p_0 \tag{17}$$

(14) makes sure that one player has no incentive to shirk in the first period. (15) requires that no player has incentive to deviate in the second period on the equilibrium path. (16) and (17) make sure that each player has no incentive to deviate off the equilibrium path in the second period.

Because all these conditions have to be satisfied for both players. It follows that the least stringent sharing rule is $s_i = 0.5$ for $i = 1, 2$. Similar to the Principal-agent setting, (17) is redundant if other three conditions are satisfied. Then the set of conditions are boiled down to:

$$(p_4 - p_1)S \geq 2; (p_4 - p_3)S \geq 1; \text{and } (p_2 - p_1)S \leq 1$$

It follows that if

$$\max\{\frac{2}{(p_4 - p_1)}, \frac{1}{(p_4 - p_3)}\} \leq S \leq \frac{1}{p_2 - p_1} \tag{18}$$

then the mutual monitoring equilibrium exists, and the efficient action can be implemented in a perfect equilibrium. Note that by assumption (2), $(p_4 - p_1) > 2(p_2 - p_1)$ and $(p_4 - p_3) > (p_2 - p_1)$. Therefore, there exists a range of $S$ that satisfies (18).

How about the case $S > \frac{1}{p_2 - p_1}$, can the efficient action be implemented with mutual monitoring? Apparently, trigger strategies are no longer credible. But notice that $\frac{2}{p_4 - p_2} < \frac{1}{p_2 - p_1}$ by assumption (2). Recall that in the last subsection, we showed that if $S \geq \frac{2}{p_4 - p_2}$, then the efficient action can be implemented without mutual monitoring. Of course, with mutual monitoring, if $S \geq \frac{2}{p_4 - p_2}$, the efficient action still can be implemented: players simply ignore their observation of the other's effort. Actually, if $S > \frac{1}{p_2 - p_1} > \frac{2}{p_4 - p_2}$, players' incentive to free-ride total disappear; so the efficient action can be implemented in any case. Therefore, we have the following lemma.

**Lemma 1** *With mutual monitoring, if* $S \geq \max\{\frac{2}{(p_4-p_1)}, \frac{1}{(p_4-p_3)}\}$*, then efficient action can be implemented. In particular, if* $\max\{\frac{2}{(p_4-p_1)}, \frac{1}{(p_4-p_3)}\} \leq S \leq \frac{2}{p_4-p_2}$*, the efficient action can only be implemented by trigger strategies. If* $S \geq \frac{1}{p_2-p_1}$*, then the efficient action can only be implemented by strategies ignoring first period efforts. For* $\frac{2}{p_4-p_2} \leq S \leq \frac{1}{p_2-p_1}$*, the efficient action can be implemented both by trigger strategies and strategies ignoring first period efforts.*

Compared to the case without mutual monitoring, mutual monitoring reduces the range of inefficiency. If $\max\{\frac{2}{(p_4-p_1)}, \frac{1}{(p_4-p_3)}\} \leq S < \frac{2}{p_4-p_2}$, then the efficient action cannot be implemented without mutual monitoring, but it can be implemented with mutual monitoring. Notice that mutual monitoring does not eliminate the inefficiency completely: if $\frac{2}{p_4-p_0} < S < \max\{\frac{2}{(p_4-p_1)}, \frac{1}{(p_4-p_3)}\}$, then the efficient action still cannot be implemented. Moreover, the comparative statics result of the previous section applies to the partnership setting as well: the more convex is the technology, the lower the low bound of $S$ that the efficient action can be implemented ($\max\{\frac{2}{(p_4-p_1)}, \frac{1}{(p_4-p_3)}\}$ decreases as the technology becomes more convex), thus the smaller the range of inefficiency. The following proposition summarizes the above discussion:

**Proposition 3** *Compared to the case without mutual monitoring, the presence of mutual monitoring makes the efficient action more likely to be implemented by simple sharing rules. The more convex is the technology, the smaller the range of inefficiency.*

The intuition of this result is the same as those in the principal-agent setting. With mutual monitoring, each player can at most free-ride once, which reduces the gain from free-riding; and increasing marginal returns makes sure that players have less incentive to free-ride in the last period on the equilibrium path. This puts less restriction on the return of the project for the efficient action to be implemented. As a result, the range of inefficiency shrinks.

## 4 Players interact more frequently

In previous sections, we assume that there are two periods needed to complete a project. More realistically, team members may interact more often before the deadline of a project. If we define the span between players' two consecutive interactions as a period, then more frequent interactions means that period length becomes smaller, and it takes more periods to complete a project. In this section, we study how the frequency of interactions between team members affects the effectiveness of the mutual monitoring.

We adopt the principal-agent setting (the partnership setting is similar). The model is essentially the same as the model in section 2. The only difference is that now players needs $n \geq 2$ periods

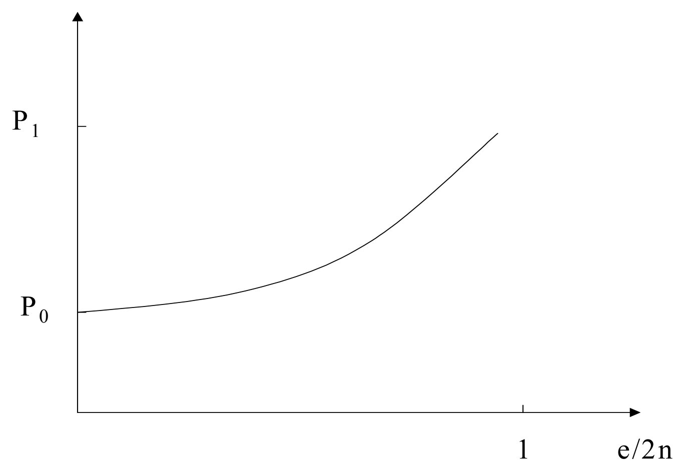


Figure 1:

to complete a project. For simplicity, we assume that only a combined project is available. To make comparative statics analysis reasonable, we fix the technology. Specifically, the probability of success only depends on the ratio of total exerted effort to full effort. Denote $P_{\frac{e}{2n}}$ as the probability of success given that two players total effort is $e$. Note that if two players exert effort in each period, then the probability of success is $P_1$, which is independent of $n$. Also note that if neither players exert effort in any period, then the probability of success is $P_0$, which is also independent of $n$. Basically, $P$ is a function mapping from $[0, 1]$ to $[0, 1]$. We fix function $P$, and assume it is strictly increasing and convex. The $P$ function is plotted in figure 1. Note that the convexity assumption is a generalization of assumption (2).

Given $n$, the relevant $P_{\frac{e}{2n}}$ for $e \in \{0, 1, 2, ..., 2n\}$ are determined from the function $P$. As $n$ increases, the domain of relevant $P_{\frac{e}{2n}}$ expands. This captures the idea that, as agents interact more often ($n$ increases), a unit of effort in one period decreases in absolute value and the number of possible partial effort increases.

Let $w_i$ be the wage payment of player $i$ when the project succeeds, player $i$'s utility function is generalized to:

$$u_i(w_i, e_i) = Ew_i - \frac{\sum_{j=1}^{n} e_i^j}{n}$$

In words, player $i$'s disutility of effort is just the average of his total efforts.[12] We also assume that

$$(P_1 - P_0)S - 2 > 0 \tag{19}$$

[12] Adopting discounting would not change the qualitative results.

(19) implies that the efficient action is for both workers to exert effort in each period. To see this, take any action profile in which the total effort is $m$, $0 < m < 2n$. The total surplus under this action profile is $P_{\frac{m}{2n}} S - \frac{m}{n}$. The total surplus under full effort is $P_1 S - 2$. The difference between the total surpluses is:

$$\begin{aligned}
(P_1 - P_{\frac{m}{2n}})S - (2 - \frac{m}{n}) &= [(P_1 - P_0) - (P_{\frac{m}{2n}} - P_0)]S - (2 - \frac{m}{n}) \\
&> [(P_1 - P_0) - \frac{m}{2n}(P_1 - P_0)]S - (2 - \frac{m}{n}) \\
&= (1 - \frac{m}{2n})(P_1 - P_0)S - (2 - \frac{m}{n}) > 2(1 - \frac{m}{2n}) - (2 - \frac{m}{n}) = 0
\end{aligned}$$

The first inequality comes from the assumption of convexity, and the second inequality comes from assumption (19). The intuition is straightforward: the cost of effort is linear, while the marginal return of effort is increasing; given that exerting full effort is more efficient than exerting zero effort, exerting full effort dominates exerting any partial effort. In the following analysis, we first fix $n$ and derive the optimal contract. Then we carry out comparative statics analysis: how does the optimal contract change as $n$ varies.

## 4.1 Team production without mutual monitoring

In this subsection, we assume that efforts exerted by players are not mutually observable in any period. Under this setting, to implement the efficient action the following conditions have to be met:

$$(P_1 - P_{\frac{n+m}{2n}})w_i - (1 - \frac{m}{n}) \geq 0 \text{ for all } m,\ 0 \leq m < n \tag{20}$$

That is, given that the other player exerts effort in every period, player $i$ should not have incentive to deviate in any possible combination of periods. Lemma 2 shows that the most stringent condition is $m = 0$, that is, one player shirks in all periods.

**Lemma 2** *If* $(P_1 - P_{\frac{n}{2n}})w_i - 1 \geq 0$*, then all the other conditions in (21) are satisfied.*

**Proof.** pick any $0 < m < n$.

$$\begin{aligned}
& [(P_1 - P_{\frac{n+m}{2n}})w_i - (1 - \frac{m}{n})] \\
> \ & (1 - \frac{m}{n})(P_1 - P_{\frac{1}{2}})w_i - (1 - \frac{m}{n}) \\
= \ & (1 - \frac{m}{n})[(P_1 - P_{\frac{1}{2}})w_i - 1] \geq 0
\end{aligned}$$

The first inequality comes from the convexity assumption, while the second inequality comes from the assumption specified in lemma 2. ■

Following lemma 2, the principal's optimal contract is to offer $w_i^* = \frac{1}{(P_1 - P_{\frac{1}{2}})}$ if the project succeeds. Note that the optimal contract is independent of $n$.

## 4.2 Team production with mutual monitoring

Now suppose that at the end of each period, players can observe each other's effort in that period. As in the two-period model, this creates the possibility of mutual sanction: if one player shirks in some period, the other player may punish him by shirking in all later periods. The $n$-period model is different from the two-period model in that in later periods players might not have incentive to carry out punishment even if they observe some shirking. But this is not a big problem, since it simply means that players' incentive to free-ride disappears in later periods. However, this does make the task of finding a subgame perfect equilibrium harder, because trigger strategies may not be supported as a subgame perfect equilibrium.

Note that any subgame in this game is characterized by $(t, e(t))$, where $t$ is the period index and $e(t)$ is the total effort that has been accumulated at the beginning of period $t$. $e(t) \in \{0, 1, 2, ..., 2t-2\}$ and is like a state variable. Consider the class of subgames starting from period $t$. We are interested in whether shirking in all subsequent periods is a Nash equilibrium in the subgame. Of course, this depends on $e(t)$. To proceed, we first prove a useful lemma.

**Lemma 3** *At the beginning of any period $t$, for any accumulated effort $m$ ($0 \leq m \leq 2(t-1)$), given that the other player is going to shirk in all subsequent periods, exerting partial effort in subsequent periods is dominated by either exerting 0 effort or by exerting full effort.*

**Proof.** Given that the other player shirks in all subsequent periods, exerting total effort $k$ in later periods ($0 \leq k \leq n-t+1$) yields a payoff (net of the sunk cost of efforts exerted in previous periods ): $P_{\frac{m+k}{2n}} w_i - \frac{k}{n} \equiv f(k)$. Then

$$f(k+1) - f(k) = [P_{\frac{m+k+1}{2n}} - P_{\frac{m+k}{2n}}] w_i - \frac{1}{n}$$

Given the convexity assumption, $f(k+1) - f(k)$ is increasing in $k$. Let $k'$ be the smallest $k$ such that $f(k+1) - f(k)$ is positive. Overall, there are three possibilities: a) $k' = 0$. Then $f(n-t+1)$ is the largest among $f(k)$, $0 \leq k \leq n-t+1$. b) $0 < k' < n-t+1$. Then either $f(0)$ or $f(n-t+1)$ is the largest among $f(k)$. c) $k'$ does not exist. Then $f(0)$ is the largest among $f(k)$. In all the cases, one of the following must be true:

$$\begin{aligned} f(k) &\leq f(0) \\ \text{or } f(k) &\leq f(n-t+1) \text{ for all } 1 \leq k < n-t+1 \end{aligned}$$

■

Denote $E(t)$ as the set of $e(t) \in \{0, 1, 2, ..., 2t-2\}$ such that shirking in all subsequent periods is not a Nash equilibrium in the subgame. By lemma 3, $e(t) \in E(t)$ if and only if $e(t) \in \{0, 1, 2, ..., 2t-2\}$ and,

$$(P_{\frac{e(t)+n-t+1}{2n}} - P_{\frac{e(t)}{2n}})w_i - \frac{n-t+1}{n} \geq 0$$

That is, given the other player shirks in all subsequent periods, the player still have incentive to work in all subsequent periods. By the convexity, if $e(t) \in E(t)$, then $e(t)+1$ also belongs to $E(t)$. Note that $E(t)$ may be empty. In this case, shirking in all subsequent periods constitute a Nash equilibrium in all the subgame starting from $t$. Let $k(t)$ be the smallest $e(t)$ that belongs to $E(t)$ and $k(t) \leq 2t-3$. Note that $k(t)$ may not exist. In that case, shirking in all subsequent periods is a Nash equilibrium of all the subgames starting from $t$ and at least one player shirks once in previous history. The following lemma establishes the relationship of $k(t)$ in different periods.

**Lemma 4** *(i) $k(t)+1 \in E(t+1)$ and $k(t)$ is weakly increasing in $t$; (ii) if $k(t)$ does not exist, then $k(t-1)$ does not exist.*

**Proof.** Property (i). By the fact $k(t) \in E(t)$,

$$(P_{\frac{k(t)+n-t+1}{2n}} - P_{\frac{k(t)}{2n}})w_i - \frac{n-t+1}{n} \geq 0 \tag{21}$$

$$(P_{\frac{k(t)+1+n-t}{2n}} - P_{\frac{k(t)+1}{2n}})w_i - \frac{n-t}{n} > \frac{n-t}{n-t+1}(P_{\frac{k(t)+n-t+1}{2n}} - P_{\frac{k(t)}{2n}})w_i - \frac{n-t}{n} \geq \frac{n-t}{n} - \frac{n-t}{n} = 0$$

The first inequality comes from the convexity assumption and the second inequality comes from (21). Thus, $k(t)+1 \in E(t+1)$.

Suppose $k(t) < k(t-1)$. By the definition of $k(t)$, (21) is satisfied. Then,

$$(P_{\frac{k(t)+n-t+2}{2n}} - P_{\frac{k(t)}{2n}})w_i - \frac{n-t+2}{n} > \frac{n-t+2}{n-t+1}(P_{\frac{k(t)+n-t+1}{2n}} - P_{\frac{k(t)}{2n}})w_i - \frac{n-t+2}{n} \geq 0$$

The first inequality comes from the convexity assumption and the second inequality comes from (21). This means that $k(t) \in E(t-1)$, and it contradicts the fact that $k(t-1) > k(t)$ is the smallest $e \in E(t-1)$. Therefore, $k(t)$ is weakly increasing in $t$.

Property (ii). Since $k(t)$ does not exist,

$$(P_{\frac{2t-3+n-t+1}{2n}} - P_{\frac{2t-3}{2n}})w_i - \frac{n-t+1}{n} < 0 \tag{22}$$

$$(P_{\frac{2t-5+n-t+2}{2n}} - P_{\frac{2t-5}{2n}})w_i - \frac{n-t+2}{n} < \frac{n-t+2}{n-t+1}(P_{\frac{2t-3+n-t+1}{2n}} - P_{\frac{2t-3}{2n}})w_i - \frac{n-t+2}{n} < 0$$

The first inequality comes from the convexity assumption and the second inequality comes from (22). Therefore, $k(t-1)$ does not exist. ■

Now we are ready to describe the strategy of mutual sanction, which is essentially a modified trigger strategy. In the first period, each worker exerts effort. In any other period $t$, each worker exerts effort in that period if one of the three conditions holds: (1) nobody shirks in previous history; (2) $e(t) \geq k(t)$; (3) $e(t)+1 \geq k(t+1)$; otherwise, shirk in period $t$. This strategy is different from trigger strategy in that if $e(t) \geq k(t)$ or $e(t)+1 \geq k(t+1)$, players are still going to exert effort in period $t$ even if shirking happened in previous history. The essence of this strategy is to carry out the harshest possible punishment if somebody deviates. From lemma (4), it is in later periods (as more effort accumulates) that players' incentive to carry out punishment decreases. Depending on the degree of the convexity of the technology, the optimal wage payment have two different values.

**Lemma 5** *If $(P_1 - P_{\frac{1}{2n}}) \geq n(P_1 - P_{\frac{2n-1}{2n}})$, then the optimal contract is $w_i^* = \frac{1}{n(P_1 - P_{\frac{2n-1}{2n}})}$. And under this contract, the strategy profile we described constitutes a perfect equilibrium.*

**Proof.** We first prove that under the optimal contract, $k(n)$ does not exist. For $m = 2n-3$,

$$(P_{\frac{2n-3+1}{2n}} - P_{\frac{2n-3}{2n}})w_i^* - \frac{1}{n} = \frac{1}{n}[\frac{P_{\frac{2n-2}{2n}} - P_{\frac{2n-3}{2n}}}{P_1 - P_{\frac{2n-1}{2n}}} - 1] < 0$$

The inequality comes from the convexity assumption. Therefore, $k(n)$ does not exist. By lemma (4), $k(t)$ does not exist for all $t$. This means that trigger strategy is credible in all subgames, and the strategy we described becomes trigger strategy.

Second, we prove that under the contract, nobody has incentive to deviate on the equilibrium path. That is,

$$(P_1 w_i^* - 1) - [P_{\frac{2(t-1)+1}{2n}} w_i^* - \frac{(t-1)}{n}] \geq 0 \text{ for all } t,\ 1 \leq t \leq n \tag{23}$$

(nobody has incentive to deviate in any period $t$). (23) includes $n$ conditions. If we minus the left hand side of $t^{th}$ condition by the left hand side of $(t+1)^{th}$ condition, then we get

$$(P_{\frac{2t+1}{2n}} - P_{\frac{2t-1}{2n}})w_i^* - \frac{1}{n} \tag{24}$$

Note that (24) is increasing in $t$. Suppose that (24) is greater than 0 for $t = 1$, then (24) is greater than 0 for all $t$. As a result, if (23) is satisfied for $t = n$, i.e., $(P_1 - P_{\frac{2n-1}{2n}})w_i^* - \frac{1}{n} \geq 0$, then all the conditions in (23) are satisfied. Now suppose that (24) is less than 0 for all $t < T$ and it is greater than 0 for all $t \geq T$. Then if (23) is satisfied for $t = 0$, i.e., $(P_1 - P_{\frac{1}{2n}})w_i^* - 1 \geq 0$, all the conditions in (23) with $t \leq T-1$ are satisfied. Similarly, if (23) is satisfied for $t = n$, i.e., $P_1 - P_{\frac{1}{2n}})w_i^* - 1 \geq 0$, then all the conditions in (23) with $t \geq T$ are satisfied. The last possibility is that (24) is less than

0 for all $t$, then if (23) is satisfied for $t = 0$, i.e., $(P_1 - P_{\frac{1}{2n}})w_i^* - 1 \geq 0$, all the conditions in (23) are satisfied. To sum up, in any case given that

$$\begin{aligned} (P_1 - P_{\frac{1}{2n}})w_i^* - 1 &\geq 0 \quad (25) \\ (P_1 - P_{\frac{2n-1}{2n}})w_i^* - \frac{1}{n} &\geq 0 \quad (26) \end{aligned}$$

are satisfied, all the conditions in (23) are satisfied. It is readily verified that (25) and (26) are satisfied given $(P_1 - P_{\frac{1}{2n}}) \geq n(P_1 - P_{\frac{2n-1}{2n}})$.

We next prove that nobody has incentive to deviate off the equilibrium path. That is,

$$(P_1 w_i^* - 1) - [P_{\frac{2(t-1)+1+l}{2n}} w_i^* - \frac{(t-1)+l}{n}] \geq 0 \text{ for all } t,\ 1 \leq t \leq n \text{ and all } k,\ 1 \leq l \leq n-t+1 \quad (27)$$

following equilibrium strategy is better than shirking in period $t$ and working in any combination of later periods. First we fix $t$. By lemma (3), we only need to worry about $l = 0$ and $l = n - t + 1$ for condition (27). When $l = 0$, (27) becomes one condition in (23). So we should only worry about (27) for $k = n - t + 1$. It is equivalent to:

$$\begin{aligned} (P_1 w_i^* - 1) - [P_{\frac{n+t-1}{2n}} w_i^* - \frac{n-1}{n}] &\geq 0 \text{ for all } t,\ 1 \leq t \leq n \\ \Leftrightarrow\ (P_1 - P_{\frac{n+t-1}{2n}})w_i^* &\geq \frac{1}{n} \text{for all } t,\ 1 \leq t \leq n \quad (28) \end{aligned}$$

It follows immediately that if $(P_1 - P_{\frac{2n-1}{2n}})w_i^* \geq \frac{1}{n}$, then (28) is satisfied.

Finally, it is obvious that $w_i^*$ is the lowest possible wage payment to make (23) satisfied, since (25) has to hold. ■

This case corresponds to the situation where the convexity of the technology is not strong enough. The optimal wage just deters workers from deviating in the last period on the equilibrium path. As a result, mutual sanction about deviation is always credible in all periods given the convexity. Under trigger strategy, player can at most free-ride in one period. Since player's incentive to free-ride decreases as total effort accumulates, deterring shirking in first period is enough. But since the convexity is not so strong, shirking is deterred in first period if player has no incentive to deviate in the last period.

**Lemma 6** *If $(P_1 - P_{\frac{1}{2n}}) \leq n(P_1 - P_{\frac{2n-1}{2n}})$, then the optimal contract is $w_i^* = \frac{1}{P_1 - P_{\frac{1}{2n}}}$. And under this contract, the strategy we described is a perfect equilibrium.*

**Proof.** We first prove that nobody has incentive to deviate on the equilibrium path. By lemma (4), there is a $t'$ such that for all $t < t'$ $k(t)$ does not exist and all $t \geq t'$ $k(t)$ exists. For $t \leq t' - 2$, any deviation will cause perpetual shirking, because $k(t' - 2)$ and $k(t' - 1)$ do not exist. Therefore, to deter deviation in $t \leq t' - 2$, the following conditions are necessary and sufficient:

$$(P_1 w_i^* - 1) - [P_{\frac{2(t-1)+1}{2n}} w_i^* - \frac{(t-1)}{n}] \geq 0 \text{ for all } t,\ 1 \leq t \leq t' - 2 \tag{29}$$

According to the proof of the last lemma, all the conditions in (29) are satisfied if (25) and (26) are satisfied. It is readily verified that (25) and (26) are satisfied under $w_i^*$, given $(P_1 - P_{\frac{1}{2n}}) \leq n(P_1 - P_{\frac{2n-1}{2n}})$.

By the fact that on the equilibrium path $e(t) \geq k(t)$ for $t \geq t'$, neither player has incentive to deviate.

In period $t' - 1$, if one player deviates, punishment will not happen in later periods, since $k(t')$ exists. To deter this deviation, the following condition should be satisfied:

$$P_1 w_i^* - 1 \geq P_{\frac{2n-1}{2n}} w_i^* - \frac{n-1}{n} \Leftrightarrow n(P_1 - P_{\frac{2n-1}{2n}}) w_i^* \geq 1 \tag{30}$$

Obviously, (30) is satisfied under the optimal contract $w_i^*$.

Next we prove that nobody has incentive to deviate off the equilibrium path. In any case, following equilibrium strategy is optimal. To see this, if $e(t) \geq k(t)$, working in next period is optimal since even the other worker shirks in all subsequent periods the worker still have incentive to work. If $e(t) + 1 \geq k(t + 1)$, then working in next period is optimal. To see this, the payoff difference between working and not working in next period is the following:

$$\begin{aligned}(P_{\frac{e(t)+1+2n-2t}{2n}} - P_{\frac{e(t)}{2n}}) w_i^* - \frac{n-t+1}{n} &\geq (P_{\frac{e(t)+n-t+2}{2n}} - P_{\frac{e(t)+1}{2n}}) w_i^* - \frac{n-t+1}{n} \\ &\geq \frac{n-t+1}{n-t}[(P_{\frac{e(t)+n-t+1}{2n}} - P_{\frac{e(t)+1}{2n}}) w_i^* - \frac{n-t}{n}] \geq 0\end{aligned}$$

The last inequality comes from the fact that $e(t)+1 = k(t+1)$. If $e(t)+1 < k(t+1)$ and $e(t) < k(t)$, then shirking in next period is clearly optimal, because working in next period would not induce the other worker to work in later periods (this comes from $e(t) + 1 < k(t + 1)$) and working alone is not optimal (this comes from $e(t) < k(t)$).

Finally, it is obvious that $w_i^*$ is the lowest possible wage payment to deter deviation in the first period. Therefore, $w_i^*$ is the optimal wage. ■

This case corresponds to the situation where the convexity is strong. The optimal wage just deters workers from deviating in the first period on the equilibrium path. Since the convexity is strong, mutual sanction is not credible in later periods. But in early periods mutual sanction is still credible. Now the problem is reduced to motivate workers to exert effort in earlier periods. Given that player's incentive to free-ride decreases as total effort accumulates, deterring shirking in the first period is enough. More intuitively, under strong convexity mutual sanction is still effective to discourage shirking in early periods; when mutual sanction becomes not credible (in later periods), workers' incentive to free-ride already disappeared. The following proposition summarizes the results of lemma (5) and (6).

**Proposition 4** *The optimal contract is* $w_i^* = \max\{\frac{1}{(P_1 - P_{\frac{1}{2n}})}, \frac{1}{n(P_1 - P_{\frac{2n-1}{2n}})}\}$. *The corresponding strategy is either a trigger strategy (convexity is not so strong ) or a modified trigger strategy (strong convexity).*

Now we are ready to do the comparative statics. As $n$ increase (the frequency of interactions between players increases), $(P_1 - P_{\frac{1}{2n}})$ apparently increases. Moreover, as $n$ goes to infinity, $(P_1 - P_{\frac{1}{2n}})$ converges to $(P_1 - P_0)$.

**Lemma 7** $n(P_1 - P_{\frac{2n-1}{2n}})$ *is increasing in* $n$*, and it goes to infinity as* $n$ *goes to infinity.*

**Proof.** Let $n(P_1 - P_{\frac{2n-1}{2n}}) = z(n)$. Then

$$\begin{aligned} z(n+1) - z(n) &= (n+1)(P_1 - P_{\frac{2n+1}{2n+2}}) - n(P_1 - P_{\frac{2n-1}{2n}}) \\ &= (P_1 - P_{\frac{2n+1}{2n+2}}) - n(P_{\frac{2n+1}{2n+2}} - P_{\frac{2n-1}{2n}}) \end{aligned}$$

$\frac{2n+1}{2n+2} - \frac{2n-1}{2n} = \frac{1}{n(2n+2)}$. By the assumption of convexity, $(P_{\frac{2n+1}{2n+2}} - P_{\frac{2n-1}{2n}}) < \frac{1}{n}(P_1 - P_{\frac{2n+1}{2n+2}})$. Therefore,

$$(P_1 - P_{\frac{2n+1}{2n+2}}) - n(P_{\frac{2n+1}{2n+2}} - P_{\frac{2n-1}{2n}}) > (P_1 - P_{\frac{2n+1}{2n+2}}) - n\frac{1}{n}(P_1 - P_{\frac{2n+1}{2n+2}}) = 0$$

Therefore, $z(n)$ is increasing in $n$. Moreover, as $n$ goes to infinity, $(P_1 - P_{\frac{2n-1}{2n}})$ converges to the derivative of $P$ evaluated at 1, which is positive since $P$ is an increasing and convex function. Therefore, as $n$ goes to infinity, $n(P_1 - P_{\frac{2n-1}{2n}})$ also goes to infinity. ■

Following lemma (7), the optimal wage $w_i^*$ decreases as $n$ increases. Since in the case of without mutual monitoring the optimal wage payment is independent of $n$, mutual monitoring becomes more effective in deterring free-riding as $n$ increases. Moreover, as $n$ goes to infinity, $w_i^*$ converges to $\frac{1}{P_1 - P_0}$, which is the optimal wage in the absence of free-riding problem. In other words, free-riding problem completely disappears if workers interact infinitely often. The following proposition summarizes the comparative statics result.

**Proposition 5** *The optimal wage payment* $w_i^*$ *is strictly decreasing in* $n$*, and it converges to* $\frac{1}{P_1 - P_0}$ *as* $n$ *goes to infinity. Mutual monitoring becomes more effective in deterring free-riding as workers interact more often.*

This comparative statics result is quite intuitive. With mutual monitoring, one player can at most free-ride for one period. As the frequency of interaction increases, each period becomes shorter. As a result, player's gain from free-riding decreases. Therefore, the principal only needs to pay a smaller wage to discourage free-riding. Similar result applies to the partnership setting:

as the frequency of interaction among partners increases, team members' incentive to free-ride decreases, thus the range of inefficiency shrinks; as partners interact infinitely often, the range of inefficiency completely disappears. It is natural to think that complex projects need more interactions among team members. Then proposition (5) implies that teams are more likely to be adopted for technologically more complex projects.

## 5 Conclusions

This paper shows that the free-riding problem in short-lived teams is not as severe as previously thought. When the technology exhibits increasing marginal return or has a "completion benefit," mutual monitoring can effectively alleviate the free-riding problem. This extends and complements the results of CY. In principal-agent settings, mutual monitoring reduces the necessary wage to motivate workers in team production, thus making teams more attractive relative to individual production. In partnership settings, mutual monitoring can reduce partners' incentive to free-ride, making socially productive partnership more likely to be implemented (reduce the range of inefficiency).

We also derived two comparative statics results. When the technology is more convex, has a bigger "completion benefit," or teams members are required to interact more often, mutual monitoring is more effective in alleviating the free-riding problem. These results imply that technologically more complex projects are more likely to adopt teams. Some existing empirical evidence supported this prediction. But the fact that teams are more likely adopted for complex projects might be explained by another factor: the gain from complementarities in production among workers is bigger for complex projects. We do admit that this factor contributes to the popularity of teams for complex projects. But we also do believe that, due to mutual monitoring, free-riding problem is less severe for teams with more complex technology, and this effect also contributes to the popularity of teams for complex projects. More empirical works need to be done to distinguish these two effects.

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