Lecture 4
Interest Rates

• Interest Rate Mechanics
  – M&B 6

• Real vs. Nominal Rates
  – M&B 8, M&I 4.1

• Loanable Funds Model
  – M&B 19, pp. 1-3

• Debtor-Creditor Redistribution
  – M&I 7.1
Present Value (PV)

$PV$ now invested for $m$ years at interest rate $i$ has Future Value (FV):

$$FV = PV (1 + i)^m$$

⇒ Future payment of $FV$ to be paid in $m$ yrs has PV:

$$PV = \frac{FV}{(1+i)^m}$$

E.g., $FV = $100, $i = 5\%$ (.05), $m = 1$ yr:

$$PV = \frac{100}{1.05} = 95.24.$$ 

or, if $m = 20$ yr,

$$PV = \frac{100}{(1.05)^{20}} = \frac{100}{2.6533} = 37.69.$$ 

Need $x^y$ or ^ key on calculator to compute!

Note: nominal interest rate “$i$” is “$R$” in M&B, M&I.
• PV = \frac{FV}{(1+i)^m} implies that holding FV constant,

\[ i \uparrow \rightarrow PV \downarrow, \quad i \downarrow \rightarrow PV \uparrow, \quad \text{and,} \quad m \uparrow \rightarrow PV \downarrow, \quad m \downarrow \rightarrow PV \uparrow. \]

• Also, effect of \( \Delta i \) on PV grows proportionately stronger with \( m \):

\[ \Delta PV/PV \approx - \Delta i \cdot m \]

- Examples:

  \( m = 1 \text{ yr}, \ i \ \text{rises from 5\% to 6\%,} \ \Delta \ i = +1\%, \ FV = 100: \)
  - \( \Delta i \cdot m = - (+1\%)(1\text{yr}) = -1\% \)
  (Actual \( \Delta PV/PV = (94.34 - 95.24)/95.24 = - .0094 = - 0.94\%) \)

  \( m = 20 \text{ yrs,} \ i \ \text{rises from 5\% to 6\%:} \)
  - \( \Delta i \cdot m = - (+1\%)(20\text{YR}) = -20\% \)
  (Actual \( \Delta PV/PV = (31.18 - 37.69)/37.69 = - .173 = - 17.3\% \)

- Leads to Interest Rate Risk when banks or thrifts lend long, borrow short. (More later)
\[ i \text{ from } FV / PV: \]

\[
PV = \frac{FV}{(1+i)^m} \Rightarrow (1+i)^m = \frac{FV}{PV},
\]

\[ 1+i = \left( \frac{FV}{PV} \right)^{1/m}, \text{ so} \]

\[ i = \left( \frac{FV}{PV} \right)^{1/m} - 1 \]

E.g., \( FV = \$100, PV = \$50, m = 10 \text{ yrs.}, \)

\[ i = \left( \frac{100}{50} \right)^{1/10} - 1 \]

\[ = 2^{0.1} - 1 = 1.0718 - 1 = 0.0718 = 7.18\%. \]

Note: 0.01\% = one “Basis Point”.
Bonds

Face Value $F$ to be paid at maturity $m$
Coupons $C$ paid each year for $m$ years.
(Assume annual for simplicity – most semiannual)

Bond Present Value ($PV_B$)

$$PV_B = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \ldots + \frac{C+F}{(1+i)^m}$$

$$\Rightarrow i \uparrow \rightarrow PV_B \downarrow, \quad i \downarrow \rightarrow PV_B \uparrow$$

(but $m \uparrow$ could have either effect because payments are being added)
Yield to Maturity (YTM)

= the value of $i$ that gives back market price of bond, holding $C$, $F$, $m$ constant.

If

- $P_{BV} = F$, bond is “at par”, $YTM = C / F$
- $P_{BV} > F$, bond is “above par”, $YTM < C / F$
- $P_{BV} < F$, bond is “below par”, $YTM > C / F$

E.g.

- $F = 100$, $C = 4$, $P_{BV} = 100 \Rightarrow YTM = 4$
- $F = 100$, $C = 3$, $P_{BV} = 110 \Rightarrow YTM < 3$
- $F = 100$, $C = 6$, $P_{BV} = 90 \Rightarrow YTM > 6$
Bond Duration*

Effect of $\Delta i$ on $\text{PV}_B$ again proportionally stronger, the longer its $m$. However, now,

$$\frac{\Delta \text{PV}_B}{\text{PV}_B} \approx - \Delta i \cdot D,$$

where the bond’s *Duration* $D$ equals the **present-value-weighted average maturity** of its payments:

$$D = \left( \frac{(1)C}{(1+i)} + \frac{(2)C}{(1+i)^2} + \ldots + \frac{(m)(C+F)}{(1+i)^m} \right) / \text{PV}_B$$

Generally,

- $D = m$ if $C = 0$,
- $D < m$ if $C > 0$,
- $D$ increases with $m$

* aka *Macaulay Duration*
Consols (Perpetuities)

Pay $C / yr. forever

Exist in UK, conceptually important

\[ PV_C = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots \quad \infty \]

\[ = \frac{C}{1+i} + \frac{1}{(1+i)} \left[ \frac{C}{1+i} + \frac{C}{(1+i)^2} + \ldots \quad \infty \right] \]

\[ = \frac{C}{1+i} + \frac{1}{(1+i)} [PV_C] \]

\[ \Rightarrow (1+i) PV_C = C + PV_C \]

\[ PV_C = C / i \]

E.g., \( C = $100, i = 4\% \) \( \Rightarrow PV_C = 100/.04 = $2500 \)
Consol Duration

\[ D_C = \frac{1}{i} \]

E.g., \( i = 4\% / \text{yr} \) \( \Rightarrow D_C = 1 / .04 = 25 \text{ yrs.} \)

\( D_C \) finite despite infinite final maturity!

\[ \Delta PV_C / PV_C \approx - \Delta i \cdot D_C \text{ as for bonds} \]
Real vs. Nominal Interest Rates

\[ i = \text{nominal interest rate} \]

\[ r = \text{real interest rate} \]

on $\text{-denominated loans, not indexed for inflation.}$

on purchasing-power-power-denominated loans, with payments indexed for inflation

Note: nominal interest rate \( i \) is “\( R \)” in M&B, M&I. “\( R \)” will also be used for bank reserves, so “\( i \)” less ambiguous.
**US Treasury Inflation-Protection Securities (TIPS)**

- All payments indexed to CPI-U (2.5 mo. lag)
- Provide direct observation of real rate $r$
- First issued Jan. 1997
- 5, 10, 20, 30-year initial maturities
- Now $472$ B (10.4% of marketable Treasury debt) (2009)

- Monthly TIPS yield curves on my webpage:
Present Values with P-indexed loans

Same formulas, with \( r \) in place of \( i \)

\[
PV = \frac{FV}{(1+r)^m}, \text{ etc.}
\]

e.g. Indexed Consol:

\[
C = \text{Real coupon payment (today's $)}
\]

\[
PV_c = \frac{C}{r} \text{ (today's $)}
\]

application:

Parcel now pays $100,000 rent per year, future rent assumed to grow in proportion to P. \( r = 2\% \).

\[
\Rightarrow PV = \frac{100,000}{.02} = 5,000,000
\]
• Real YTM $r$ on 10-yr TIPS has been between 0.5% and 4.5% since their introduction in 1997. Recent values under 0.5% very unusual.

• Nominal YTM $i$ on conventional Treasuries higher, to compensate for likely future inflation. Also more volatile, because of fluctuations in inflation premium.
\( i \) minus \( r \) gives “Break-Even” inflation rate, at which returns on real and nominal bonds are equal.

According to the **Fisher Equation** (next slide), this is the market’s **expectation of future inflation** \( \pi^e \) over the life of the loans.
Determination of $r$, $i$

- **Loanable Funds Model**
  
  real rate $r$ primarily determined by savings, investment decisions, S and D for Credit.

- **Fisher Equation**
  
  \[ i = r + \pi^e, \]

  where $\pi^e$ is *expected inflation* over life of loan

  (sometimes written $\pi^a$ for *anticipated* inflation)

- **Adaptive Learning (AL)**

  $\pi^e$ mostly determined by past $\pi$,
  
  with biggest weights on recent past.

  Coefficients may change slowly over time.
• Before 1997, \( r \) not directly observed.

• But Adaptive Learning allows us to infer \( r \) from \( i \), using Fisher Eq’n:

\[
r = i - \pi^e
\]

plus recent inflation.

• Simple model for 1-yr horizon using recent experience:

\[
\pi^e = 1.24 + .64 \bar{\pi}
\]

where \( \bar{\pi} \) is avg. inflation over past 12 mo. (Coefficients change slowly over time.)

Currently (using 3.8% 8/11 \( \bar{\pi} \)), this gives

\[
\pi^e = 3.6\% \text{ (not plotted)}
\]
Changes in $\pi^e$ account for much of the movement in nominal rates over past 50 years ...
... but inferred real rates have not been constant:
1-yr $r$ typically about 2%,
but was 0-1% in 1970s,
5-6% in early 80s,
negative 2003-5, 2008-
2011 (not plotted).
Loanable Funds Model of $r$ (M&I 19, pp. 1-3)

$(1+r)^m$ is price of present goods in terms of future goods

- $r \uparrow \Rightarrow$ present goods more costly (rel. to future goods)
- $r \downarrow \Rightarrow$ present goods less costly.

“Credit” = command over present goods

= what you get in exchange for your IOU when you borrow
= what you give up in exchange for someone else’s IOU when you lend.

Non-monetary equilibrium $r$ determined by Demand & Supply of Credit.
At low \( r \), borrowers want more credit.

At high \( r \), borrowers want less credit.

\( \Rightarrow \) \( D_{NM}(r) \) slopes down.
At high \( r \), lenders willing to give up more credit

At low \( r \), lenders give up less credit.

\[ \Rightarrow S_{NM}(r) \text{ slopes up} \]
Credit Market Equilibrium
(non-monetary economy)

\[ r_0 = \text{Non-Monetary Equilibrium real interest rate} \]
Credit Market Equilibrium

(non-monetary economy)

\[ r_0 = \text{Non-Monetary Equilibrium real interest rate} \]

Increase in D for Credit (rightward shift in \( D_{NM}(r) \)) increases \( r_0 \) to \( r_0' \).
Credit Market Equilibrium
(non-monetary economy)

$r_0 = \text{Non-Monetary Equilibrium real interest rate}$

**Decrease in D for Credit** (leftward shift in $D_{NM}(r)$) decreases $r_0$ to $r_0''$
Credit Market Equilibrium

(non-monetary economy)

$r_0 = \text{Non-Monetary Equilibrium real interest rate}$

Increase in $S$ of Credit
(rightward shift in $S_{NM}(r)$)
Decreases $r_0$ to $r_0'''$

(corrected 10/5/11)
Credit Market Equilibrium
(non-monetary economy)

\[ r_0 = \text{Non-Monetary Equilibrium real interest rate} \]

Decrease in S of Credit (leftward shift in \( S_{NM}(r) \)) increases \( r_0 \) to \( r_0''' \)
Debtor-Creditor redistribution (M&I 7.1)

- Nominal Debt paying $i = r + \pi^e$
  1. $\pi = \pi^e$
     $\Rightarrow i - \pi = r$, No transfer.
  2. $\pi > \pi^e$ (as in 1970s)
     $\Rightarrow i - \pi < r$. Creditors lose, Debtors gain.
  3. $\pi < \pi^e$ (1930’s, 1980’s)
     $\Rightarrow i - \pi > r$. Debtors lose, Creditors gain.*

- if they can collect – Bankruptcies & foreclosures rise!
• Transfer may be eliminated with Price-Level Indexed Debt.
  – Payments indexed to CPI-U or other index
  – Real return independent of inflation
  – TIPS since 1997

• Nominal debt = safe indexed debt + lottery ticket on CPI.
  – Serves no function for risk-averse investors, borrowers
  – But still no private indexed securities to speak of!
• Next:
  – Velocity and the Quantity Equation
    • M&I 3, 4, 7.4