Velocity × Quantity Eq: \( M + 1 = 3,4 \)

**Quantity Theory of Money** (Review)

\[ P \times V = \frac{M^S}{m^0} \text{ in L.R. eq.} \]

"\( M^S \): \( M \) set by gov't, banks.

\( m^0 \)

- Increases with volume of real transactions/yr (proportional to real income)

- Increases with avg time each $ held before transactions

- Or, decreases w/ # times/yr each $ changes hands, aka "Velocity of Money"
Income Velocity of M

\[ Y = \text{Nominal Income} \ [\$/yr] \]
\[ M = \text{Nominal Money Stock} \ [\$] \]
\[ \frac{M}{Y} \ [\text{yrs}] \leq \text{avg time each \$ is held} \]

\[ V = \frac{Y}{M} \ [\text{yrs}^{-1}] = \text{Income Velocity of M} \]

\[ \leq 5 \text{ times each \$ changes hands/yr.} \]

\[ \text{"Velocity" for short.} \]

\[ \text{Transaction Velocity} \]
\[ T = \text{total transactions/yr} \]
\[ V_T = \frac{T}{M} = \text{actual } 5 \text{ times avg \$ changes hands/yr.} \]

\[ \text{but } T \text{ harder to measure, so rarely used.} \]

\[ T \gg Y, \text{ so } V \ll V_T \]
Quantity Equation
\[ V = \frac{Y}{M} = \frac{Py}{M}, \quad y = \frac{Y}{P} \]

\[ \Rightarrow \quad MV = Py \quad \text{(Eqn)} \]

\[ \Rightarrow \quad P = \frac{MV}{Y} \]

\[ \circ \quad MP \rightarrow P \uparrow \]
\[ \circ \quad V \uparrow \rightarrow P \uparrow \]
\[ \circ \quad y \uparrow \rightarrow P \uparrow \]

* Identically true since \( V := \frac{Y}{M} \)

* Reflects QTM if \( V \rightarrow V^* = \frac{Y}{M_0} \)
  
  since then \( P \rightarrow P^* = \frac{M}{M_0} \)
Dynamic Form of Q-Eq'n.

Time 0:

\[
MV = PY = (a+b)(c+d) = ac + ad + bc + bd
\]

Time 1:

\[
(M+\Delta M)(V+\Delta V) = (P+\Delta P)(Y+\Delta Y)
\]

\[
\Rightarrow MV + M\Delta V + V\Delta M + \Delta M \Delta V = MV + P\Delta Y + Y\Delta P + \Delta P \Delta Y
\]

\[
\Rightarrow \frac{\Delta M}{M} + \frac{\Delta V}{V} + \frac{M\Delta V}{MV} = 0
\]

\[
\Rightarrow \frac{\Delta P}{P} + \frac{\Delta Y}{Y} + \frac{ Folio}{Y} = 0
\]

\[
\Rightarrow \frac{\Delta M}{M} + \frac{\Delta V}{V} = \frac{\Delta P}{P} + \frac{\Delta Y}{Y}
\]

Q-Eq'n

(Dynamic Form)
Q-Eq'n: \[ \frac{\Delta M}{M} + \frac{\Delta V}{V} = \frac{\Delta P}{P} + \frac{\Delta Y}{Y} \]

Implications of Dynamic Q Eq'n.

\[ \frac{\Delta P}{P} = \frac{\Delta M}{M} + \frac{\Delta V}{V} - \frac{\Delta Y}{Y} \]

Inflation =

Money growth

+ Velocity growth

- Real income growth.

Eg

\[ \frac{\Delta M}{M} = 5\% \]

\[ \frac{\Delta V}{V} = 3\% \]

\[ \frac{\Delta Y}{Y} = 0 \quad (V: \text{const}) \]

\[ \Rightarrow \frac{\Delta P}{P} = (5\%) + (0) - (3\%) \]

\[ = +2\% \]

But if \( \frac{\Delta V}{V} = -4\% \), some \( \frac{\Delta M}{M}, \frac{\Delta Y}{Y} \)

\[ \Rightarrow \frac{\Delta P}{P} = (5\%) + (-4\%) - (3\%) \]

\[ = -2\% \]
Similarly,

\[
\frac{\Delta V}{V} = \frac{\Delta P}{P} + \frac{\Delta Y}{Y} - \frac{\Delta M}{M}
\]

\[
\frac{\Delta M}{M} = \frac{\Delta P}{P} + \frac{\Delta Y}{Y} - \frac{\Delta V}{V}
\]

Q. Eq'n \Rightarrow

If \( V = \text{const.} \left( \frac{\Delta V}{V} = 0 \right) \),

need \( \frac{\Delta M}{M} = \frac{\Delta Y}{Y} \) for \( \frac{\Delta P}{P} = 0 \)

— Milton Friedman "Monetarist" prescription for P-stability.
Inflationary Dynamics

Actual inflation driven by

1. Excess S or D for M
   Leads to Q.T. in Long Run.

2. Public's inflationary expectations
   Causes π inertia

3. Microeconomic S&D shocks
   Makes month-to-month π very noisy.

② and or ③ can lead P away from Q.T. equilibrium, but eventually
① will pull it back.
Price Controls

M+I 3.5, 3.6

- Why not fix prices by law?

Price Ceiling

- World War II

- Vietnam
  1971-74

- Gas Prices
  1972-74, 1975

Price Floors

- New Deal 1930's
Price Ceiling
\[ \bar{P} < P_0 \]
\[ \Rightarrow Q^D > Q^S @ \bar{P} \]

\[ \Rightarrow \text{Suppressed Inflation} \]
- Shortages \( Q^D - Q^S \)
- Quality Deterioration
- Rationing (WWII)
- Black Markets

Some consumers willing to pay \( > P_0 \)

WWII - Office of Price Admin. (OPA)

Vietnam / Nixon 1971-74
- Gas controls briefed again 1974.
Nixon Price Control Period
1971-74

1972 -
Measured to hold low, shortage build.

1973 -
\( p < p^* \), shortage acute, esp. grain in Fall,

1974 -
\( p \) catches up to \( p^* \). Measured \( \epsilon > 10\% \), then falls abruptly after \( p \) catches up.
Price Floors

\[
P > P_0
\]

\[
\Rightarrow \quad Q^s > Q^d
\]

\[
\overline{\text{Suppressed Deflation}}
\]

- Unsold Output
- Unemployment

New Deal, 1930's.
Price Floors in Great Depression (1929-1941)

Herbert Hoover (29-33)
- Successfully urged industry not to cut wages
  \[ \Rightarrow \text{Real wages} \uparrow \text{as} \ P \downarrow, \text{Unemployment?} \]
- Encouraged Trade Union.
  
  Maintained Price, Cut Output.

(See Ohanian working paper, "What or Who caused the Great Depression?"

For New Deal (33-1941)
- Nat'l Recovery Admin. (NRA) 1933-35.
  - Imposed industrial cartels
    set min prices, cut output
- Ag. Adjustment Act 1933-36, 1938-
  created "Market Orders" (farm cartels)
  set min prices, cut output
- Wagner Act (1935)
  - Majority of workers can form unions
    union wages on all existing, new workers
- Minimum Wage (1938)

Result = U remained high, output depressed throughout decade of 30's.
Apart from transitory deviations
from Q.T. equilibrium \( P = \frac{M}{m^0} \),
does \( V \) change?

If so, why?
\[ V = \frac{Y}{M} = \frac{P_Y}{M} = \frac{Y}{m} \]
M1 velocity = \frac{\gamma}{M}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{chart.png}
\caption{Graph showing M1 velocity over time from 1950 to 2000.}
\end{figure}
Why does $V$ change?

1. Nominal interest rate

   $i \uparrow \Rightarrow V \uparrow$, $m^d \downarrow$

2. Economics of scale?

   $\gamma \uparrow \Rightarrow \frac{m^d}{\gamma} \downarrow \Rightarrow V \uparrow$?
Overlay of V, i shows-

- Uptrend in i accounts for uptrend in V, 1950s - 1980

- Short moves in i often not reflected in V.

- Down trend in i, 1980-95 reverses uptrend in V

- V responds to swings in i, 1980-95.

- But V does not return to 1950's levels as i does.

\[ \Rightarrow \]

1. V rises with i

2. Economies of scale make V rise with V over time as well.
Why does \( i \) change?

\[
i = r + \tau^e
\]

\( i \) = Nominal interest rate
\( r \) = real interest rate
\( \tau^e \) = expected inflation

- \( r \) determined in L.R. by savings, investment, independently of \( \tau, \tau^e \) (Loanable Funds Model)
- \( \tau^e \) determined by past \( \tau \) by Adaptive Learning

\( \Rightarrow \) feedback from \( \tau \) to \( i \), \( V \).
Money Demand $m^0(y, i)$
(for real money balances)

$i = \text{nominal interest rate on loans} = \text{opportunity cost of holding } M$
if $M$ pays no interest.

$m^0 \uparrow$ with $y$,
$m^0 \downarrow$ with $i$.

\[\begin{array}{c}
\text{i} \\
\text{m}^0 \text{ (given high } y) \\
\end{array}\]

\[\begin{array}{c}
\text{m}^0 \text{ (given low } y) \\
\text{m/p} \\
\end{array}\]
McCulloch (1997) $m^0$ estimates (M1)

$$m^0 = (\text{const}) \ y^{.42} \ \exp(-.028 \ i)$$

($i$ = %/yr)

$$\Rightarrow V = \frac{Y}{M} = \frac{Y}{m} = (\text{const}) \ y^{.58} \ \exp (.028 \ i)$$

so $V \uparrow \Rightarrow y \uparrow \Rightarrow i \uparrow$,

up trend in $y \Rightarrow$ up trend in $V$
Real M1 Demand $m^0(y, i)$

![Graph showing the relationship between real M1 demand and M/P, with labels for Y1973, Y1993, and Y1954.](image)

M/P, $\$ billions (1987)$

$\dot{y}, \dot{I}$ (3-mo T Bill rate)
Velocity Boost Inflation

\[ \pi = \frac{\Delta m}{m} + \frac{\Delta V}{V} - \frac{\Delta Y}{Y} - \Phi_0 \sim' \]

\[ \text{if } \frac{\Delta m}{m} \uparrow \]

\[ \rightarrow \pi \uparrow \]

\[ \rightarrow \pi_0 \uparrow \text{ (by Adaptivity Learning)} \]

\[ \rightarrow i \uparrow \text{ (by Fisher's ![image](https://via.placeholder.com/150))} \]

\[ \rightarrow m_0 \downarrow \]

\[ \rightarrow V \uparrow \text{ (since } V = \frac{Y}{m} \text{)} \]

\[ \rightarrow \frac{\Delta V}{V} > 0 \text{ as } V \uparrow \]

\[ \rightarrow \text{extra inflation during transition} \]

\[ \pi > \frac{\Delta m}{m} - \frac{\Delta Y}{Y} \text{ as } V \uparrow. \]
Velocity Drag Deflation

\[ \frac{\Delta m}{m} \downarrow \rightarrow \pi \downarrow \rightarrow \pi e \downarrow \quad (\text{by AE}) \]

\[ \rightarrow \zeta \downarrow \quad \text{by Fischer Eqn} \]

\[ \Rightarrow m \uparrow \]

\[ \rightarrow V \downarrow \]

\[ \Rightarrow \frac{\Delta V}{V} < 0 \quad \text{as } V \downarrow \]

\[ \Rightarrow \pi < \frac{\Delta m}{m} - \frac{\Delta y}{y} \quad \text{as } V \downarrow. \]

May result in unintended deflation.
Self - Generating Inflation

Possible in theory with fixed $M$,

Suppose $\frac{\Delta M}{M} = \frac{\Delta Y}{Y}$, so $\Pi = 0$ if $V; \text{const}$, but supply shock pushes $P^\Pi$.

Vicious Circle of

Self - Generating $\Pi$.

when

- $\Pi^e$ responds quickly to $\Pi$

- and

- $\Pi^D$ responds strongly to $\Pi^e$, i
Self-Generating Inflation

Possible in theory with fixed $M$.

Suppose $\frac{\Delta M}{M} = \frac{\partial y}{y}$, so $\Pi = 0$ if $V = \text{const}$, but supply shock pushes $P^\uparrow$.

$P^\uparrow \rightarrow X \rightarrow \Pi^e \uparrow \rightarrow V \uparrow \rightarrow m^0 \uparrow$

$P$ stable when

- $\Pi^e$ responds slowly to $\Pi$
- $m^0$ responds weakly to $i$

In practice, runaway $\Pi$ always associated with runaway $\Delta M/M$. 
Why do gov'ts often allow \( \frac{\Delta M}{M} > \frac{\Delta Y}{Y} \)?

\( \pi > 0 \) ?

3 Motives for \( \frac{\Delta M}{M}, \pi \):

1. Inflationary Finance
   \[ \frac{M + I_5}{M} \]

2. Stimulate \( Y \), reduce Unemployment
   \[ \frac{M + I_6}{M} \]

3. Reduce \( r \) and/or \( i \).
   \[ \frac{M + B}{M + B} 19, 21 \]