

Is Antitrust Beneficial?  
Increasing Returns to Scale with Sunk Costs

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## Introduction

U. S. antitrust policy prohibits any one firm from obtaining a 100% share of the market in a given industry, and also prohibits price collusion by the incumbents in the industry. It has been objected that this policy is internally inconsistent: If firms do not compete as vigorously as possible, they may be punished for tacitly colluding with other firms. But at the same time, if they *do succeed* at competition, by taking away *all* the customers of other firms, they will surely be punished for monopolization.<sup>1</sup>

If factor markets are competitive, there are no legal barriers to entry, and each firm has an identical production function that exhibits constant returns to scale, antitrust is superfluous, since the market outcome will tend to the zero-profit, perfectly competitive equilibrium, even if one firm happens to end up with 100% of the market. Essentially the same will be true if there are a large number of potential entrants, each of which has an identical U-shaped average cost schedule whose minimum is at a quantity that is a small fraction of total demand at a price equal to minimum average cost. In this case, the market outcome will be a large number of firms operating very near minimum average cost and as nearly perfect price-takers (Viner 1931).

The case of increasing returns to scale is more problematic. However, Baumol (1982), working with Panzar and Willig, has shown that if markets are "contestable," in the sense that "entry is

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<sup>1</sup> Brown Shoe ( ), Alcoa ( ).

absolutely free and exit is absolutely costless," and a large number of firms have an identical production function exhibiting increasing returns to scale, the only equilibrium is one in which there is a single firm, which charges the quasi-competitive zero-profit price. This is not the ideal outcome, but it is the second-best optimum, subject to the constraint that profits be non-negative. In this case, antitrust could only be counter-productive, since it would inefficiently force the break-up of the large firm, and would raise prices for consumers.

Nevertheless, if there are some sunk costs, the case of increasing returns to scale no longer leads to the Baumol-Panzar-Willig "Contestable Markets" equilibrium. If entry requires an irreversible, industry-specific investment, this in itself can act as a barrier to exit by interlopers, and therefore as a barrier to their entry. The first firm to enter will have an advantage, allowing it to exploit some monopoly power. It is important that these be truly sunk costs and not merely "fixed costs," however. The latter may indicate merely a positive level of costs in the limit as output goes to zero. If these costs can be eliminated by exiting the industry, they will not serve as a barrier to entry, any more than do variable costs. It is by removing an incumbent firm's option to exit costlessly that sunk costs give the incumbent an edge over merely potential entrants. In this case, the market outcome will tend to be one large firm with some market power. Even so, the threat of new entry will not enable it to act like a simple monopolist.

In this case it is not at once clear whether antitrust is beneficial or counter-productive. On the one hand, by forcing the presence of at least two firms that may not cooperate, antitrust will make firms behave more like price-takers, thus increasing economic efficiency. On the other hand, by denying the economy the full potential advantages of the increasing returns to scale exhibited by the production function, it will to that extent reduce economic efficiency.

An alternative to antitrust that has been proposed and that does not share its internal inconsistency is laissez-faire. We take this to mean the absence of legal barriers to entry, with no restrictions against either monopolization or non-fraudulent coordination of price and quantity decisions.<sup>2</sup>

This paper investigates the relative inefficiency of antitrust versus laissez-faire in the presence of increasing returns to scale with some sunk costs. Which is more desirable (or less undesirable, if you will) will depend on the global properties of the demand and production functions. We cannot consider all possibilities, but must limit our investigation to a restricted class of parametric functions, and consider several different illustrative settings of the relevant parameters, in the hope that this sheds some light on the issue.

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<sup>2</sup> Coordinating ostensibly competitive bids, as was alleged in the Iron Pipe case ( ), constitutes fraud, and would presumably be prohibited as such under laissez-faire. The related issue of whether purchasers of a commodity produced under increasing returns to scale should insist on multiple independent bids is important, particularly when the purchaser is a public agency, but is one which we do not consider here.

## Demand and Production

At time 0 an industry comes into being. Flow demand for the homogeneous output  $Q$  has the constant elasticity form

$$Q^D = D(P) = P^{-\eta}. \quad (1)$$

The demand curve does not shift over time. Without loss of generality, the demand curve is normalized so that unit quantity is demanded at unit price. Price discrimination is not practical.

In order to introduce an element of market stability, I assume that consumers are *lexicographic shoppers*. This means that once they begin to buy from a given firm, they will continue to purchase from that firm, until either that firm raises its price or another firm offers a perceptibly lower price, i.e. a price that is lower by some very small but unspecified amount  $\delta > 0$ .

There are two factors of production, a sunk factor  $K$  and a variable factor  $V$ . " $K$ " may represent industry-specific physical plant, human capital, computer software, brand recognition, or firm organization. " $V$ " may represent industry non-specific labor, leasable equipment, energy, or materials. To the extent the sunk factor depreciates, it will begin to act like a variable factor. I therefore assume, without any important loss of generality, that it does not depreciate at all.

Output may be produced by means of the Cobb-Douglas production function<sup>3</sup>

$$Q = (bK^\alpha V^{1-\alpha})^\epsilon, \quad (2)$$

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<sup>3</sup> For the present, we restrict our analysis to the case of a unitary elasticity of substitution, as implied by (2). A more general analysis might also consider a Constant Elasticity of Substitution (CES) production function.

where  $0 < \alpha < 1$  and  $1 < \varepsilon < 1/(1-1/\eta)$ . The upper bound on the scale elasticity  $\varepsilon$  prevents a single firm from being able to make an infinite profit by selling infinite output at zero price.

$K$  and  $V$  are both in infinitely elastic supply at unit price. Firms may borrow unlimited amounts for *bona fide* investment in  $K$  at a constant real interest rate  $r$ . The real interest cost of  $K$  is therefore  $rK$ . Investments in  $K$ , once made, are not reversible, and may not be extended. A firm that wishes to increase its capacity can only do this by opening a second plant, on the same terms as a new entrant.

The long-run (or, more precisely, *ex-ante*) total cost function  $C(Q)$  is found by finding  $K^*$  and  $V^*$  that minimize

$$K + \int_0^{\infty} V e^{-rt} dt, \quad (3)$$

or equivalently,  $rK + V$ , subject to (2), to obtain

$$\begin{aligned} K^*(Q) &= \frac{1}{b} \left( r \frac{1-\alpha}{\alpha} \right)^{\frac{1}{1-\alpha}} Q^{1/\varepsilon} \\ V^*(Q) &= \frac{1}{b} \left( r \frac{1-\alpha}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} Q^{1/\varepsilon}, \end{aligned} \quad (4)$$

whence

$$C(Q) = rK^* + V^* = \frac{r^\alpha}{b} \left[ \left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} + \left( \frac{\alpha}{1-\alpha} \right)^\alpha \right] Q^{1/\varepsilon}. \quad (5)$$

Without loss of generality,<sup>4</sup> we may set

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<sup>4</sup> This substitution makes  $r$  disappear from the calculations. This does not matter, since  $r$  is held constant, and leads to no loss of generality, since a the effect of a different  $r$  is equivalent to a change in  $\alpha$ .

$$b = r^{\alpha} \left[ \left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} + \left( \frac{\alpha}{1-\alpha} \right)^{\alpha} \right], \quad (6)$$

so that (5) becomes

$$C(Q) = Q^{1/\varepsilon}. \quad (7)$$

This normalization implies that the demand curve and average cost curve (AC) intersect at unit output and unit price, as shown in Figure 1. We will use this zero-profit outcome as a reference point for our calculations. The upper bound on the scale elasticity  $\varepsilon$  implies that the demand schedule cuts the average cost schedule from above, as shown.

Figure 2 adds the marginal revenue (MR) and marginal cost (MC) schedules to the demand and average cost schedules of Figure 1. The ideal, Pareto optimal outcome in such a market is the price-quantity pair  $(P_I, Q_I)$  at which MC intersects D. There are two difficulties with this outcome, however. The first is that it requires producing at a loss, since D lies below AC at  $P_I$ . This in turn implies that distortionary taxes must be levied on the rest of the economy in order to make up this loss, creating deadweight losses elsewhere in the economy. The second, and more fundamental, problem is that the cost schedule (5) is not directly observable. If residual profits accrue to a private firm that is managing production, we may assume that we are observing points on the minimized cost schedule. But if instead residual profits (here negative) accrue to taxpayers who are very remote from production decisions, the principal-agent problem, already present in any large firm, is enormously aggravated, and there is

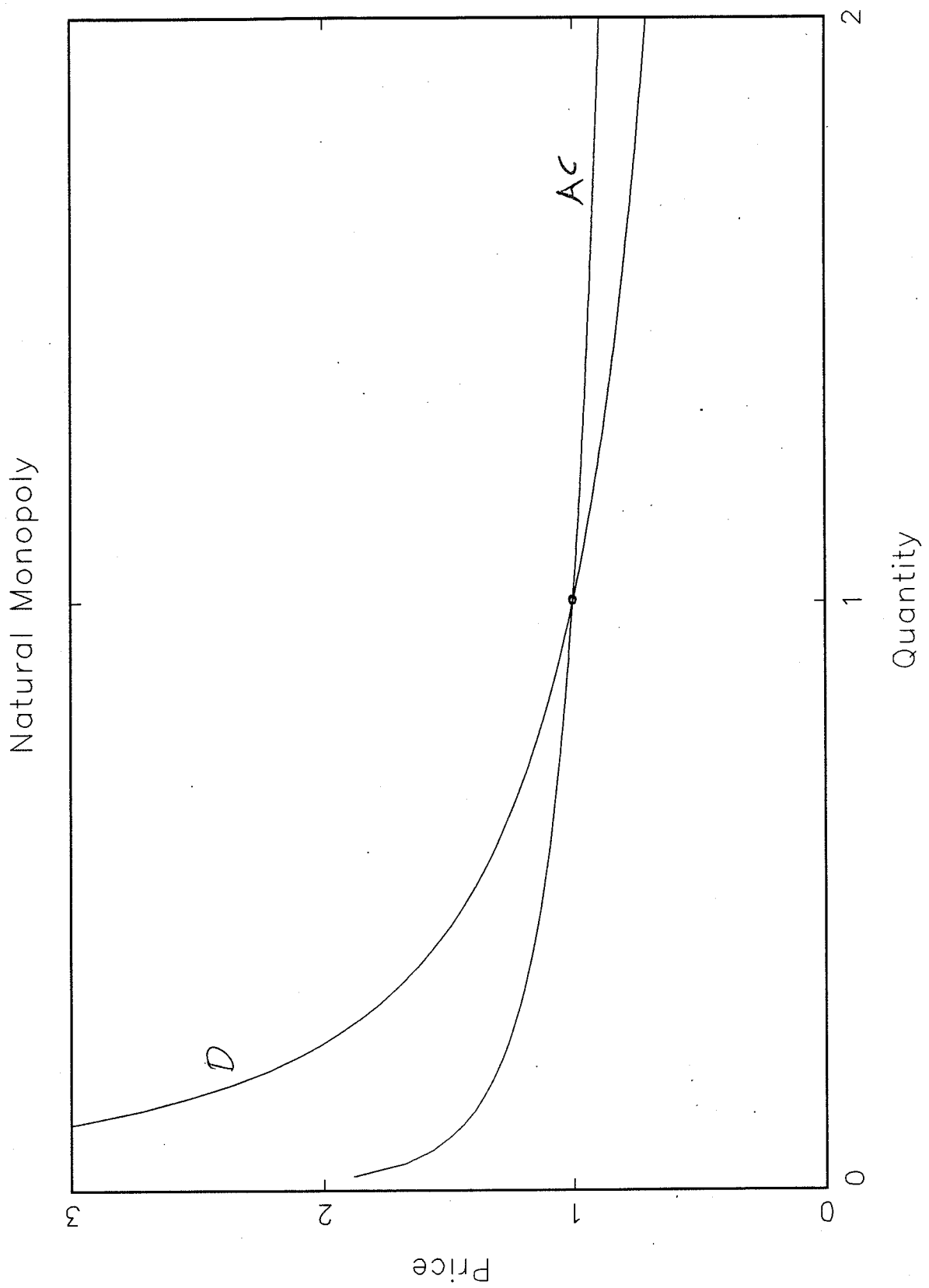


Figure 1

# Monopoly, Zero-Profit, and Ideal Pricing

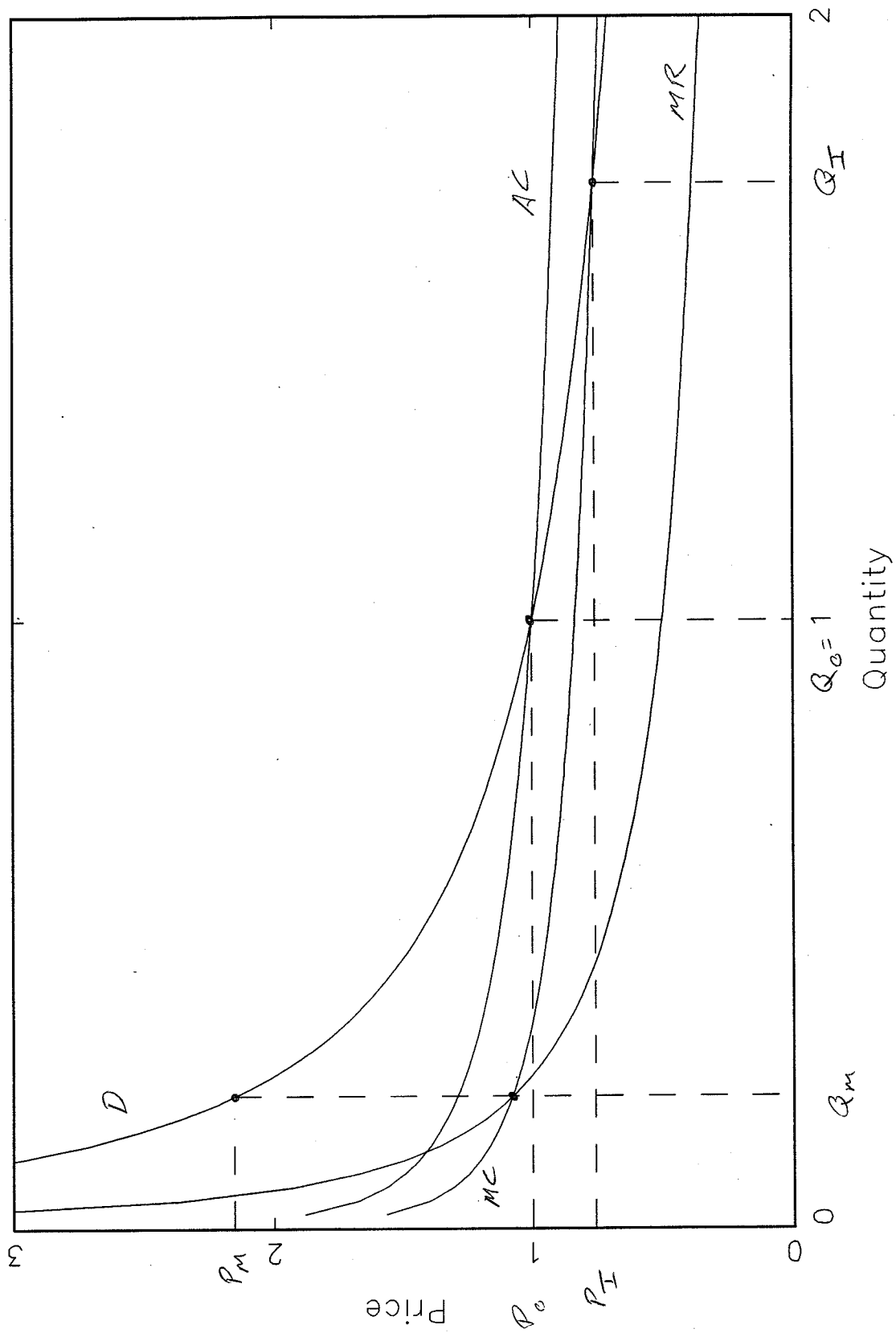


Figure 2

no reason to expect costs to be minimized to the extent that would be feasible under private ownership. Instead, management may be expected to indulge in expense preference behavior.

The zero-profit single-firm outcome  $P_0 = 1$ ,  $Q_0 = 1$  eliminates the first problem, but retains the second: If a regulated monopoly is constrained to make zero profit over and above the cost of servicing the debt it incurs to acquire its capital, its management has an incentive to extract monopoly rents on its own behalf in the form of utility-generating expenses. Individual customers, who have little knowledge of the actual production process, have only negligible incentive to monitor these expenses and to exert control on the regulatory process as voters. Antitrust and laissez-faire, on the other hand, both encourage cost-minimization.<sup>5</sup> Neither is ideal, but both are at least feasible.

We may measure the economic welfare gain or loss from any outcome relative to our zero-profit reference point as the change in the area under the demand curve, minus the change in the cost of production.<sup>6</sup> Let  $\Delta W_I$  be the (positive) gain in welfare under the ideal outcome relative to the zero-profit outcome. This may

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<sup>5</sup> We will see that neither antitrust nor laissez-faire induces firms to employ the cost-efficient mix of  $K$  and  $V$ . Nevertheless, they will use the amount of  $K$  and  $V$  they do purchase effectively and efficiently. There is no reason to think this to be done if profits are constrained to be zero or if losses are subsidized.

<sup>6</sup> The Marshallian consumer's surplus measure implicit in our welfare measure has been severely criticized by Silberberg (1972) as being entirely indeterminate. However, Willig (1976) has shown that so long as price variation is monotonic, income elasticities are moderate, and the budget share of the good in question is small, the Hicksian compensating and equivalent variations are close to one another, and the line integral defining the change in Marshallian consumer's surplus is a close approximation to both.

be measured in Figure 2 as the area lying vertically between the D and MC schedules and horizontally between  $Q = 1$  and  $Q = Q_I$ .

Figure 2 is drawn for  $\eta = 2.0$  and  $\varepsilon = 1.2$ . For these parameters,  $P_I = 0.761$  and  $Q_I = 1.728$ . Profit

$$\begin{aligned}\Pi_I &= P_I Q_I - C(Q_I) \\ &= -0.263\end{aligned}$$

and welfare gain

$$\Delta W_I = 0.052$$

are both measured as a fraction of total zero-profit sales, which have been normalized to unity. Note that despite the huge (72.8%) increase in output that would be implied by moving from the zero-profit outcome to the ideal outcome, the increase in welfare is rather small, being a mere 5.2% of zero-profit industry sales.

A monopolist who is sheltered from potential entrants will produce the quantity  $Q_M$  at which MC intersects MR, and then charge

$$P_M = D^{-1}(Q_M),$$

as shown in Figure 2. Using the parameters of Figure 2,

$$\begin{aligned}Q_M &= 0.216 \\ P_M &= 2.152 \\ \Pi_M &= 0.186 \\ \Delta W_M &= -0.349\end{aligned}$$

Although the sheltered monopoly generates a profit equal to 18.6% of zero-profit sales, these profits are far exceeded by the attendant loss in consumer surplus, for a net welfare loss

(relative to our zero-profit benchmark) of 34.9% of zero-profit sales.

Once  $K$  has been selected and installed, the interest cost on it becomes a sunk cost and is no longer relevant for the firm's decisions. Equation (2) may be solved for  $V$  to give variable cost (VC) as a function of  $Q$ , given  $K$ :

$$VC(Q; K) = K^{\frac{-\alpha}{1-\alpha}} b^{\frac{-1}{1-\alpha}} Q^{\frac{1}{\varepsilon(1-\alpha)}} \quad (8)$$

Short run (or, more precisely, ex post) total cost of producing  $Q$ , given  $K$ , is therefore

$$SRTC(Q; K) = VC(Q; K) + rK$$

These schedules are shown in Figure 3, along with the long run or ex ante total cost schedule  $C$ , for  $K = K^*(1)$ .  $SRTC$  and  $C$  are tangent at the level of  $Q$  (here unity) for which the sunk level of  $K$  is ex ante cost-efficient, and elsewhere  $SRTC$  lies above  $C$ . We define this tangency level of output to be the output capacity corresponding to the underlying level of  $K$ . Note that capacity is not an upper limit on output, as is often over-restrictively assumed in this literature, but rather is merely the ex ante cost-efficient level of output, which may or may not be desirable ex post.  $C(Q)$  is the envelope of  $SRTC(Q; K)$  for all values of  $K$ .

Figure 4 shows average and marginal cost schedules corresponding to the schedules of Figure 3. Average short-run total cost  $ASRTC$  is necessarily tangent to average long-run cost  $AC$  at capacity, here taken as unity. Marginal variable cost  $MVC$  necessarily crosses  $ASRTC$  at the minimum of the latter, and crosses marginal long run cost  $MC$  at capacity.

Long-Run, Short-Run, and Variable Cost

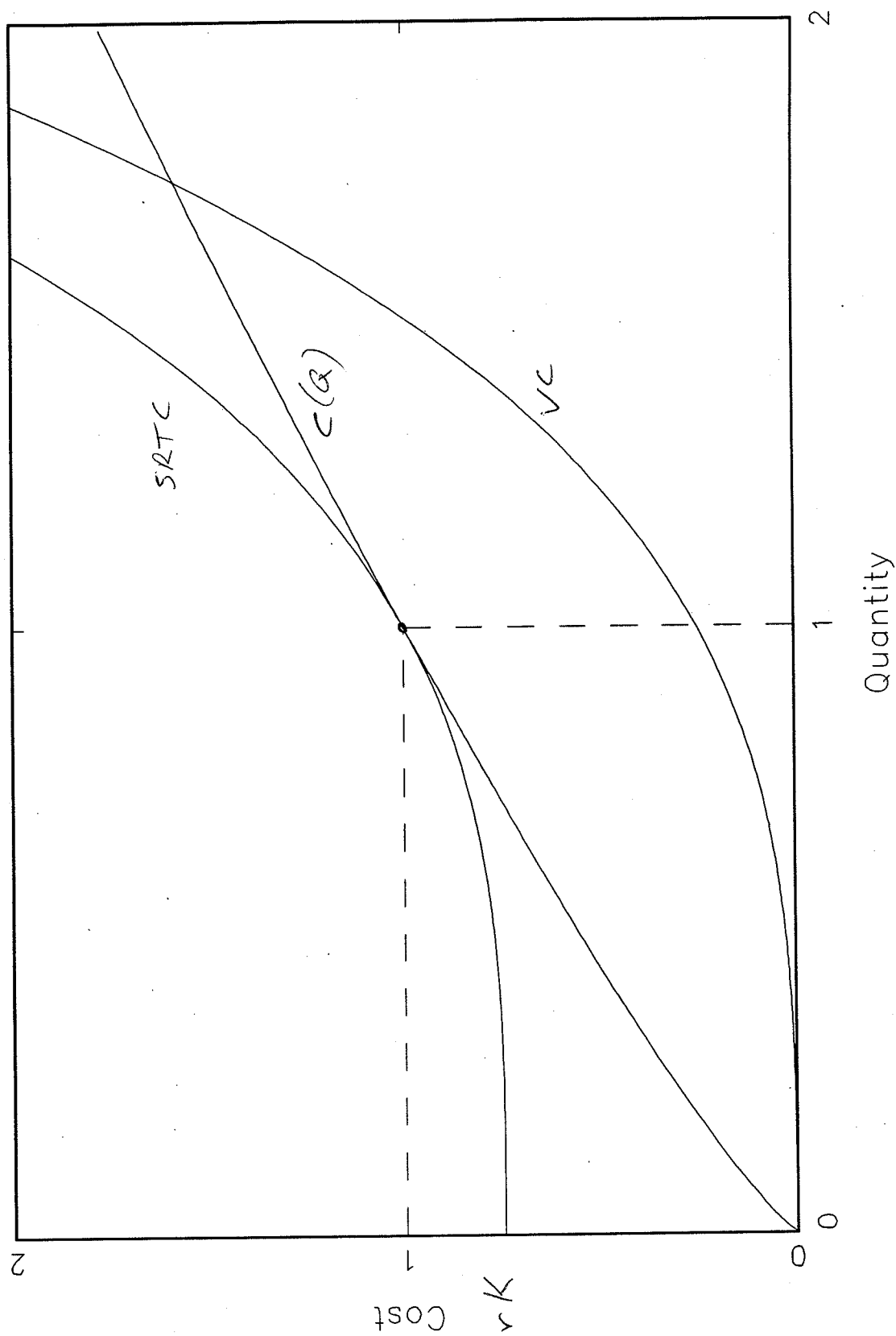
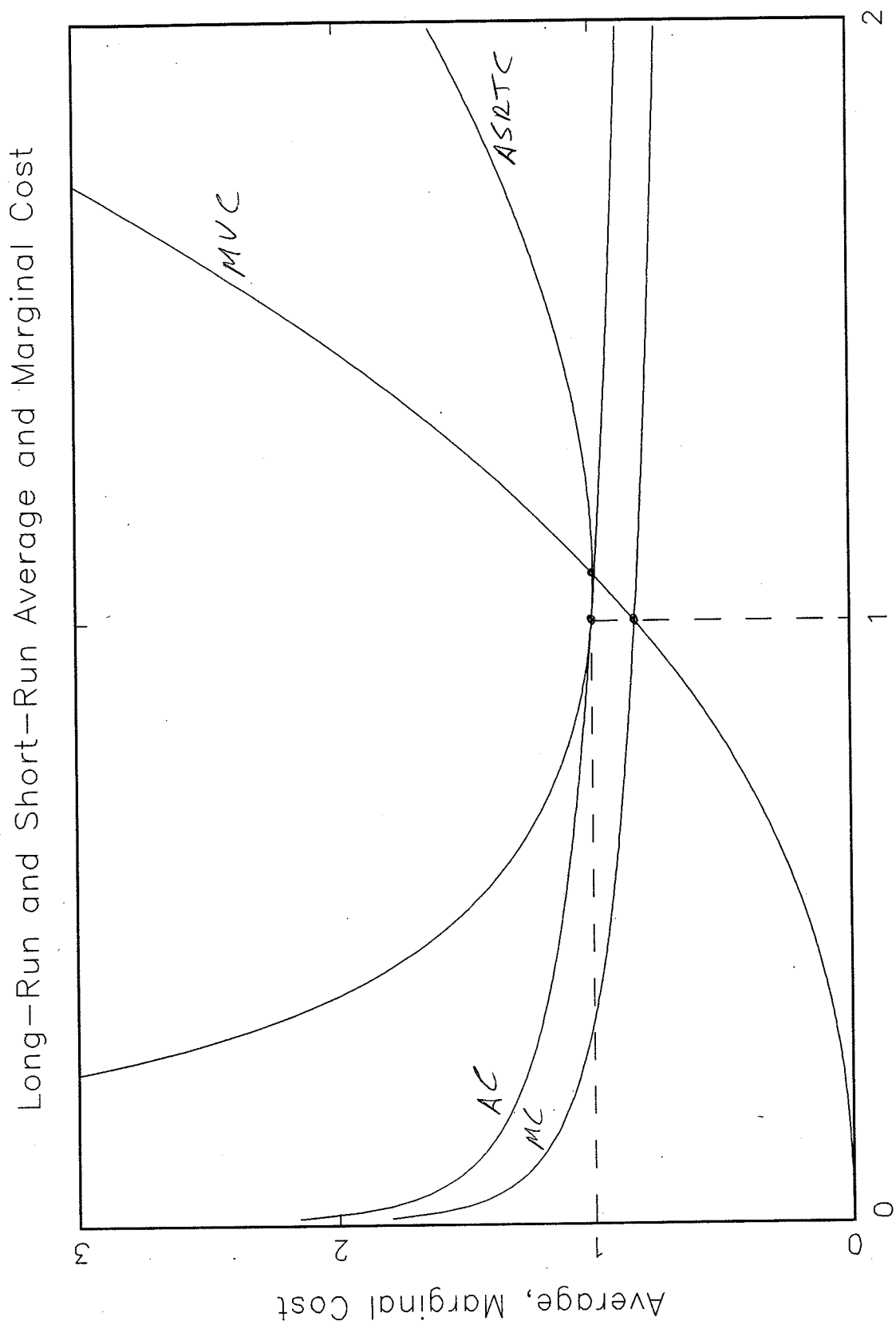


Figure 3



Quantity  
Figure 4

If a sheltered monopolist should, for any reason, has chosen a level of capacity different from  $Q_M$ , it will no longer choose the long-run (or ex-ante) monopoly price and quantity combination shown in Figure 2. Instead, it will select its quantity by equating MVC to MR, as in Figure 5, which is drawn for  $K$  corresponding to unit capacity (as evidenced by the intersection of MVC and MC). This ex post, or short-run monopolist will elect to produce below capacity since its capacity is greater than monopoly capacity., but still will produce a much larger quantity at a much lower price than will the ex ante monopolist. Using the parameters for which Figures 2 and 5 are drawn,

$$Q_M^{SR} = 0.835$$

$$P_M^{SR} = 1.094$$

The solid line in Figure 6 shows how the short-run monopoly profits of a sheltered monopolist vary with its choice of  $K$ , plotted as a fraction of its zero-profit setting  $K_0$ . Note that if this ratio is unity, short-run monopoly profits will be positive, since the short-run monopolist will not select the cost-efficient ratio of  $V$  to  $K$  assumed in Figure 5, but rather will cut back on  $V$  so as to raise price and profits.

To the extent the threat of potential entry can induce the first firm into the industry to select a large value of  $K$ , the loss in consumer's surplus and in total welfare from monopolization of the industry will be greatly reduced.

At time 0, when this industry comes into being, there are an infinite number of potential entrants, each with access to the

# Short Run Monopolist

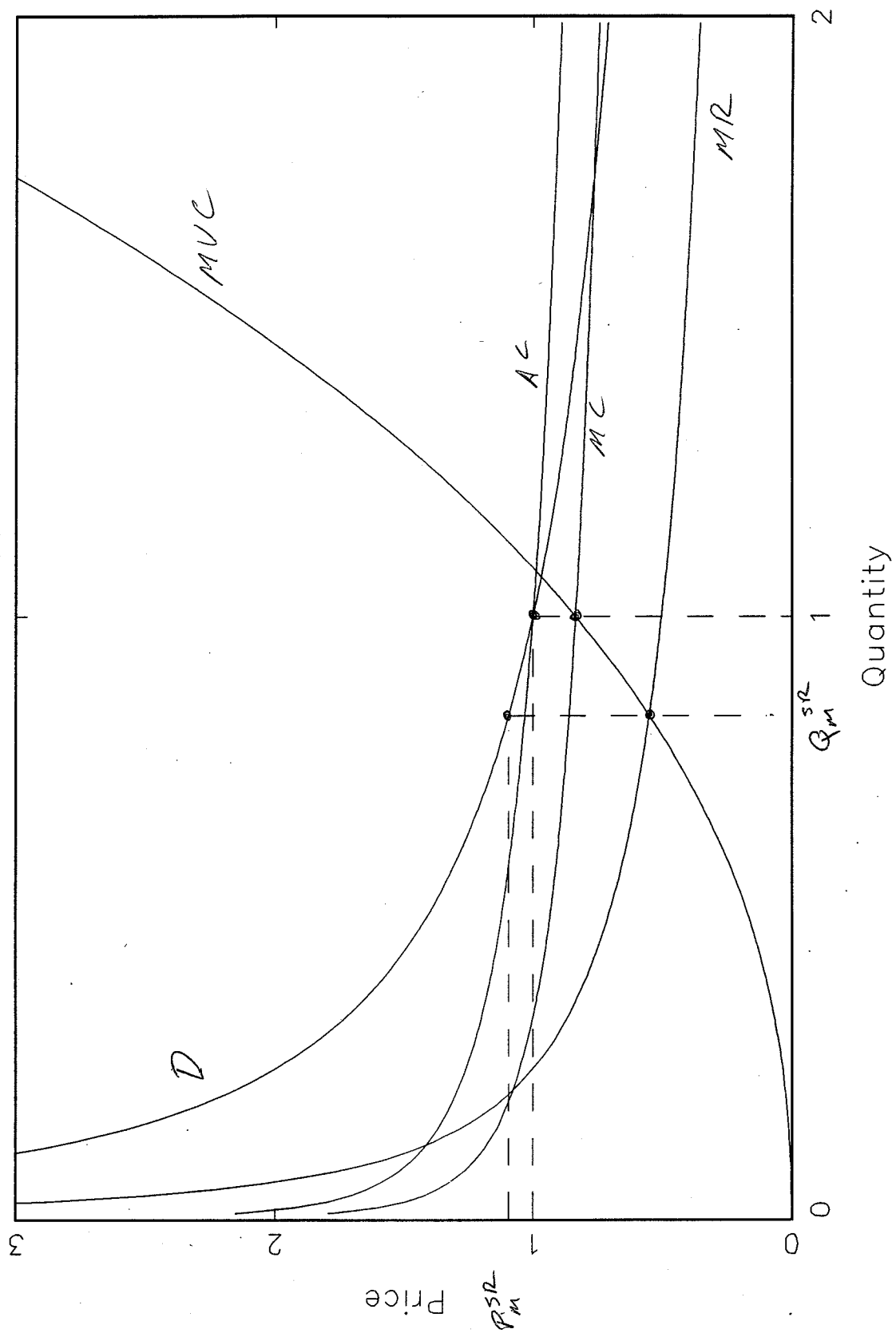


Figure 5

# Firm 1's K Decision

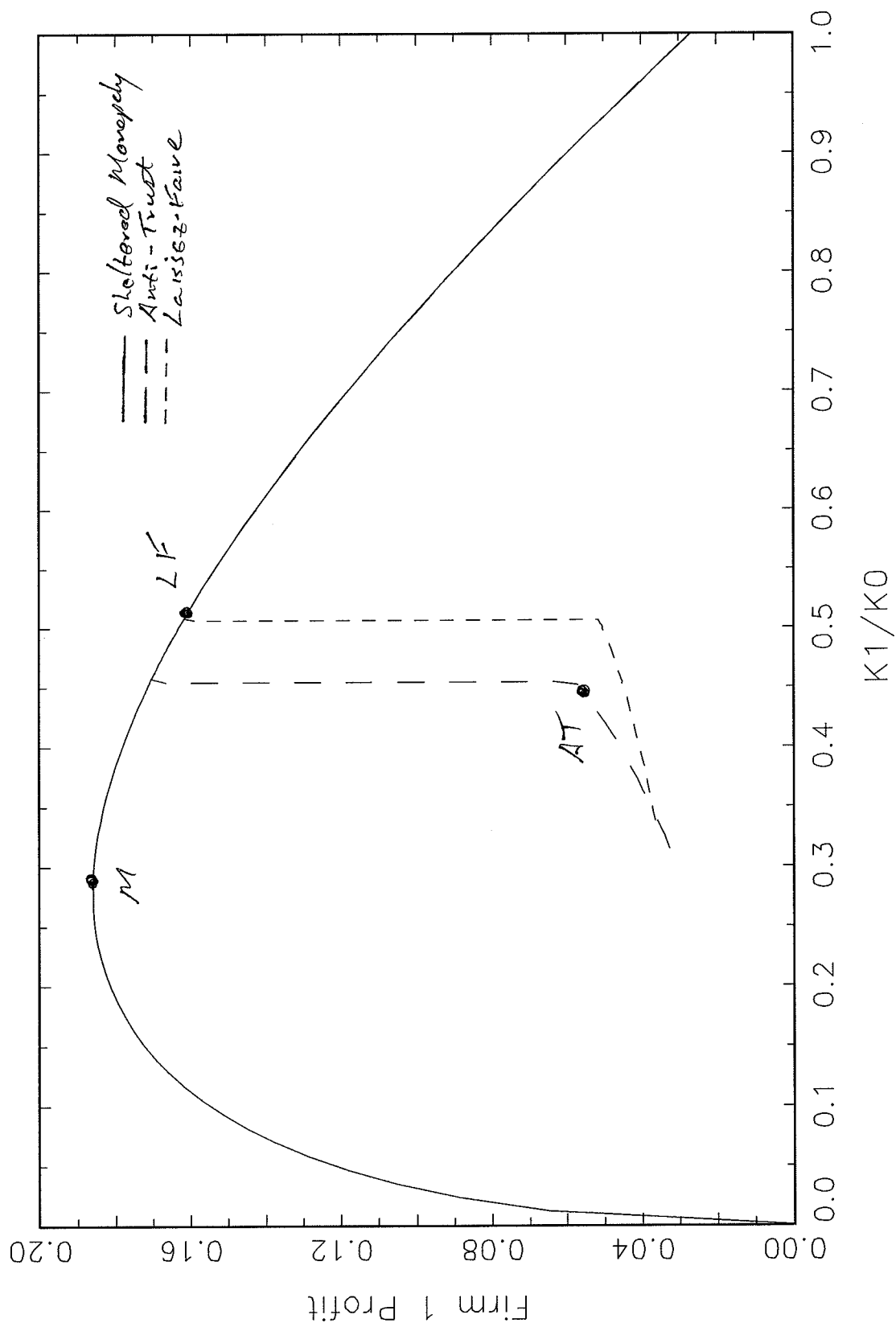


Fig. 6

production function (2), who would like to enter the industry by acquiring some of the sunk factor. They simultaneously reach for their telephones to place their orders. They succeed in placing their calls in a negligible amount of time, but in a randomly determined order. Let Firm  $i$  be the  $i$ -th firm in this sequence, and  $K_i$  be the amount of the sunk factor it orders and instantaneously installs. Each firm sees to it that subsequent firms in the sequence know how much it has ordered, in order to discourage market-spoiling overinvestment. Let firm 1 install  $K_1$ , giving it marginal variable cost schedule  $MVC_1$  in perpetuity.

### The Antitrust Duopoly

Under antitrust, Firm 1 must select  $K_1$  so as to ensure that at least one more firm, with non-zero capacity, will co-exist with it. With increasing returns to scale, this second firm will ordinarily have a smaller, yet significant share of the market, and we assume that just one such additional firm is sufficient to meet antitrust requirements.

In most of the cases we will encounter, it is reasonable to assume that once the second firm has selected its  $K_2$ , the industry will behave like a *price-leader duopoly*.<sup>7</sup> The larger firm, which will ordinarily be Firm 1, will behave as the price leader.

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<sup>7</sup> This follows from our assumption that consumers are lexicographic shoppers, when MVC is increasing and when one firm is unambiguously larger than the other. When the two firms are equal in size, each would like the other to be the price leader, which may result in a standoff. In the present model, however, Firm 1 will always choose more capacity than Firm 2. If  $\alpha < 1 - 1/\varepsilon$ , MVC is downward sloping, and Firm 2 cannot be a price taker. This case is discussed further below.

Firm 2 will take the duopoly price  $P_D$  set by Firm 1 as given and maximize its profits by setting its output  $Q_2$  equal to  $MVC_2^{-1}(P_D)$ . Firm 1 may thus take Firm 2's MVC schedule as its supply schedule. This gives Firm 1 a net demand schedule  $D_1$  equal to the industry demand schedule  $D$  minus the quantity supplied at each price by Firm 2, and a corresponding marginal revenue schedule  $MR_1$ , as shown in Figure 7. Firm 1 then selects the  $P_D$  that maximizes its profits by finding its output  $Q_1$  at the intersection of  $MVC_1$  with  $MR_1$ , and then reading  $P_D$  off the  $D_1$  schedule at this  $Q_1$ , as indicated.

Figure 8 shows Firm 1's total profit under such a duopoly (net of the interest cost of  $K$ ) as a function of  $K_1 / K_0$  and  $K_2 / K_0$ , where  $K_0 = K^*(1)$  is the level of the sunk factor that gives unit output capacity. The small wrinkle in the surface is caused by the reversal in their roles as price leader and price follower should Firm 2, for whatever reason, become larger. Apart from this wrinkle, Firm 1 profits increase as Firm 2 capacity decreases, for each level of Firm 1 capacity. As Firm 2 capacity goes to 0, Firm 1 behaves like a short-run monopolist, as already shown by the solid line in Fig. 6.

Figure 9 shows Firm 2's total profit under this duopoly, again as a function of  $K_1 / K_0$  and  $K_2 / K_0$ . (This is, in fact, the same surface shown in the preceding diagram, but with the roles of the two firms reversed.) Figure 10 shows a contour map of this same surface.

Under antitrust, Firm 1 must choose its own capacity small enough that Firm 2 will choose to enter with some positive level

# Price-Leader Duopoly

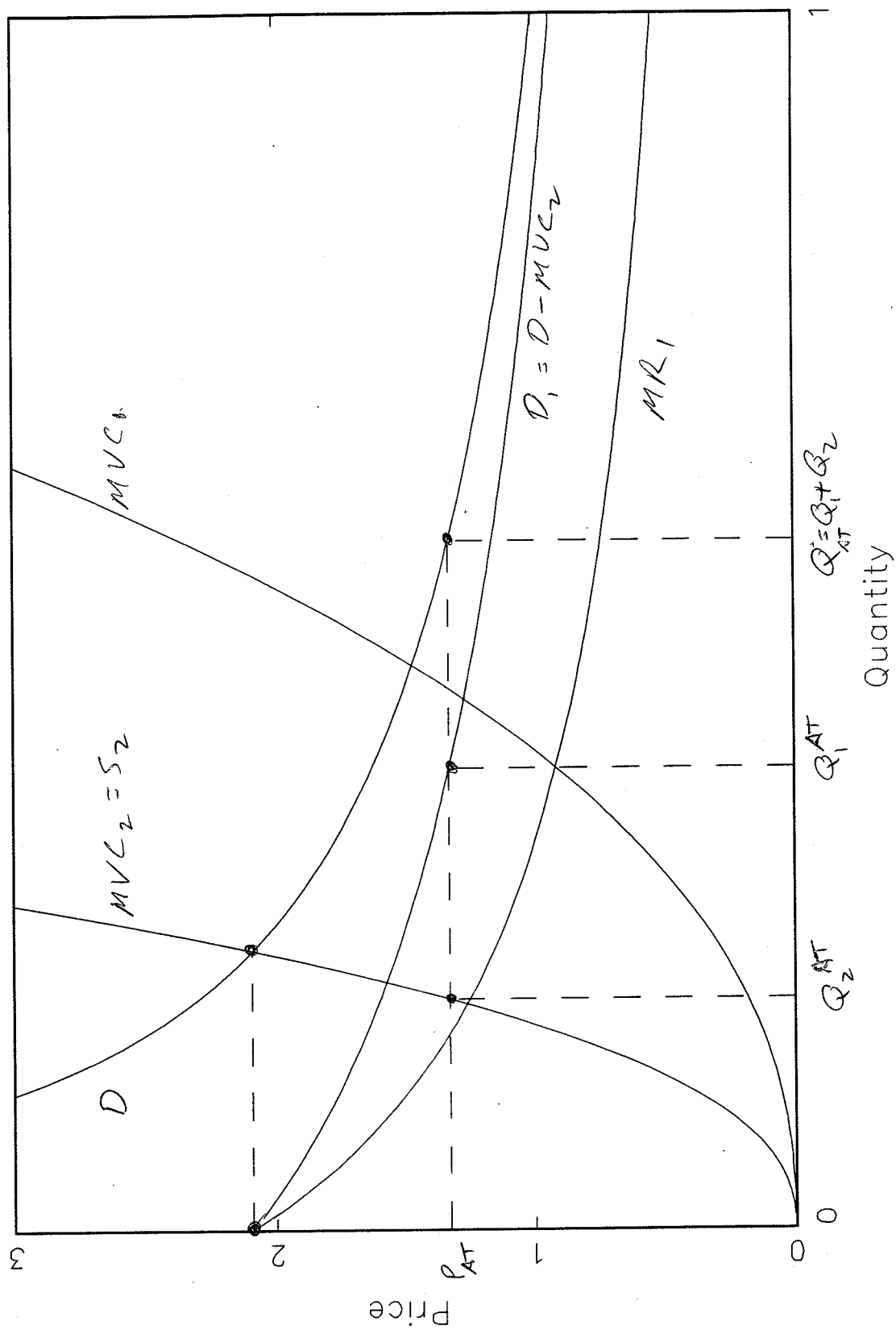


Figure 7.

Firm 1 Duopoly Profit

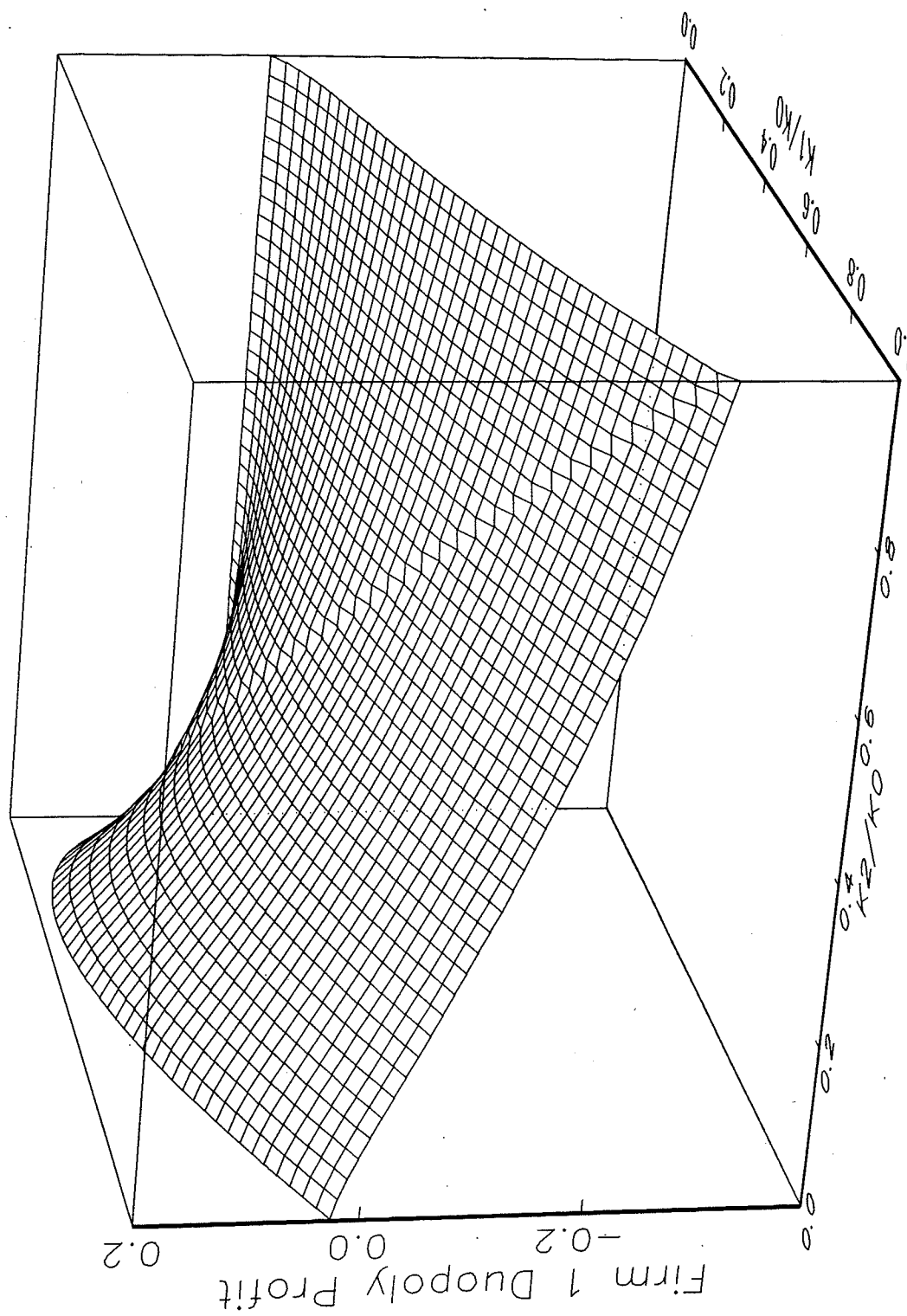


Figure 8

Firm 2 Duopoly Profit and Firm 1 Antitrust Strategy

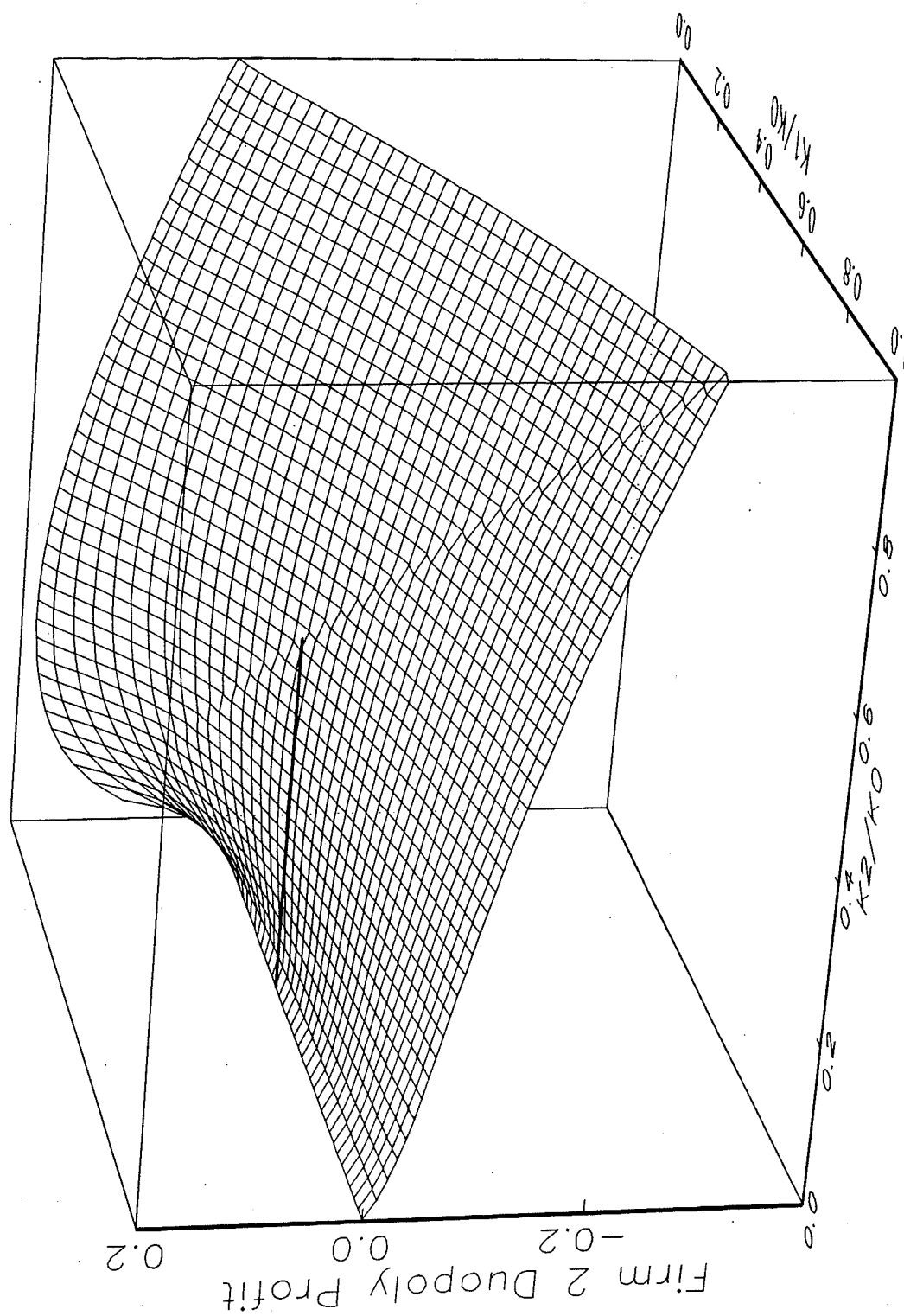


Figure 9

Firm 2 Duopoly Profit Contours and Firm 1 Antitrust Strategy

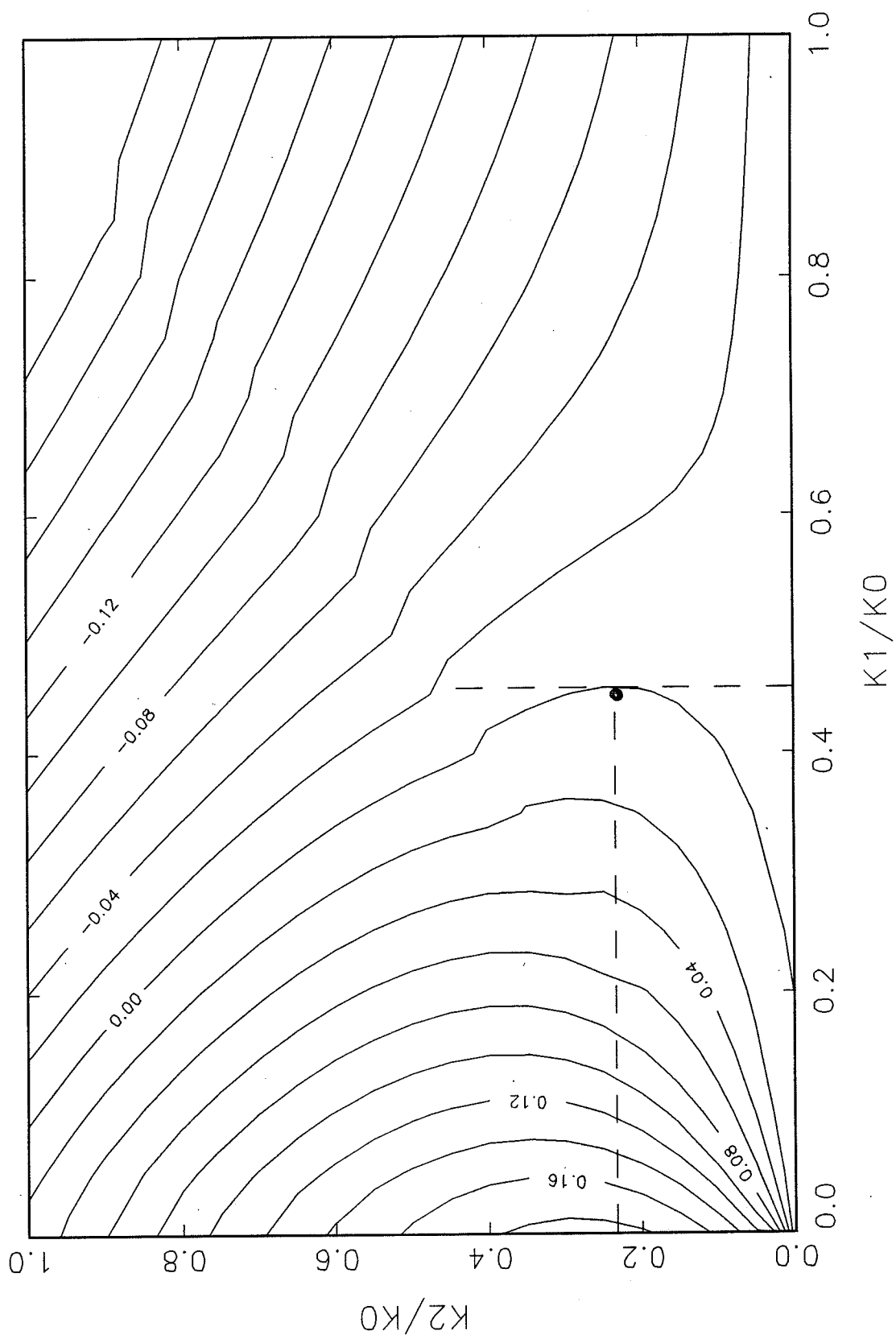


Figure 10

of capacity. For this purpose, Firm 1's pricing strategy during the brief moment that it holds a monopoly of production is irrelevant to Firm 2's decision, because Firm 2 knows that Firm 1 will change from a short-run monopolist to a price-leader duopolist once Firm 2 enters the market, and that price and profits will thereafter be determined solely by the two firms' investment in the sunk factor. Firm 1's antitrust strategy will ordinarily be to set its own capacity as high as possible, consistent with Firm 2 being guaranteed a positive profit. This can be visualized in Figure 10 by finding the point of tangency between Firm 2's 0-profit contour and a vertical line. (The same line is also shown in Figure 9.)

For the illustrative parameters we have been using, ( $\eta = 2$ ,  $\varepsilon = 1.2$ ,  $\alpha = .75$ , this optimal antitrust strategy yields

$$K_1^{AT} / K_0 = 0.455^-$$

$$K_2^{AT} / K_0 = 0.234$$

$$Q_1^{AT} = 0.378$$

$$Q_2^{AT} = 0.188$$

$$Q_{AT} = 0.566$$

$$P_{AT} = 1.329$$

$$\Pi_1^{AT} = 0.058$$

$$\Pi_2^{AT} = 0.000^+$$

$$\Delta W_{AT} = -0.190$$

If a second firm just breaks even, a third firm would be even less profitable. Under this antitrust duopoly outcome, there will therefore be no incentive for a third firm to enter.

### **Laissez-Faire**

Under laissez-faire, Firm 1 will instead choose its capacity  $K_1$  so as to just guarantee that Firm 2 is *ex ante* unprofitable. Having thus eliminated Firm 2, Firm 1 will then act like a short-run monopolist, as in Figure 5, but with an MVC schedule determined by  $K_1$ .

Firm 2's profit prospects are now somewhat increased, however, by the possibility, precluded under antitrust, of cooperating with Firm 1, either as a cartel or by either firm purchasing the other and operating as a *two-plant monopolist*, as illustrated in Figure 11. Given two MVC schedules, a cartel or two-plant monopolist can improve on the price-leader duopoly outcome by equalizing marginal variable costs in the two plants, and then equating the common MVC to total MR, as illustrated. Because Firm 2, if it enters, can demand a cut in these increased profits from Firm 1, Firm 2 is actually more profitable than in the antitrust case illustrated Figures 9 and 10, and so Firm 1 will invest in significantly more capacity under laissez-faire than under antitrust. This will significantly mitigate the rise in price that would otherwise occur from Firm 2's absence.

If the two firms do not cooperate or merge, the outcome will be a price-leader duopoly. If they do cooperate, their mutual profits will be increased. This mutual benefit does not begin to

# Cartel / Two-Plant Monopolist

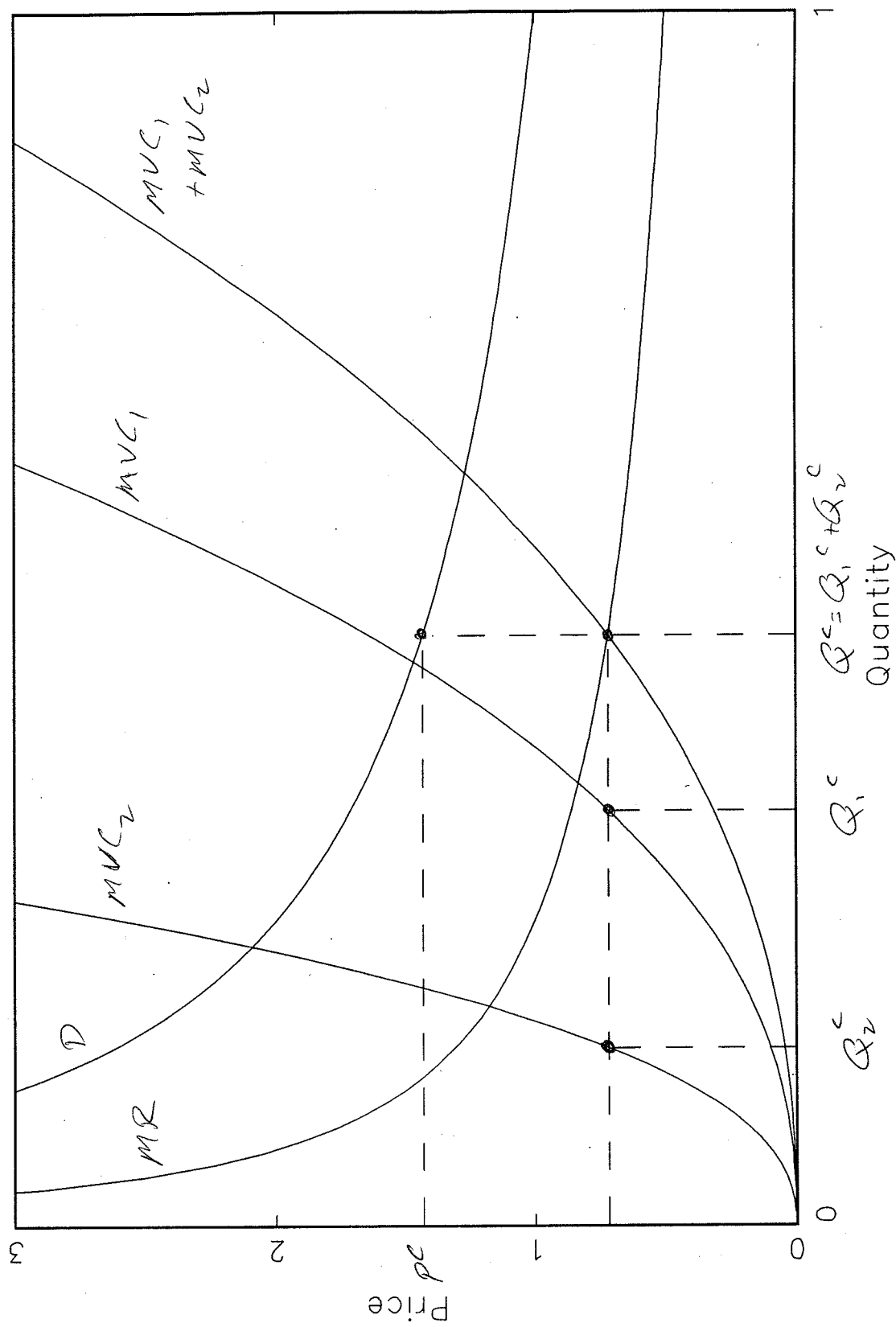


Figure 11

accrue until they have both agreed to cooperate. It does not matter whether Firm 1 buys Firm 2, Firm 2 buys Firm 1, or they simply cooperate in exchange for a side payment. It does make a big difference, how the surplus from merging or cooperating is divided between the two firms.

Ariel Rubenstein (1982) has solved this bargaining problem in the case of discrete time. It follows immediately from his result, in the limit of continuous time, that if two bargainers having continuously compounded discount rates  $r_1$  and  $r_2$  must agree on how to divide a "pie" before the benefits begin to accrue to either, that the unique subgame perfect bargaining equilibrium is that bargainer 1's share will be  $r_2 / (r_1 + r_2)$  and bargainer 2's share will be  $r_1 / (r_1 + r_2)$ .<sup>8</sup> In the present model, both bargainers have the same discount rate,  $r$ , and so will divide the surplus equally. Their bargaining power thus depends only on their relative discount rates, and not on their relative sizes.

Figure 12 shows a contour map of Firm 2's prospective profits from entering the market and thus participating in a cartel or merger. Again, Firm 1's strategy is to find the point of tangency between a vertical line and Firm 2's 0-profit contour.

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<sup>8</sup> Rubenstein in fact considers a discrete time situation in which the bargainers take turns making proposals for splitting the "pie," with bargainer 1 going first. In this case his result is that bargainer 1 gets  $(1 - \delta_1) / (1 - \delta_1\delta_2)$ , and bargainer 2 gets 1 minus this, where the  $\delta_i$  are the respective 1-period discount factors. This implies that if the two discount rates are equal, bargainer 1 gets slightly more than 50% of the pie. However, setting  $\delta_i = \exp(-r_i\Delta t)$  and taking the limit  $\Delta t \rightarrow 0$ , eliminates the first-move advantage and gives the result in the text.

Firm 2 Cartel Profit Contours and Firm 1 Laissez-Faire Strategy

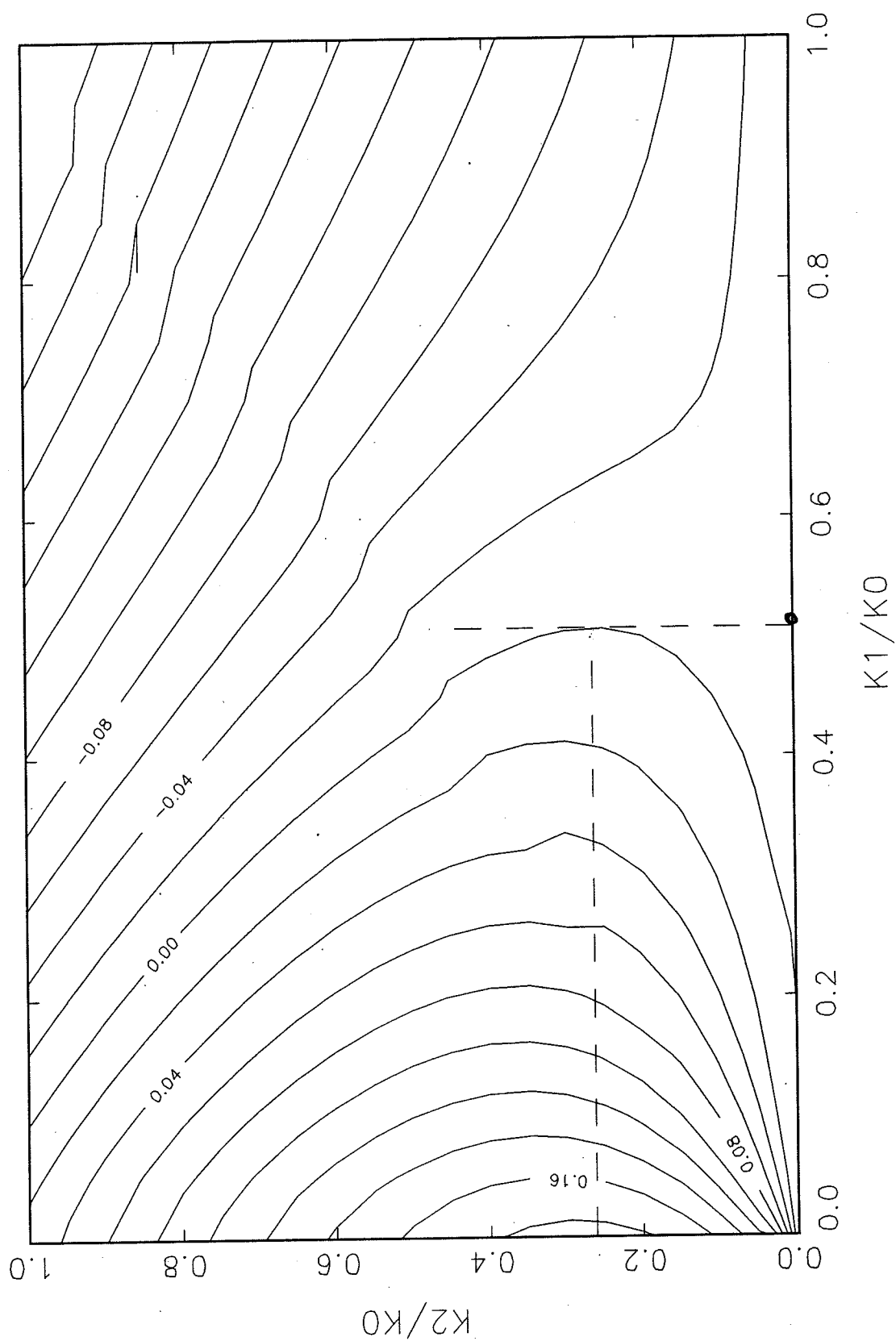


Figure 12

However, Firm 1 now sets its capacity slightly *higher* than this level, so as to discourage Firm 2 from entering.

With  $K_1$  thus determined and Firm 2 out of the way, Firm 1 will be free to operate as a short-run monopolist, as shown in Figure 13. The welfare loss from antitrust (relative to our zero-profit reference case) can be visualized as the sum of two areas: first, the area between D and MC from  $Q = 1$  back to Firm 1's capacity output at the intersection of MC and MVC, reflecting the loss caused by the reduction in both factors in equal proportions to this point, and second, the area between D and MVC from this capacity output on back to  $Q_{LF}$ , reflecting the loss caused by the further reduction in V alone.

Using our illustrative parameters, the laissez-faire outcome is:

$$\begin{aligned} K_1^{LF} / K_0 &= 0.507^+ \\ Q_1^{LF} &= 0.407 \\ P_{LF} &= 1.568 \\ \Pi_1^{LF} &= 0.162 \\ \Delta W_{LF} &= -0.200 \end{aligned}$$

For these parameters, at least, we see that antitrust gives slightly less welfare cost than does laissez-faire (both taken relative to our zero-profit reference point). The difference, however, is slight, being only 0.010, or 1.0% of zero-profit sales volume.

#### Variation in $\alpha$

Laissez-Faire Outcome

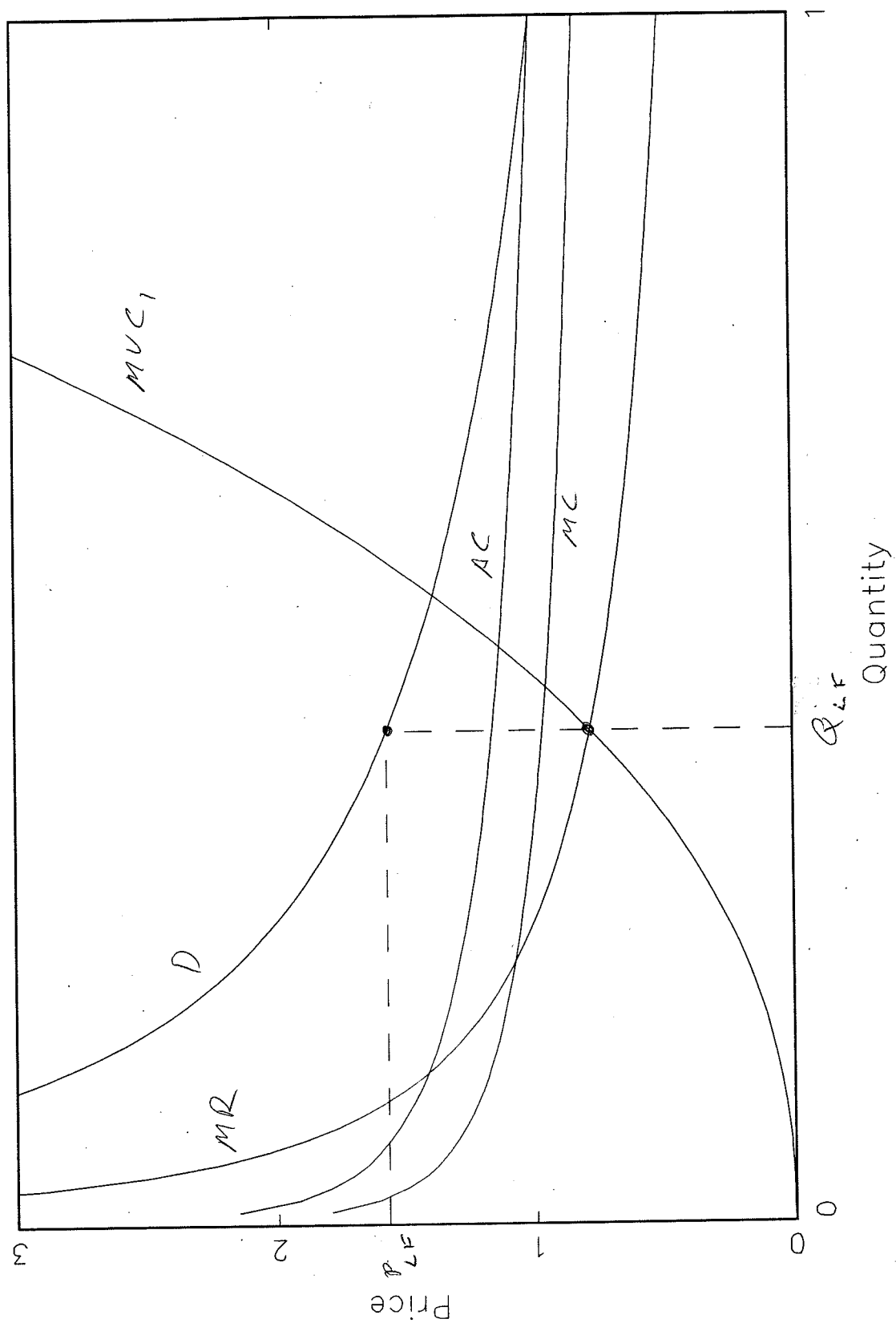


Figure 13

Table 1 investigates changes in  $\alpha$ , the share of the interest cost of the sunk factor in total cost when factors are used in their cost-efficient proportions, holding the other two parameters constant at their illustrative values.

As  $\alpha$  is raised above .75 to .90, LF continues to be slightly less efficient than AT. As  $\alpha$  is lowered to .50, LF gains a slight edge over AT.

At  $\alpha = .20$ , however, an unexpected development occurs. Figure 14 shows Firm 1's AT strategy on a contour plot of Firm 2's AT duopoly profits, while Figure 15 shows Firm 1's AT duopoly profits relative to the same coordinates. It is apparent from the two figures that Firm 1's profits are negative for any value of Firm 1 capacity that allows Firm 2 to be profitable. Under AT, Firm 1 will therefore never enter, nor will any other firm. The industry therefore shuts down, quantity is 0, price is infinite, and the welfare loss reflects the entire area under the demand curve from 0 quantity out to unit quantity, minus the unit costs of producing unit quantity. This welfare loss from AT is 1.000 using the illustrative value of the demand elasticity.

LF, on the other hand, is at least operational, producing a welfare loss of only .250. In this case and <sup>its</sup> surrounding region, therefore, AT is far inferior to LF. (This case appears to set in at about  $\alpha = 0.27$ .)

With constant elastic demand and scale economies, there is always a range of capacities at which two firms can both be profitable, even if both are price takers, so our finding that AT can force the closing of the industry is surprising. This unexpected

Table 1

Variation in  $\alpha$  (holding  $\eta = 2$ ,  $\varepsilon = 1.2$ )

$\alpha$	$\Delta W_I$	$P_M$	$Q_M$	$\Delta W_M$	$P_{AT}$	$Q_1^{AT}$	$Q_{AT}$	$\Delta W_{AT}$	$P_{LF}$	$Q_{LF}$	$\Delta W_{LF}$
.20	.052	2.152	.216	-.349	$\infty$	0	0	-1.000	1.551	.416	-.250
.50	.052	2.152	.216	-.349	1.316	.362	.540	-.201	1.522	.432	-.196
.75	.052	2.152	.216	-.349	1.329	.378	.566	-.190	1.568	.407	-.200
.90	.052	2.152	.216	-.349	1.320	.383	.574	-.187	1.595	.393	-.206

Table 2

Variation in  $\varepsilon$  (holding  $\eta = 2$ ,  $\alpha = .75$ )

$\varepsilon$	$\Delta W_I$	$P_M$	$Q_M$	$\Delta W_M$	$P_{AT}$	$Q_1^{AT}$	$Q_{AT}$	$\Delta W_{AT}$	$P_{LF}$	$Q_{LF}$	$\Delta W_{LF}$
1.01	.000	2.008	.248	-.255	1.0(?)	(?)	1.0(?)	0(?)	1.0(?)	1.0(?)	0(?)
1.02	.000	2.015	.246	-.261	>1.072	<.841	<.871	<-.050	1.065	.882	-.055
1.05	.003	2.038	.241	-.276	>1.139	<.705	<.770	<-.057	1.163	.739	-.059
1.1	.011	2.076	.232	-.302	1.220	.558	.672	-.095	1.305	.587	-.098
1.2	.052	2.152	.216	-.349	1.329	.378	.566	-.190	1.568	.407	-.200
1.3	.140	2.226	.202	-.393	1.407	.264	.505	-.277	1.833	.298	-.303
1.4	.316	2.298	.189	-.434	$\infty$	0	0	-1.000	2.140	.218	-.403

Table 3

Variation in  $\eta$  (holding  $\varepsilon = 1.2$ ,  $\alpha = .75$ )

$\eta$	$\Delta W_I$	$P_M$	$Q_M$	$\Delta W_M$	$P_{AT}$	$Q_1^{AT}$	$Q_{AT}$	$\Delta W_{AT}$	$P_{LF}$	$Q_{LF}$	$\Delta W_{LF}$
1.1	.021	15.07	.051	-1.697	>1.670	<.521	<.569	<-.300	1.793	.526	-.434
1.5	.033	3.393	.160	-.588	1.407	.465	.599	-.194	1.602	.493	-.211
2.0	.052	2.152	.216	-.349	1.329	.378	.566	-.190	1.568	.407	-.200
3.0	.122	1.562	.262	-.213	$\infty$	0	0	-.500	1.557	.265	-.212
4.0	.355	1.372	.284	-.166	$\infty$	0	0	-.333	1.372	.282	-.166

Firm 2 Duopoly Profit Contours and Firm 1 Antitrust Strategy

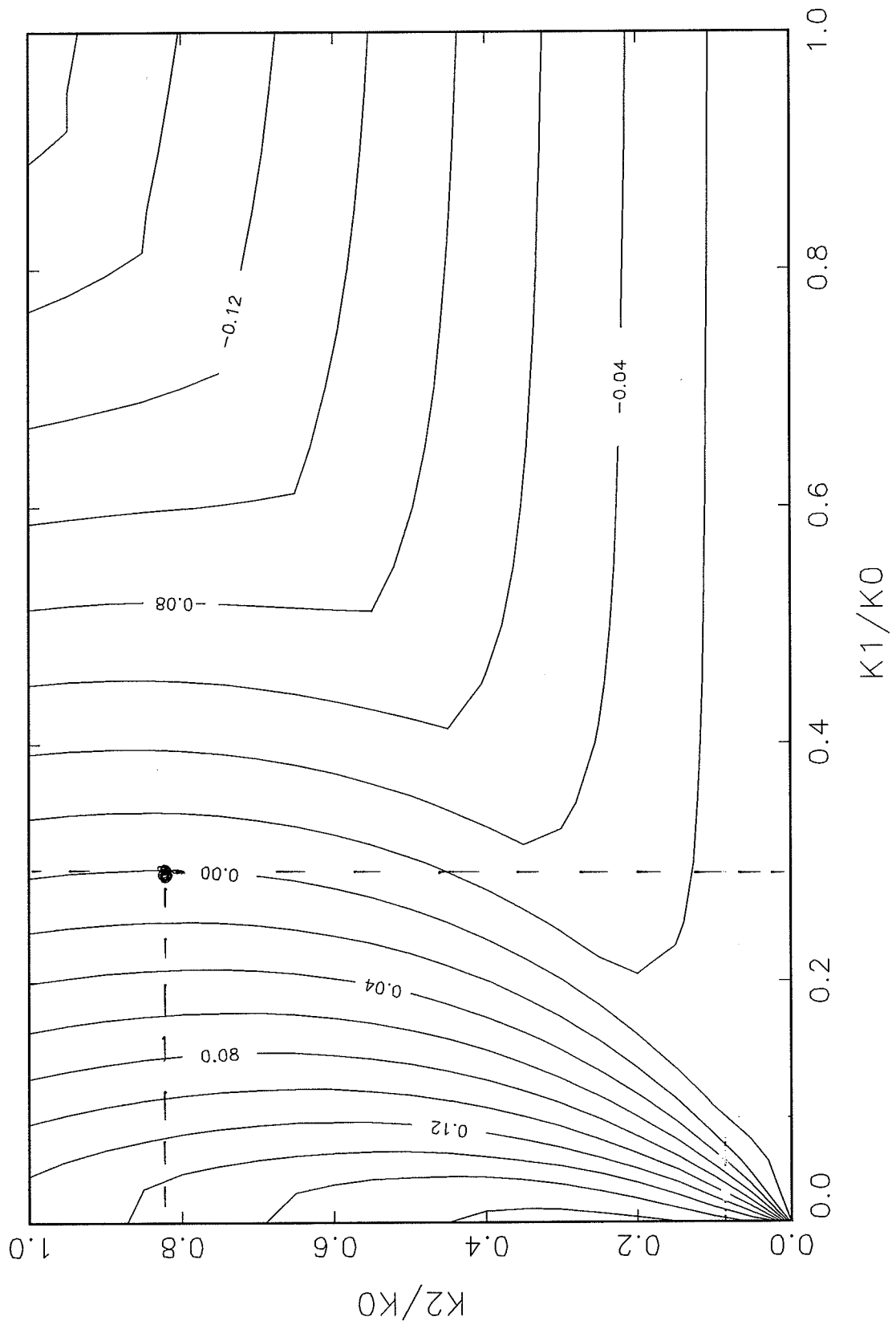
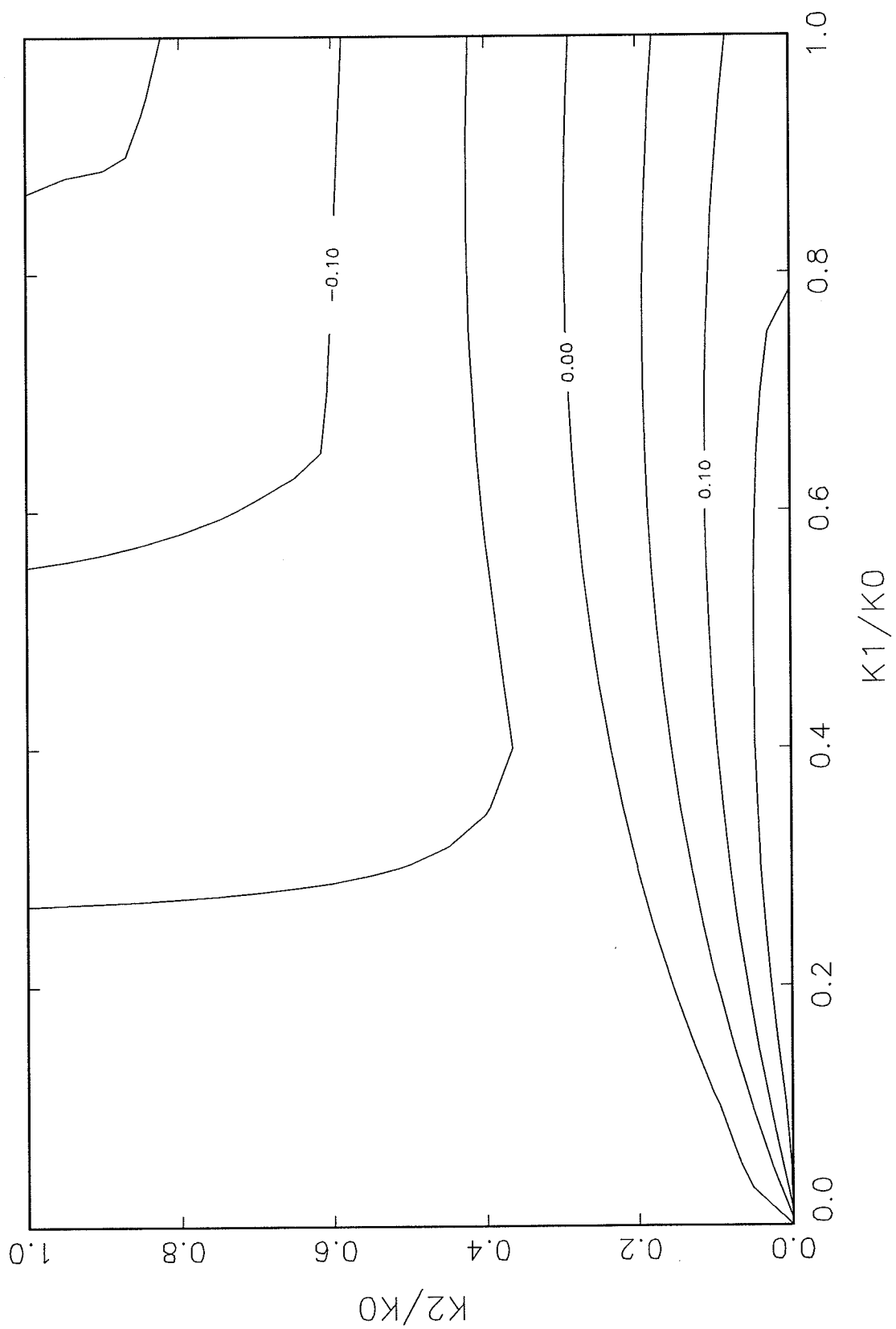


Fig. 14

$\alpha = 1.2$   
 $\gamma = 2$   
 $\epsilon = 1.2$

Firm 1 Duopoly Profit Contours



$\alpha = 2$   
 $\gamma = 2$   
 $\epsilon = 1.2$

Fig. 15

result occurs because of the sequential nature of capacity choice. Firm 1 could choose a very low level of capacity that would enable both it and Firm 2 to be profitable, provided Firm 2 also chooses a low capacity. However, Firm 2 would rather increase its profits by increasing its capacity to such a high level that Firm 1 is no longer profitable. Firm 1 cannot exit, so Firm 2 need not fear becoming a monopolist. Even if Firm 1 goes bankrupt because it cannot pay the interest cost of its sunk factor, its creditors will seize its capacity and continue to operate it in competition with Firm 2.

In order for the smaller firm to be a price taker as we have assumed in these calculations, its MVC curve must be upward sloping. This in turn requires that  $\alpha$  be greater than  $1 - 1/\varepsilon$ . For  $\varepsilon = 1.2$  as in Table 1, this requires  $\alpha > 1/6 = 0.167$ . It is not at present clear to the author what happens for  $\alpha$  below this critical value. Probably the smaller firm must take the larger firm's quantity as well as price as given, and then the larger firm optimizes over both these variables rather than just price. It is expected that LF will continue to dominate AT in this region.

#### **Variation in $\varepsilon$**

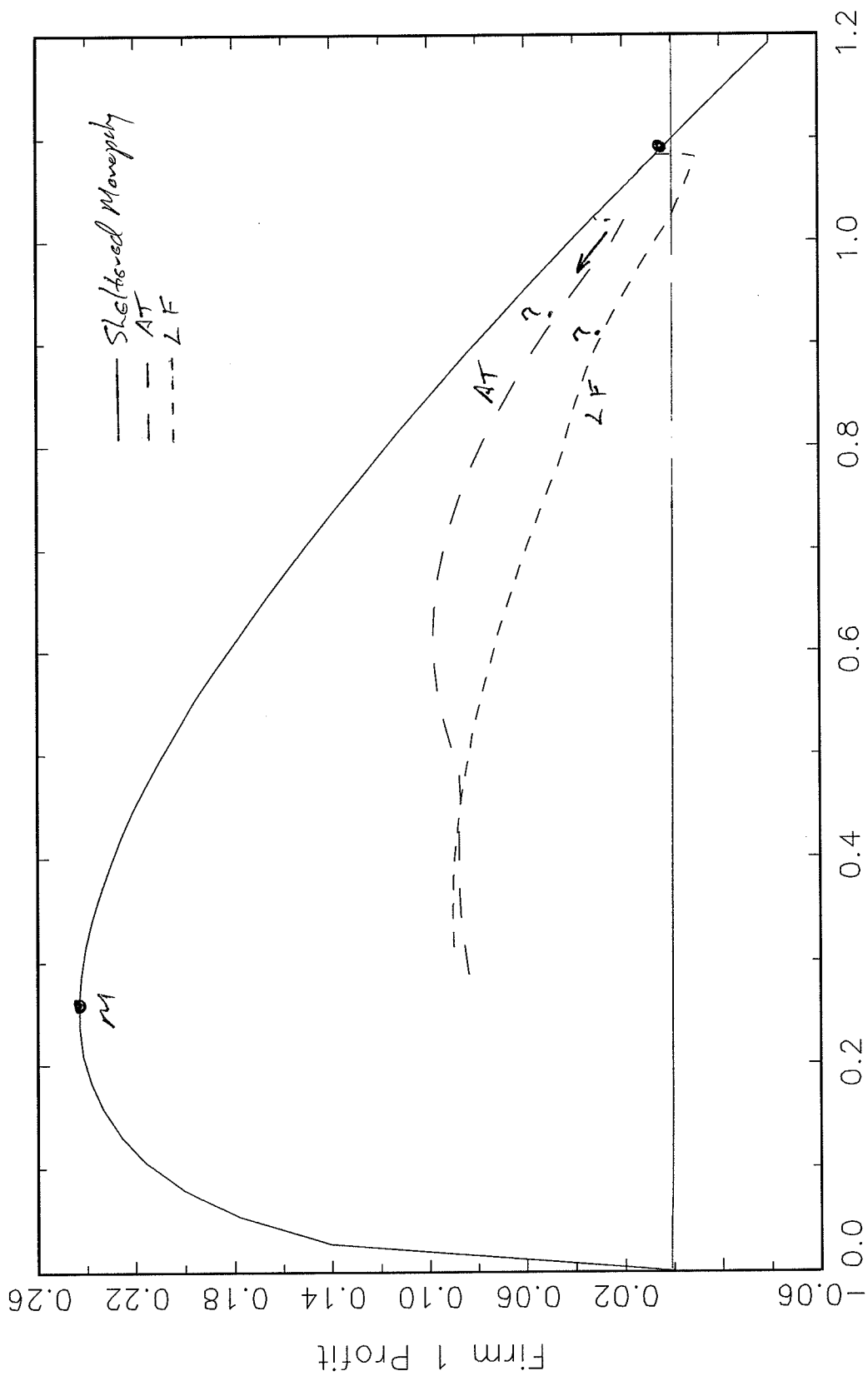
Table 2 investigates variation in the scale elasticity  $\varepsilon$ , holding the other two parameters constant at their illustrative values. As  $\varepsilon$  rises above 1.2 to 1.3, the small edge of AT over LF grows to .026, or 2.6% of zero-profit sales. But when  $\varepsilon$  rises to 1.4, AT once again breaks down, for the same reason as above;

there is no level of Firm 1 capacity that ensures that it will be profitable. This situation continues as  $\varepsilon$  rises to its maximum permissible value, given  $\eta$ , of 2.000. In this range AT is clearly worse than LF.

As  $\varepsilon$  falls from 1.2 to 1.1, AT's edge over LF falls to a mere .003. At  $\varepsilon = 1.05$  and 1.02, another unexpected phenomenon develops, illustrated in Figure 16 for  $\varepsilon = 1.02$ : Under AT, if Firm 1 just excludes Firm 2, Firm 1's capacity will very very high relative to Firm 2's. If Firm 1 then reduces its capacity further, Firm 2 will grow, but not by so much as to reduce Firm 1's profits. Firm 2 therefore gains by making Firm 2 profitable, and the corner solution that usually is binding is no longer valid. We make no attempt here to calculate how far this will go; at some point, a third firm will become profitable and there will be a further discontinuous ~~fall~~ fall in Firm 1's profits. The long-dashed line in Figure 16 is calculated under the assumption that there are only two firms. All we can say for the moment, therefore, is that the welfare cost of AT is somewhat greater than at the calculated corner solution. This probably eliminates AT's slight edge over LF in these cases.

The short-dashed line Figure 16, showing Firm 1's LF profits under the assumption that there are only two firms in question, also rises as Firm 1 capacity falls below its critical, Firm 2-excluding value, and eventually rises well above LF monopoly profits. Again, it is not entirely clear to the author at the moment just what the implications of this are, particularly given the potential entry of additional firms, but it is possible that

# Firm 1's K Decision



$K1/K0$

$$\begin{aligned} \epsilon &= 1.02 \\ \gamma &= 2.0 \\ \alpha &= .75 \end{aligned}$$

Fig. 16

it indicates that LF will break down into a 0-profit solution, giving it a clear welfare advantage over AT.

When the scale elasticity drops even further, to 1.01, the profit schedules shown in Figure 16 look similar, but now the local maximum for LF at the critical capacity level gives *negative* profits. (See Figure 17). Here, the LF strategy clearly breaks down, degenerating into a 0-profit quasi-competitive equilibrium. Presumably LF dominates AT in this case.

### Variation in $\eta$

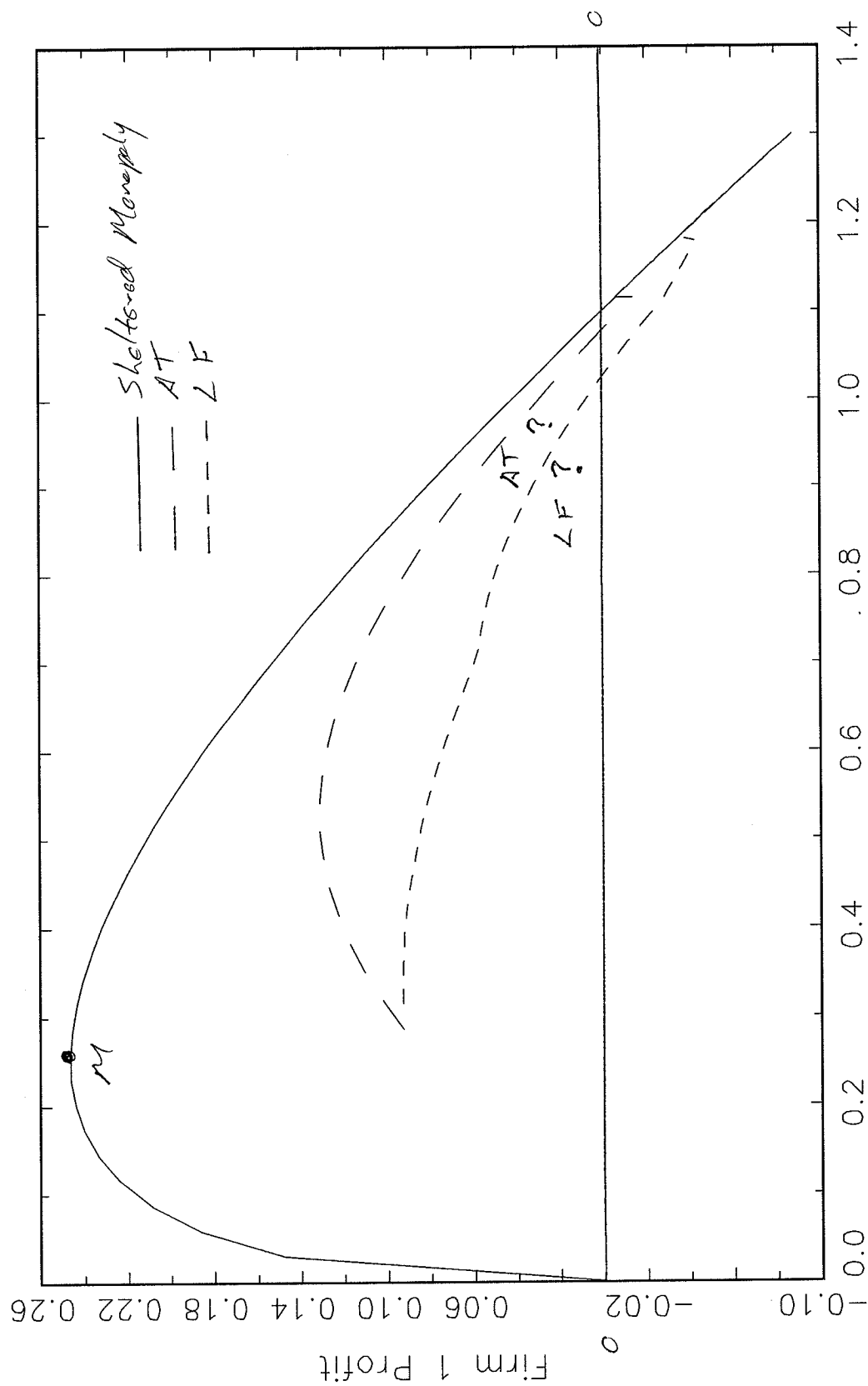
Table 3 investigates variation in the demand elasticity  $\eta$  about our illustrative value of 2.0, holding the other two parameters constant at their illustrative values. The highest the demand elasticity can go, given the scale elasticity, is 5.00. We do not at present consider values below or equal to 1.00, simply because our monopoly calculation would blow up in these cases. (The LF single-firm outcome would not yield these infinite monopoly profits, however!)

As  $\eta$  rises above 2.00, AT soon breaks down again as in Table 1 and the lower part of Table 2, because Firm 1 cannot find a capacity that guarantees both its and Firm 2's profitability simultaneously. AT is clearly dominated by LF in these cases.

As the demand elasticity falls below 2.00 to 1.50, the slight edge of AT over LF grows a little. ~~But as it falls further, as shown for fig 1, because figures above~~

As  $\eta$  falls further, to 1.10, the AT profit schedule once again begins to invert, so that just including Firm 2 is no

Firm 1's K Decision



$K1/K0$

Fig. 17

$$\epsilon = 1.01$$

$$\eta = 2.0$$

$$\alpha = .75$$

longer optimal. Instead, Firm 1 will move back, in all likelihood to the point illustrated in Figure 18. The welfare loss from AT will therefore be somewhat greater (in absolute value) than that calculated in Table 3 for the critical value of Firm 1 capacity. This will reduce, though probably not reverse, the otherwise rather large apparent edge of AT over LF for these parameters.

## **Conclusion**

TBA

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