The Ohio State University
Department of Economics
First Midterm Questions and Answers

Econ 5001
Spring 2020
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Directions: *Answer all questions, show all work, and label all figures.*

Note: There were 4 versions of the exam, A, B, C, and D, based on the label of player 1's left initial choice on question 1. The questions are repeated here for exam A. For the game tree, see the exam which is posted separately.

1. (20 points) *Convert the following extensive form game into normal form, by drawing the payoff matrix, labeling the strategies corresponding to the rows and columns, and filling in the payoffs.*

exam A:

player 1 \ player 2	DF	DG	EF	EG
A	1, 12	1, 12	2, 11	2, 11
B	3, 10	4, 9	3, 10	4, 9
C	5, 8	6, 7	5, 8	6, 7

exam B:

player 1 \ player 2	XS	XT	YS	YT
B	0, 1	0, 1	2, 3	2, 3
C	4, 5	4, 5	6, 7	6, 7
D	8, 9	10, 11	8, 9	10, 11

exam C:

player 1 \ player 2	WY	WZ	XY	XZ
C	6, 12	6, 12	5, 11	5, 11
D	4, 10	3, 9	4, 10	3, 9
E	2, 8	1, 7	2, 8	1, 7

exam D:

player 1 \ player 2	AY	AZ	BY	BZ
D	10, 9	10, 9	8, 7	8, 7
E	6, 5	6, 5	4, 3	4, 3
F	2, 1	0, 0	2, 1	0, 0

2. (20 points) Consider the following game.

		a	b	c
			player 2	
player 1	w	$5,1$	$4,6$	$5,3$
	x	$2,10$	$10,2$	$3,2$
	y	$4,3$	$5,7$	$4,12$

Find all of the values of p for which the corresponding mixed strategy for player 1, $\sigma_1 = (p, 1-p, 0)$, dominates the strategy, y.

Answer:

Exam A: For σ_1 to dominate y, the expected payoff must be higher under σ_1 when player 2 plays a, which requires $5p + 2(1-p) > 4$, which simplifies to $p > 2/3$. The expected payoff must be higher under σ_1 when player 2 plays b, which requires $4p + 10(1-p) > 5$, which simplifies to $p < 5/6$. The expected payoff must be higher under σ_1 when player 2 plays c, which requires $5p + 3(1-p) > 4$, which simplifies to $p > 1/2$. Since all of these inequalities must be satisfied, the values of p for which σ_1 dominates y are: $\frac{2}{3} < p < \frac{5}{6}$.

Exam B: For σ_1 to dominate y, the expected payoff must be higher under σ_1 when player 2 plays a, which requires $3p + 6(1-p) > 5$, which simplifies to $p < 1/3$. The expected payoff must be higher under σ_1 when player 2 plays b, which requires $11p + 5(1-p) > 6$, which simplifies to $p > 1/6$. The expected payoff must be higher under σ_1 when player 2 plays c, which requires $4p + 6(1-p) > 5$, which simplifies to $p < 1/2$. Since all of these inequalities must be satisfied, the values of p for which σ_1 dominates y are: $\frac{1}{6} < p < \frac{1}{3}$.

Exam C: For σ_1 to dominate y, the expected payoff must be higher under σ_1 when player 2 plays a, which requires $5p + 3(1-p) > 4$, which simplifies to $p > 1/2$. The expected payoff must be higher under σ_1 when player 2 plays b, which requires $5p+2(1-p) > 4$, which simplifies to $p > 2/3$. The expected payoff must be higher under σ_1 when player 2 plays c, which requires $4p+10(1-p) > 5$, which simplifies to $p < 5/6$. Since all of these inequalities must be satisfied, the values of p for which σ_1 dominates y are: $\frac{2}{3} < p < \frac{5}{6}$.

Exam D: For σ_1 to dominate y, the expected payoff must be higher under σ_1 when player 2 plays a, which requires $2p + 4(1-p) > 3$, which simplifies to $p < 1/2$. The expected payoff must be higher under σ_1 when player 2 plays b, which requires $p+4(1-p) > 3$, which simplifies to $p < 1/3$. The expected payoff must be higher under σ_1 when player 2 plays c, which requires $9p+3(1-p) > 4$, which simplifies to $p > 1/6$. Since all of these inequalities must be satisfied, the values of p for which σ_1 dominates y are: $\frac{1}{6} < p < \frac{1}{3}$.

3. (20 points) Consider the following game with 8 players. Each player simultaneously decides whether to attend a dinner party (strategy A) or to not attend the dinner party (strategy N). Let M denote the number of players who decide to attend the dinner party. Any player who decides not to attend the dinner party receives a payoff of zero, so for all i, we have

$$u_i(s) = 0 \quad \text{if} \quad s_i = N.$$

The following information gives the payoff to each player who attends the dinner party:

· Player 7's payoff from attending is equal to the total number of attendees, M.

· Player 5's payoff from attending is 1 if player 2 attends, and is -1 if player 2 does not attend.

· Player 1's payoff from attending is -1.

· Player 8's payoff from attending is $2M - 15$.

· Player 2's payoff from attending is 1 if player 5 attends, and is -1 if player 5 does not attend.

· Player 3's payoff from attending is 1 if player 8 does not attend, and is -1 if player 8 attends.

· For players 4 and 6, their payoff from attending is 1 if $M \geq 3$ (at least 3 attend), and is -1 if $M < 3$.

Find the set of rationalizable strategies for each player. Equivalently, iteratively eliminate dominated strategies until no more strategies can be eliminated, and report which strategies are left for each player.

Answer:

Exam A: Player 7's only rationalizable strategy is A. Player 1's only rationalizable strategy is not to attend. Given that player 1 must choose N, the number attending is at most 7, so player 8's payoff from attending is negative, and the only rationalizable strategy for player 8 is N. Given the player 8 does not attend, player 3 will choose A. Given that players 7 and 3 choose A, players 4 and 6 are guaranteed a positive payoff from attending, so they each choose A. Players 2 and 5 are playing a little coordination game with each other; both A and N are rationalizable. Summarizing, the rationalizable strategies have an "x" in the following table:

Exam A:

	1	2	3	4	5	6	7	8
A		x	x	x	x	x	x	
N	x	x			x			x

Exam B:

	1	2	3	4	5	6	7	8
A	x	x	x	x		x	x	
N	x		x		x			x

		1	2	3	4	5	6	7	8
Exam C:	A	x	x		x		x	x	x
	N	x		x		x		x	

		1	2	3	4	5	6	7	8
Exam D:	A	x	x	x	x		x		x
	N		x			x		x	x

4. (20 points) *Consider the following game.*

		player 2				
		P	Q	R	S	T
player 1	B	$1,-4$	$3,4$	$1,5$	$4,3$	$5,1$
	C	$2,5$	$1,-1$	$2,2$	$0,2$	$3,4$
	D	$-2,3$	$4,2$	$4,4$	$4,-5$	$3,1$
	E	$4,2$	$-5,1$	$4,5$	$0,2$	$5,3$
	F	$1,1$	$2,5$	$2,-4$	$1,3$	$2,3$

(i) (15 points) Find all of the (pure strategy) Nash equilibria of this game, and report your answer here:

Exam A: The Nash equilibria are (D,R) and (E,R).
Exam B: The Nash equilibria are (C,R) and (E,R).
Exam C: The Nash equilibria are (W,R) and (Z,R).
Exam D: The Nash equilibria are (D,X) and (A,X).

(ii) (5 points) Is the profile, (D,R), efficient? Briefly explain your reasoning.

Exam A: No, because (E,R) gives at least as high a payoff to player 1 and more to player 2.

Exam B: (E,R) is not efficient, because (C,R) gives at least as high a payoff to player 1 and more to player 2, than (E,R).

Exam C: (W,R) is not efficient, because (Z,R) gives at least as high a payoff to player 1 and more to player 2, than (W,R).

Exam B: (D,X) is not efficient, because (A,X) gives at least as high a payoff to player 1 and more to player 2, than (D,X).

5. (20 points) Two firms are playing a game of Cournot (quantity) competition. Denoting the quantity chosen by firm 1 as q_1 and the quantity chosen by firm 2 as q_2, the market price is given by the inverse demand equation

$$p = 420 - 3q_1 - 3q_2.$$

Each firm has a production cost of 60 per unit of output. Each firm's payoff is defined to be its profit.

(a) (15 points) Find the Nash equilibrium strategy profile, and show your work.

(b) (5 points) In the Nash equilibrium, what is the market price and what are the profits of each firm?

Answer:

Exam A: Firm 1 chooses q_1 to solve the following profit maximization problem:

$$\max(420 - 3q_1 - 3q_2)q_1 - 60q_1$$

The solution is found by differentiating with respect to q_1, setting the expression equal to zero, and solving for q_1.

$$\begin{aligned} 420 - 6q_1 - 3q_2 - 60 &= 0 \\ q_1 &= 60 - \frac{q_2}{2}. \qquad (1) \end{aligned}$$

This is firm 1's best response function. Going through the same steps for firm 2, we have firm 2's best response function,

$$q_2 = 60 - \frac{q_1}{2}. \qquad (2)$$

Simultaneously solving (1) and (2) yields the Nash equilibrium, $(40, 40)$. That is, each firm produces a quantity of 40. Therefore, the price is \$180, and each firm receives profit of \$4800.

Exam B: Going through the same steps, the Nash equilibrium is (30,30) and each firm receives a profit of \$2700.

Exam C: Going through the same steps, the Nash equilibrium is (20,20) and each firm receives a profit of \$1200.

Exam D: Going through the same steps, the Nash equilibrium is (10,10) and each firm receives a profit of \$300.