The Ohio State University
Department of Economics
Second Midterm Examination Answers

Econ 5001
Fall 2015
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Note: *There were three versions of the exam. The versions had slightly different numbers or labeling of strategies, so they all were of the exact same level of difficulty. Version 1 has a cost of 20 and a denominator in the price function of 100 in question 1; version 2 has a cost of 20 and a denominator in the price function of 200; and version 3 has a cost of 50 and a denominator in the price function of 100. In the answers below, I will show work to derive the answer for version 1, then give the final answer without showing work for versions 2 and 3.*

1. (25 points) Consider the following Cournot game in which two firms simultaneously choose a non-negative output quantity. Letting q_1 denote the quantity chosen by firm 1, q_2 denote the quantity chosen by firm 2, and Q denote the total quantity, $q_1 + q_2$, the market price is given by

$$p = 80 - \frac{Q}{100}.$$

Each firm has a production cost of 20 per unit of output.

Find the Nash equilibrium of this game. What is the equilibrium payoff received by each firm?

Answer: The profit function for firm 1 is given by

$$u_1(q_1, q_2) = (80 - \frac{q_1 + q_2}{100})q_1 - 20q_1.$$

To find firm 1's best response function, set the derivative with respect to q_1 equal to zero and solve for q_1. We have

$$\begin{aligned} 80 - \frac{2q_1}{100} - \frac{q_2}{100} - 20 &= 0 \\ q_1 &= 3000 - \frac{q_2}{2}. \qquad (1) \end{aligned}$$

Because of the symmetry of the problem, firm 2's best response function is $q_2 = 3000 - \frac{q_1}{2}$ (which you can check by doing the computation). To find the Nash equilibrium, simultaneously solve the two best response functions. Because of the symmetry, I will solve (1) after imposing $q_1 = q_2$, giving $q_1^* = q_2^* = 2000$. Therefore, the price is 40 and the profit for each firm is $40,000$.

In version 2, we have $q_1^* = q_2^* = 4000$, the price is 40, and the profit for each firm is $80,000$.

In version 3, we have $q_1^* = q_2^* = 1000$, the price is 60, and the profit for each firm is $10,000$.

2. (25 points) For the following normal form game, find the mixed strategy Nash equilibrium. Show all of your work, and clearly indicate the equilibrium mixed strategy profile.

		player 2		
		a	b	c
player 1	w	$5, 6$	$2, 3$	$1, 5$
	x	$8, 2$	$1, 5$	$4, 1$
	y	$0, 3$	$6, 1$	$7, 2$

Answer:

First we need to find which two strategies each player is mixing over. For player 2, c is dominated by a, so player 2 will be mixing between a and b. For player 1 w is dominated by a mixture between x and y, of the form $(0, r, 1 - r)$. To see that there is a value of r that works, first suppose player 2 plays a. Then we want the payoff of the mixed strategy, $8r + 0(1 - r)$, to be greater than 5, or $r > \frac{5}{8}$. Now suppose player 2 plays b. Then we want the payoff of the mixed strategy, $r + 6(1 - r)$, to be greater than 2, or $r < \frac{4}{5}$. So r near $\frac{7}{10}$ will work.

Next we need to find the mixed strategy NE, of the form $\sigma_1 = (0, p, 1 - p)$ and $\sigma_2 = (q, 1 - q, 0)$. For player 1 to be willing to mix between x and y, his payoff from x must equal his payoff from y. This requires

$$8q + 1(1 - q) = 0q + 6(1 - q)$$

which we can solve for $q = \frac{5}{13}$.

For player 2 to be willing to mix between a and b, her payoffs from the two choices must be equal. This requires

$$2p + 3(1 - p) = 5p + 1(1 - p)$$

which we can solve for $p = \frac{2}{5}$. So the mixed strategy NE is $\sigma_1 = (0, \frac{2}{5}, \frac{3}{5})$ and $\sigma_2 = (\frac{5}{13}, \frac{8}{13}, 0)$.

In version 2, the mixed strategy NE is $\sigma_1 = (0, \frac{1}{3}, \frac{2}{3})$ and $\sigma_2 = (\frac{7}{15}, \frac{8}{15}, 0)$.

In version 3, the mixed strategy NE is $\sigma_1 = (0, \frac{2}{5}, \frac{3}{5})$ and $\sigma_2 = (\frac{4}{11}, \frac{7}{11}, 0)$.

3. (25 points) Consider the extensive form game shown below:
(a) (10 points) Find all pure strategy Nash equilibria.
(b) (15 points) Find all subgame perfect Nash equilibria, and briefly explain.

Answer:
The game in matrix form is given by

		player 2: C	D
	AE	$3, 3$	$3, 3$
player 1	AF	$3, 3$	$3, 3$
	BE	$4, 2$	$1, 1$
	BF	$3, 3$	$2, 4$

We see that there are 3 NE in pure strategies: (AE,D), (AF,D), and (BE,C).

In the subgame in which it is player 2's turn, there are 2 NE: (E,C) and (D,F). Then of the 3 NE of the whole game, (AE,D) is not subgame perfect, because (E,D) is not a NE of the subgame. The SPNE are (AF,D), and (BE,C).

In version 2, the NE are: (CZ,B), (DY,A), and (CY,B). The SPNE are (CZ,B) and (DY,A).

In version 3, the NE are: (XU,S), (YV,T), and (YU,T). The SPNE are (XU,S) and (YV,T).

4. (25 points) Consider the repeated game in which the following stage game is played twice. A player's payoff in the repeated game is the sum of his/her payoff in period 1 and period 2.

		player 2		
		X	Y	Z
player 1	A	$10, 10$	$2, 12$	$0, 13$
	B	$12, 2$	$5, 5$	$0, 0$
	C	$13, 0$	$0, 0$	$1, 1$

(a) (8 points) What is the lowest payoff that player 1 can receive in any subgame perfect Nash equilibrium (SPNE) of the repeated game? Clearly and completely specify the corresponding SPNE strategy profile.

(b) (8 points) What is the highest payoff that player 1 can receive in any subgame perfect Nash equilibrium (SPNE) of the repeated game? Clearly and completely specify the corresponding SPNE strategy profile.

(c) (9 points) Is there a SPNE in which the action profile (C, Y) is played in period 1? If yes, then clearly and completely specify the corresponding SPNE strategy profile. If no, then explain your reasoning.

Answer:

(a) The lowest payoff is 2. This is achieved with the following SPNE: Player 1 plays C in period 1 and after every history. Player 2 plays Z in period 1 and after every history. To see that a lower payoff is not possible, either (B,Y) or (C,Z) must be played in all period 2 subgames, so player 1 must receive at least a payoff of 1 in the second period, no matter what he does in period 1. Therefore, he cannot receive a payoff of 0 in the first period and 1 in the second period and be best responding to player 2's strategy in the repeated game.

(b) The highest payoff is 18. This is achieved with the following SPNE: the profile (C,X) is played in period 1. If (C,X) is played in period 1, the profile (B,Y) is played in period 2; otherwise, the profile (C,Z) is played in period 2. Player 2 receives a payoff of 0+5 by following her strategy, and she would only receive 1+1 by deviating to Z in period 1. Since all of the subgames are in a NE, this is subgame perfect. Clearly it is impossible for player 1 to do better, since he is receiving at most 5 in the second period.

(c) No, there is no SPNE in which (C,Y) is played in period 1. If there were such a SPNE, player 1 would be receiving a payoff of at most 0+5. By deviating to B in period 1, player 1 would receive at least 5+1.

In version 2, the lowest payoff is 2 and the highest payoff is 18. There is no SPNE in which (G,N) is played in period 1.

In version 3, the lowest payoff is 2 and the highest payoff is 17. There is no SPNE in which (E,Y) is played in period 1.

(The construction of SPNE in parts (a) and (b) and the argument in part (c) are exactly the same, but the actions are labelled differently.)