The Ohio State University
Department of Economics
Second Midterm Answers

Econ 5001
Spring 2017
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There were four versions of the exam. In question 1, Exam A had a cost of 40, Exam B had a cost of 120, Exam C had a cost of 280, and Exam D had a cost of 360. Below, the questions are reproduced for Exam A, but answers for all four exams are given.

1. (25 points) Consider the following Cournot game in which three firms simultaneously choose a non-negative output quantity. Let q_1 denote the quantity chosen by firm 1, q_2 denote the quantity chosen by firm 2, q_3 denote the quantity chosen by firm 3, and Q denote the total quantity, $q_1 + q_2 + q_3$. The market price is given by

$$p = 600 - 2Q.$$

Each firm has a marginal production cost of 40 per unit of output.

(a) (12 points) What is firm 1's best response to the profile, (q_2, q_3)? That is, according to the notation used in class, what is firm 1's best response function, $BR_1(q_2, q_3)$?

(b) (13 points) Solve for the Nash equilibrium of this game.

Hint for part (b): Because all firms have the same marginal cost, the NE will be symmetric, with $q_1 = q_2 = q_3 = q^*$ for some q^*. Use your result from part (a) and the condition, $q^* = BR_1(q^*, q^*)$ to solve for q^*.

Answer:
Exam A: The payoff function for firm 1 is given by

$$(600 - 2q_1 - 2q_2 - 2q_3)q_1 - 40q_1.$$

Differentiating with respect to q_1, and setting the expression equal to zero, we have the first order condition,

$$600 - 4q_1 - 2q_2 - 2q_3 - 40 = 0.$$

Solving for q_1, we have the best response function,

$$q_1 = 140 - \frac{q_2 + q_3}{2}. \qquad (1)$$

Setting all three quantities in (1) equal to q^*, and solving for q^*, yields $q^* = 70$. Therefore, because of symmetry, the NE is $(70, 70, 70)$.

Exam B: Going through the same steps as for Exam A with the cost 120, we have the best response function,

$$q_1 = 120 - \frac{q_2 + q_3}{2}.$$

Setting all three quantities equal to q^*, and solving for q^*, yields $q^* = 60$. Therefore, because of symmetry, the NE is $(60, 60, 60)$.

Exam C: Going through the same steps as for Exam A with the cost 280, we have the best response function,

$$q_1 = 80 - \frac{q_2 + q_3}{2}.$$

Setting all three quantities equal to q^*, and solving for q^*, yields $q^* = 40$. Therefore, because of symmetry, the NE is $(40, 40, 40)$.

Exam D: Going through the same steps as for Exam A with the cost 360, we have the best response function,

$$q_1 = 60 - \frac{q_2 + q_3}{2}.$$

Setting all three quantities equal to q^*, and solving for q^*, yields $q^* = 30$. Therefore, because of symmetry, the NE is $(30, 30, 30)$.

2. (25 points) In the following game of "Hide and Seek," first player 1 either hides in the treehouse, T, or hides under the bridge in a stream, B. Then player 2 tries to find player 1, but only has time to look in the treehouse, LT, or look under the bridge, LB. Even though player 1 moves first, player 2 does not observe this choice, so we can treat this as a simultaneous move game. Player 1's payoff depends on whether or not he is found, and also on the disutility of hiding under the bridge in the stream, given by the parameter, c. Player 2's payoff only depends on whether or not she finds player 1. The payoff matrix is

		player 2	
		LT	LB
player 1	T	$0, 10$	$10, 0$
	B	$10 - c, 0$	$-c, 10$

(a) (15 points) Find the mixed strategy Nash equilibrium of this game, keeping in mind that the strategies could depend on the parameter, c.

(b) (10 points) When c goes up, (i) does the probability that player 1 hides under the bridge in the MSNE go up, go down, or stay the same, and (ii) does the probability that player 2 looks under the bridge in the MSNE go up, go down, or stay the same? Briefly explain.

Answer:

Exam A: Denote player 1's mixed strategy by $(p, 1-p)$ and player 2's mixed strategy by $(q, 1-q)$. Player 1 must be indifferent between T and B, yielding the equation

$$\begin{aligned} 10(1-q) &= (10-c)q + (-c)(1-q) \\ q &= \frac{10+c}{20}. \end{aligned}$$

Player 2 must be indifferent between LT and LB, yielding the equation

$$\begin{aligned} 10p &= 10(1-p) \\ p &= \frac{1}{2}. \end{aligned}$$

Therefore, the MSNE is given by

$$\sigma_1 = (\frac{1}{2}, \frac{1}{2}) \text{ and } \sigma_2 = (\frac{10+c}{20}, \frac{10-c}{20}). \tag{2}$$

From (2), you can see that the probability that player 1 hides under the bridge does not change as c increases. The reason is that c does not affect player 2's payoff, so the condition for player 2 to be indifferent does not change. When c increases, the probability that player 2 looks under the bridge goes down. The reason is that, for player 1 to be indifferent with the higher disutility, he must have a higher probability of not being caught.

Exam B: Going through the same steps as above, with the payoff matrix for Exam B, yields

$$\sigma_1 = (\frac{1}{2}, \frac{1}{2}) \text{ and } \sigma_2 = (\frac{5+c}{10}, \frac{5-c}{10}).$$

Same answer to part (b).

Exam C: Going through the same steps as above, with the payoff matrix for Exam C, yields

$$\sigma_1 = (\frac{1}{2}, \frac{1}{2}) \text{ and } \sigma_2 = (\frac{10+c}{20}, \frac{10-c}{20}).$$

Same answer to part (b).

Exam D: Going through the same steps as above, with the payoff matrix for Exam D, yields

$$\sigma_1 = (\frac{1}{2}, \frac{1}{2}) \text{ and } \sigma_2 = (\frac{5+c}{10}, \frac{5-c}{10}).$$

Same answer to part (b).

3. (25 points) Consider the following game in extensive form.

(a) (10 points) Find all of the Nash equilibria of this game.

(b) (15 points) Find all of the subgame perfect Nash equilibria of this game.

Exam A: The matrix corresponding to this game tree is

		player 2			
		CF	CG	DF	DG
	AY	5, 1	5, 1	3, 3	3, 3
player 1	AZ	5, 1	5, 1	3, 3	3, 3
	BY	4, 2	0, 0	4, 2	0, 0
	BZ	0, 0	4, 2	0, 0	4, 2

There are three Nash equilibria: (AY, DG), (AZ, DG), and (BY, DF). All of these profiles imply D in the subgame following A, which is a NE. In the subgame following B, (Y, F) and (Z, G) are Nash equilibria, but (Y, G) is not. Therefore, (AY, DG) is not subgame perfect, but the other two NE are subgame perfect.

Exam B: The matrix corresponding to this game tree is

		player 2			
		DF	DG	EF	EG
	BJ	10, 2	10, 2	6, 6	6, 6
player 1	BK	10, 2	10, 2	6, 6	6, 6
	CJ	0, 0	4, 8	0, 0	4, 8
	CK	8, 4	0, 0	8, 4	0, 0

There are three Nash equilibria: (BJ, EG), (BK, EG), and (CK, EF). All of these profiles imply E in the subgame following B, which is a NE. In the subgame following C, (K, F) and (J, G) are Nash equilibria, but (K, G) is not. Therefore, (BK, EG) is not subgame perfect, but the other two NE are subgame perfect.

Exam C: The matrix corresponding to this game tree is

		player 2			
		WY	WZ	XY	XZ
	CE	5, 5	5, 5	7, 3	7, 3
player 1	DF	5, 5	5, 5	7, 3	7, 3
	DE	6, 4	0, 0	6, 4	0, 0
	DF	0, 0	4, 6	0, 0	4, 6

There are three Nash equilibria: (CE, WZ), (CF, WZ), and (DE, WY). All of these profiles imply W in the subgame following C, which is a NE. In the subgame following D, (E, Y) and (F, Z) are Nash equilibria, but (E, Z) is not. Therefore, (CE, WZ) is not subgame perfect, but the other two NE are subgame perfect.

Exam D: The matrix corresponding to this game tree is

		player 2			
		WY	WZ	XY	XZ
	DA	$15, 3$	$15, 3$	$9, 9$	$9, 9$
player 1	DB	$15, 3$	$15, 3$	$9, 9$	$9, 9$
	EA	$0, 0$	$6, 12$	$0, 0$	$6, 12$
	EB	$12, 6$	$0, 0$	$12, 6$	$0, 0$

There are three Nash equilibria: (DA, XZ), (DB, XZ), and (EB, XY). All of these profiles imply X in the subgame following D, which is a NE. In the subgame following E, (B, Y) and (A, Z) are Nash equilibria, but (B, Z) is not. Therefore, (DB, XZ) is not subgame perfect, but the other two NE are subgame perfect.

4. (25 points) Two firms are engaged in a game of sequential price competition. First firm 1 chooses a non-negative price, p_1. Then firm 2 observes firm 1's price before choosing its own non-negative price, p_2. Both firms have constant marginal production cost of 200 per unit. The market demand, as a function of the (lowest) price p, is given by

$$D(p) = 1000 - p.$$

If $p_2 \leq p_1$, then firm 2 serves the entire market demand at the price p_2, and firm 1's sales are zero. If $p_1 < p_2$, then firm 1 serves the entire market demand at the price p_1, and firm 2's sales are zero. Notice that when both firms choose the same price, firm 2 serves the entire market. Each firm's payoff is the profit that it receives.

Find a subgame perfect Nash equilibrium of this game. Remember that a strategy for firm 2 is a function that specifies p_2 for every non-negative p_1.

Answer:

Exam A: First let us find the monopoly price in this market, which will be relevant for firm 2's best response. Since the quantity sold is $D(p)$, a monopoly receives payoff

$$\begin{aligned} & pD(p) - 200D(p) \\ = \ & p(1000 - p) - 200(1000 - p). \end{aligned}$$

Differentiating this payoff with respect to p, and setting the expression equal to zero, yields the first order condition

$$\begin{aligned} 1200 - 2p &= 0, \\ p &= 600, \end{aligned}$$

so the monopoly price is 600.

Now let us find firm 2's best response to each p_1, keeping in mind that firm 2 can capture the entire market by matching firm 1's price. If we have $p_1 < 200$, firm 2 would face losses unless it set a price above 200, so a best response is $p_2 = 1000$ (any price above 200 will do). If we have $200 \leq p_1 \leq 600$, the best response is $p_2 = p_1$. If we have $p_1 > 600$, firm 2 can capture the entire market when it sets the monopoly price, so the best response is $p_2 = 600$. Since firm 2 observes p_1, this paragraph defines firm 2's strategy in the SPNE.

Given firm 2's strategy defined above, firm 1 cannot receive positive profits for any p_1. Thus, any $p_1 \geq 200$ is a best response to firm 2's strategy and is part of a SPNE. (Firm 1 cannot set a price below 200, or else it would serve the market and face losses.)

Recapping, here is a SPNE

$$\begin{aligned}
p_1 &= 200 \\
p_2 &= 1000 \text{ if } p_1 < 200 \\
p_2 &= p_1 \text{ if } 200 \leq p_1 \leq 600 \\
p_2 &= 600 \text{ if } p_1 > 600.
\end{aligned}$$

Exam B: Using the same logic as in the answer for Exam A, the monopoly price is 650, and here is a SPNE

$$\begin{aligned}
p_1 &= 300 \\
p_2 &= 1000 \text{ if } p_1 < 300 \\
p_2 &= p_1 \text{ if } 300 \leq p_1 \leq 650 \\
p_2 &= 650 \text{ if } p_1 > 650.
\end{aligned}$$

Exam C: Using the same logic as in the answer for Exam A, the monopoly price is 700, and here is a SPNE

$$\begin{aligned}
p_1 &= 400 \\
p_2 &= 1000 \text{ if } p_1 < 400 \\
p_2 &= p_1 \text{ if } 400 \leq p_1 \leq 700 \\
p_2 &= 700 \text{ if } p_1 > 700.
\end{aligned}$$

Exam D: Using the same logic as in the answer for Exam A, the monopoly price is 550, and here is a SPNE

$$\begin{aligned}
p_1 &= 100 \\
p_2 &= 1000 \text{ if } p_1 < 100 \\
p_2 &= p_1 \text{ if } 100 \leq p_1 \leq 550 \\
p_2 &= 550 \text{ if } p_1 > 550.
\end{aligned}$$