The Ohio State University
Department of Economics
First Midterm Examination Answers

Econ 5001
Fall 2015
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Note: *There were three versions of the exam. The versions had slightly different numbers or labeling of strategies, so they all were of the exact same level of difficulty. Exams are called "abc," "FGH," and "XYZ," based on the labeling of player 1's strategies in question 1.*

1. (20 points) *Convert the following extensive form game into normal form, by drawing the payoff matrix, labeling the strategies corresponding to the rows and columns, and filling in the payoffs.*

player 2

		dfh	dfi	dgh	dgi	efh	efi	egh	egi
	a	$1,2$	$1,2$	$1,2$	$1,2$	$3,4$	$3,4$	$3,4$	$3,4$
player 1	b	$5,6$	$5,6$	$7,8$	$7,8$	$5,6$	$5,6$	$7,8$	$7,8$
	c	$9,10$	$11,12$	$9,10$	$11,12$	$9,10$	$11,12$	$9,10$	$11,12$

player 2

		acy	acz	ady	adz	bcy	bcz	bdy	bdz
	F	$0,1$	$0,1$	$0,1$	$0,1$	$2,3$	$2,3$	$2,3$	$2,3$
player 1	G	$4,5$	$4,5$	$6,7$	$6,7$	$4,5$	$4,5$	$6,7$	$6,7$
	H	$8,9$	$10,11$	$8,9$	$10,11$	$8,9$	$10,11$	$8,9$	$10,11$

player 2

		ace	acf	ade	adf	bce	bcf	bde	bdf
	X	$0,12$	$0,12$	$0,12$	$0,12$	$1,11$	$1,11$	$1,11$	$1,11$
player 1	Y	$2,10$	$2,10$	$3,9$	$3,9$	$2,10$	$2,10$	$3,9$	$3,9$
	Z	$4,8$	$5,7$	$4,8$	$5,7$	$4,8$	$5,7$	$4,8$	$5,7$

2. (25 points) *Consider the following game (game for Exam FGH is pictured).*

		player 2				
		F	G	H	I	J
	A	$1, 6$	$6, 2$	$3, 3$	$7, 7$	$2, 7$
player 1	B	$0, 2$	$6, 4$	$5, 5$	$4, 4$	$7, 3$
	C	$6, 9$	$5, 8$	$2, 5$	$1, 2$	$0, 5$
	D	$3, 10$	$1, 9$	$0, 4$	$5, 7$	$2, 4$
	E	$7, 5$	$8, 4$	$4, 2$	$4, 1$	$3, 4$

(a) (15 points) Find all of the (pure strategy) Nash equilibria of this game, and indicate your answer here:

For Exam abc, the Nash equilibria are: (E, U), (B, W), and (D, X).
For Exam FGH, the Nash equilibria are: (A, I), (B, H), and (E, F).
For Exam XYZ, the Nash equilibria are: (F, L), (G, N), and (I, O).

(b) (10 points) Find all of the efficient strategy profiles, and indicate your answer here:

A strategy profile is efficient if there is no other profile that provides each player with at least as high a payoff, and some player with a strictly higher payoff. A profile does not have to be a Nash equilibrium to be efficient.

For Exam abc, the efficient strategy profiles are: (A, U), (D, X), (E, V), and (C, U).
For Exam FGH, the efficient strategy profiles are: (D, F), (C, F), (A, I), and (E, G).
For Exam XYZ, the efficient strategy profiles are: (I, O), (H, L), (J, L), and (F, M).

3. (25 points) Consider the following game (game for Exam FGH is pictured).

		player 2		
		L	C	R
	X	$5, 0$	$7, 1$	$8, 1$
player 1	Y	$10, 2$	$2, 0$	$9, 0$
	Z	$6, 2$	$5, 2$	$8, 10$

Exam abc: *Is player 1's strategy, D, dominated? If yes, then name a strategy that dominates D. If no, then name a belief to which D is a best response.*

Neither pure strategy dominates D, but there are mixed strategies that do. Let p denote the probability of U and $(1-p)$ denote the probability of M. This mixed strategy yields a higher payoff than D when player 2 chooses L if we have $8p + 2(1-p) > 4$, it yields a higher payoff than D when player 2 chooses C if we have $0p + 5(1-p) > 3$, and it yields a higher payoff than D when player 2 chooses R if $p > 0$. All three of these conditions are satisfied when $\frac{1}{5} < p < \frac{2}{5}$. For example, the mixed strategy $(\frac{3}{10}, \frac{7}{10}, 0)$ dominates D.

Exam FGH: *Is player 1's strategy, Z, dominated? If yes, then name a strategy that dominates Z. If no, then name a belief to which Z is a best response.*

Neither pure strategy dominates Z, but there are mixed strategies that do. Let p denote the probability of X and $(1-p)$ denote the probability of Y. This mixed strategy yields a higher payoff than Z when player 2 chooses L if we have $5p + 10(1-p) > 6$, it yields a higher payoff than Z when player 2 chooses C if we have $7p + 2(1-p) > 5$, and it yields a higher payoff than Z when player 2 chooses R if $p < 1$. All three of these conditions are satisfied when $\frac{3}{5} < p < \frac{4}{5}$. For example, the mixed strategy $(\frac{7}{10}, \frac{3}{10}, 0)$ dominates Z.

Exam XYZ: *Is player 1's strategy, C, dominated? If yes, then name a strategy that dominates C. If no, then name a belief to which C is a best response.*

Neither pure strategy dominates C, but there are mixed strategies that do. Let p denote the probability of A and $(1-p)$ denote the probability of B. This mixed strategy yields a higher payoff than C when player 2 chooses X if we have $9p + 4(1-p) > 5$, it yields a higher payoff than C when player 2 chooses Y if we have $1p + 6(1-p) > 4$, and it yields a higher payoff than C when player 2 chooses Z if $p > 0$. All three of these conditions are satisfied when $\frac{1}{5} < p < \frac{2}{5}$. For example, the mixed strategy $(\frac{3}{10}, \frac{7}{10}, 0)$ dominates C.

4. (30 points) This is the question for Exam FGH. The other exams use different letters for the "enter" or "not enter" strategies or put the three types of firms in different order.

Consider the following strategic situation, in which ten firms simultaneously decide whether to enter a market (strategy Y) or not enter the market (strategy N). Any firm that does not enter the market receives a payoff of 0, no matter what the other firms do. If firm i enters the market, its payoff depends on how many firms enter. Letting m denote the number of firms that enter the market, firm i's payoff from entering is denoted by $u_i(Y, m)$, given by

$$\begin{aligned} u_i(Y,m) &= 11 - 3m \quad \text{for } i = 1,2,3,4 \\ u_i(Y,m) &= 11 - 2m \quad \text{for } i = 5,6 \\ u_i(Y,m) &= 11 - m \quad \text{for } i = 7,8,9,10. \end{aligned}$$

(a) (15 points) Specify the rationalizable strategies for each player, and briefly explain your reasoning.

Each of the 4 firms with payoff function $11-m$ receive a positive payoff from entering, no matter what the other 9 firms do. Therefore, entering dominates not entering for those firms (1-4 for Exam abc, 7-10 for Exam FGH, and 3-6 for Exam XYZ).

After eliminating these dominated strategies, each of the 4 firms with payoff function $11 - 3m$ realize that there will be at least 4 firms entering the market, so they would receive a negative payoff from entering. Therefore, on the second round of elimination, not entering dominates entering for these firms (5-8 for Exam abc, 1-4 for Exam FGH, and 7-10 for Exam XYZ).

Each of the two firms with payoff function $11 - 2m$ can identify 4 firms that will enter and 4 firms that will not enter. If the remaining firm also enters, the best response would be to not enter. If the remaining firm does not enter, the best response would be to enter. Therefore, neither strategy can be eliminated, so entering and not entering are both rationalizable for these two firms.

(b) (15 points) Find all of the (pure strategy) Nash equilibria of this game, and briefly explain your reasoning.

Based on the answer to part (a), there are two Nash equilibria in pure strategies. Each of the 4 firms with payoff function $11 - m$ enter, each of the 4 firms with payoff function $11 - 3m$ do not enter, one of the 2 firms with payoff function $11 - 2m$ enters, and the other firm with payoff function $11 - 2m$ does not enter.

For Exam abc, the NE are: $(E, E, E, E, N, N, N, N, E, N)$ and $(E, E, E, E, N, N, N, N, N, E)$.

For Exam FGH, the NE are: $(N, N, N, N, Y, N, Y, Y, Y, Y)$ and $(N, N, N, N, N, Y, Y, Y, Y, Y)$.

For Exam XYZ, the NE are: $(A, B, A, A, A, A, B, B, B, B)$ and $(B, A, A, A, A, A, B, B, B, B)$.