The Ohio State University
Department of Economics
First Midterm Examination Answers

Econ 5001
Fall 2016
Prof. James Peck

Note: *There were four versions of the exam. The versions had slightly different numbers or labeling of strategies, so they all were of the exact same level of difficulty. Exams are called A, W, P, and I, based on the labeling of player 1's leftmost action in the game tree of question 1.*

1. (20 points) *Convert the following extensive form game into normal form, by drawing the payoff matrix, labeling the strategies corresponding to the rows and columns, and filling in the payoffs.*

Exam A:

		player 2			
		EG	EH	FG	FH
	A	0, 1	0, 1	2, 3	2, 3
player 1	B	4, 5	4, 5	6, 7	6, 7
	C	8, 9	10, 11	8, 9	10, 11
	D	12, 13	14, 15	12, 13	14, 15

Exam W:

		player 2			
		AC	AD	BC	BD
	W	0, 10	0, 10	1, 9	1, 9
player 1	X	2, 8	2, 8	3, 7	3, 7
	Y	10, 0	9, 1	10, 0	9, 1
	Z	8, 2	7, 3	8, 2	7, 3

Exam P:

		player 2			
		AX	AY	BX	BY
	P	0, 8	0, 8	1, 9	1, 9
player 1	Q	2, 10	2, 10	3, 11	3, 11
	R	4, 12	5, 13	4, 12	5, 13
	S	6, 14	7, 15	6, 14	7, 15

Exam I:

		player 2			
		AC	AD	BC	BD
	I	15, 7	15, 7	14, 6	14, 6
player 1	J	13, 5	13, 5	12, 4	12, 4
	K	11, 3	10, 2	11, 3	10, 2
	L	9, 1	8, 0	9, 1	8, 0

2. (25 points) *(Game for Exam A is pictured). In the following game, find a strategy of player 2 that is (strictly) dominated. (Remember that a strategy can be dominated by either a pure strategy or a mixed strategy.) Show your work to receive credit.*

		player 2		
		L	M	R
player 1	U	$2,4$	$3,6$	$4,1$
	D	$4,4$	$2,3$	$3,7$

Exam A answer: M is a best response to U and R is a best response to D, so clearly neither can be dominated. We will find a mixed strategy of the form $\sigma_2 = (0, p, 1-p)$ that dominates L. In order for σ_2 to give a higher expected payoff when player 1 plays U, we must have

$$\begin{aligned} 6p + (1-p) &> 4, \text{ or} \\ p &> \frac{3}{5}. \end{aligned}$$

In order for σ_2 to give a higher expected payoff when player 1 plays D, we must have

$$\begin{aligned} 3p + 7(1-p) &> 4, \text{ or} \\ p &< \frac{3}{4}. \end{aligned}$$

Any p satisfying both inequalities works. For example, $\sigma_2 = (0, \frac{2}{3}, \frac{1}{3})$ dominates L.

Exam W answer: X is a best response to A and Z is a best response to B, so clearly neither can be dominated. We will find a mixed strategy of the form $\sigma_2 = (p, 0, 1-p)$ that dominates Y. In order for σ_2 to give a higher expected payoff when player 1 plays A, we must have

$$\begin{aligned} 7p + 2(1-p) &> 5, \text{ or} \\ p &> \frac{3}{5}. \end{aligned}$$

In order for σ_2 to give a higher expected payoff when player 1 plays B, we must have

$$\begin{aligned} 4p + 8(1-p) &> 5, \text{ or} \\ p &< \frac{3}{4}. \end{aligned}$$

Any p satisfying both inequalities works. For example, $\sigma_2 = (\frac{2}{3}, 0, \frac{1}{3})$ dominates Y.

Exam P answer: A is a best response to X and B is a best response to Y, so clearly neither can be dominated. We will find a mixed strategy of the form $\sigma_2 = (p, 1-p, 0)$ that dominates C. In order for σ_2 to give a higher expected payoff when player 1 plays X, we must have

$$\begin{aligned} 7p + 2(1-p) &> 5, \text{ or} \\ p &> \frac{3}{5}. \end{aligned}$$

In order for σ_2 to give a higher expected payoff when player 1 plays Y, we must have

$$\begin{aligned} 4p + 8(1-p) &> 5, \text{ or} \\ p &< \frac{3}{4}. \end{aligned}$$

Any p satisfying both inequalities works. For example, $\sigma_2 = (\frac{2}{3}, \frac{1}{3}, 0)$ dominates C.

Exam I answer: R is a best response to T and C is a best response to B, so clearly neither can be dominated. We will find a mixed strategy of the form $\sigma_2 = (0, p, 1-p)$ that dominates L. In order for σ_2 to give a higher expected payoff when player 1 plays T, we must have

$$\begin{aligned} 5p + 9(1-p) &> 6, \text{ or} \\ p &< \frac{3}{4}. \end{aligned}$$

In order for σ_2 to give a higher expected payoff when player 1 plays B, we must have

$$\begin{aligned} 8p + 3(1-p) &> 6, \text{ or} \\ p &> \frac{3}{5}. \end{aligned}$$

Any p satisfying both inequalities works. For example, $\sigma_2 = (0, \frac{2}{3}, \frac{1}{3})$ dominates L.

3. (25 points) (Game for Exam W is pictured). Consider the following game.

		player 2				
		A	B	C	D	E
	W	$3,5$	$2,1$	$8,0$	$7,1$	$1,2$
	X	$3,1$	$6,6$	$3,2$	$6,3$	$6,4$
player 1	Y	$8,0$	$4,3$	$6,2$	$3,0$	$6,5$
	Z	$5,2$	$3,5$	$5,7$	$1,1$	$3,5$

(a) (15 points) Find all of the (pure strategy) Nash equilibria of this game, and indicate your answer here:

For Exam A, the Nash equilibria are: (B,Y) and (C,W).
For Exam W, the Nash equilibria are: (X,B) and (Y,E).
For Exam P, the Nash equilibria are: (C,Z) and (D,W).
For Exam I, the Nash equilibria are: (F,K) and (I,N).

(b) (10 points) Find all of the efficient strategy profiles, and indicate your answer here:

A strategy profile is efficient if there is no other profile that provides each player with at least as high a payoff, and some player with a strictly higher payoff. A profile does not have to be a Nash equilibrium to be efficient.

For Exam A, the efficient strategy profiles are: (A,U), (C,X), (A,V), (B,Y), and (D,U).
For Exam W, the efficient strategy profiles are: (Y,A), (W,C), (W,D), (X,B), and (Z,C).
For Exam P, the efficient strategy profiles are: (A,X), (C,V), (A,Y), (D,W), and (B,X).
For Exam I, the efficient strategy profiles are: (G,L), (I,J), (G,M), (F,K), and (H,L).

4. (30 points) (Question for Exam P is written here.) Two firms engage in Cournot (quantity) competition. For $i = 1, 2$, firm i must choose a nonnegative quantity to produce, q_i. Production is costless, so the payoff to each firm is its revenue. The price in dollars is determined by the inverse demand curve,

$$p = 600 - 2q_1 - 2q_2.$$

(a) (10 points) Find the best response function for firm 1, which specifies the payoff maximizing q_1 as a function of q_2.

(b) (10 points) Find the Nash equilibrium of this game.

(c) (10 points) Now suppose that, prior to choosing its quantity, firm 1's lawyers determine that firm 1 could enforce a patent on its product design, which would prevent firm 2 from producing any output. If the cost to firm 1 of protecting its patent is $\$45,000$, is it in firm 1's interest to do so? In other words, would firm 1 be willing to pay $\$45,000$ to be a monopolist instead of playing the Cournot game? Show the computations that justify your answer and briefly explain.

Answer for Exam A: The demand curve is $p = 1200 - 2q_1 - 2q_2$, so the profit function for firm 1 is $(1200 - 2q_1 - 2q_2)q_1$. Differentiating with respect to q_1 gives $1200 - 4q_1 - 2q_2$. Profits are maximized when the derivative is zero, or

$$\begin{aligned} q_1 &= \frac{1200 - 2q_2}{4} \text{ or} \\ q_1 &= 300 - \frac{q_2}{2}. \end{aligned}$$

This is firm 1's best response function. Going through the same steps for firm 2 yields firm 2's best response function,

$$q_2 = 300 - \frac{q_1}{2}.$$

To find the Nash equilibrium, we need to find (q_1, q_2) where each quantity is a best response to the other. Simple algebra yields the solution, $(200, 200)$.

For part (c), let us first find firm 1's payoff in the NE. Substitute the NE quantities into the demand equation to get a price of 400, so firm 1's payoff is $\$80,000$. If firm 1 is a monopolist, we can find its optimal quantity by substituting $q_2 = 0$ into the best response function above, yielding a monopoly quantity of 300, a price of 600, and profits of $\$180,000$. Because the increase in profits is more than $\$90,000$, it is in the interest of firm 1 to enforce the patent.

Answer for Exam W: The demand curve is $p = 2400 - 2q_1 - 2q_2$, so the profit function for firm 1 is $(2400 - 2q_1 - 2q_2)q_1$. Differentiating with respect to q_1 gives $2400 - 4q_1 - 2q_2$. Profits are maximized when the derivative is zero, or

$$\begin{aligned} q_1 &= \frac{2400 - 2q_2}{4} \text{ or} \\ q_1 &= 600 - \frac{q_2}{2}. \end{aligned}$$

This is firm 1's best response function. Going through the same steps for firm 2 yields firm 2's best response function,

$$q_2 = 600 - \frac{q_1}{2}.$$

To find the Nash equilibrium, we need to find (q_1, q_2) where each quantity is a best response to the other. Simple algebra yields the solution, $(400, 400)$.

For part (c), let us first find firm 1's payoff in the NE. Substitute the NE quantities into the demand equation to get a price of 800, so firm 1's payoff is $\$320,000$. If firm 1 is a monopolist, we can find its optimal quantity by substituting $q_2 = 0$ into the best response function above, yielding a monopoly quantity of 600, a price of 1200, and profits of $\$720,000$. Because the increase in profits is more than $\$350,000$, it is in the interest of firm 1 to enforce the patent.

Answer for Exam P: The demand curve is $p = 600 - 2q_1 - 2q_2$, so the profit function for firm 1 is $(600 - 2q_1 - 2q_2)q_1$. Differentiating with respect to q_1 gives $600 - 4q_1 - 2q_2$. Profits are maximized when the derivative is zero, or

$$\begin{aligned} q_1 &= \frac{600 - 2q_2}{4} \text{ or} \\ q_1 &= 150 - \frac{q_2}{2}. \end{aligned}$$

This is firm 1's best response function. Going through the same steps for firm 2 yields firm 2's best response function,

$$q_2 = 150 - \frac{q_1}{2}.$$

To find the Nash equilibrium, we need to find (q_1, q_2) where each quantity is a best response to the other. Simple algebra yields the solution, $(100, 100)$.

For part (c), let us first find firm 1's payoff in the NE. Substitute the NE quantities into the demand equation to get a price of 200, so firm 1's payoff is $\$20,000$. If firm 1 is a monopolist, we can find its optimal quantity by substituting $q_2 = 0$ into the best response function above, yielding a monopoly quantity of 150, a price of 300, and profits of $\$45,000$. Because the increase in profits is more than $\$22,000$, it is in the interest of firm 1 to enforce the patent.

Answer for Exam I: The demand curve is $p = 1800 - 2q_1 - 2q_2$, so the profit function for firm 1 is $(1800 - 2q_1 - 2q_2)q_1$. Differentiating with respect to q_1 gives $1800 - 4q_1 - 2q_2$. Profits are maximized when the derivative is zero, or

$$\begin{aligned} q_1 &= \frac{1800 - 2q_2}{4} \text{ or} \\ q_1 &= 450 - \frac{q_2}{2}. \end{aligned}$$

This is firm 1's best response function. Going through the same steps for firm 2 yields firm 2's best response function,

$$q_2 = 450 - \frac{q_1}{2}.$$

To find the Nash equilibrium, we need to find (q_1, q_2) where each quantity is a best response to the other. Simple algebra yields the solution, $(300, 300)$.

For part (c), let us first find firm 1's payoff in the NE. Substitute the NE quantities into the demand equation to get a price of 600, so firm 1's payoff is $\$180,000$. If firm 1 is a monopolist, we can find its optimal quantity by substituting $q_2 = 0$ into the best response function above, yielding a monopoly quantity of 450, a price of 900, and profits of $\$405,000$. Because the increase in profits is more than $\$200,000$, it is in the interest of firm 1 to enforce the patent.