The Ohio State University
Department of Economics
Midterm Answers

Econ 5001
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1. Player 1 has 3 different information sets in which he is called upon to move, and 2 choices at each information set, so his total number of possible strategies is 8.

2. Player 1 has 8 strategies, and player 2 has 2 strategies (C or D). A strategy profile is a combination of 1 strategy for each player. Therefore, there are 16 possible strategy profiles. (There are only 8 possible terminal nodes or paths through the game tree, but that is not the same thing as a strategy profile.)

3-5. On the tests marked with a "plus" or a "triangle", the payoff matrix is given by

| | | player 2 | | | |
|---|---|---|---|---|---|
| | | W | X | Y | Z |
| player 1 | A | $3,2$ | $1,1$ | $4,3$ | $3,5$ |
| | B | $1,3$ | $3,0$ | $2,4$ | $4,2$ |
| | C | $2,1$ | $0,1$ | $1,2$ | $1,0$ |
| | D | $1,0$ | $2,0$ | $3,1$ | $4,0$ |

and on the tests marked with a "circle" or a "circle around plus", the payoff matrix is given by

| | | player 2 | | | |
|---|---|---|---|---|---|
| | | W | X | Y | Z |
| player 1 | A | $2,4$ | $3,0$ | $1,3$ | $4,2$ |
| | B | $4,3$ | $1,1$ | $3,2$ | $3,5$ |
| | C | $1,2$ | $0,1$ | $2,1$ | $1,0$ |
| | D | $3,1$ | $2,0$ | $1,0$ | $4,0$ |

3. Player 1 plays A with probability one half and C with probability one half. With the first matrix, player 2's expected payoff from (respectively) W, X, Y, and Z are (respectively) 3/2, 1, 5/2, and 5/2. Therefore, both Y and Z are best responses. With the second matrix, player 2's expected payoff from (respectively) W, X, Y, and Z are (respectively) 3, 1/2, 2, and 1. Therefore, W is the best response.

4. With the first matrix, A dominates C, Y dominates W, and Y dominates X. With the second matrix, B dominates C, W dominates X, and W dominates Y.

5. Let us first iteratively eliminate dominated strategies. With the first matrix, C is dominated for player 1 and W and X are dominated for player 2. In the reduced 3x2 matrix, it is easy to see that a mixed strategy of D with probability 0.99 and A with probability 0.01 dominates B. Therefore, the MSNE will be of the form $\sigma_1 = (p, 0, 0, 1-p)$ and $\sigma_2 = (0, 0, q, 1-q)$. For player 1 to be indifferent between A and D, the expected payoffs should be equal, which implies

$$4q + 3(1-q) = 3q + 4(1-q).$$

Solving, we have $q = \frac{1}{2}$. For player 2 to be indifferent between Y and Z, expected payoffs should be equal, which implies

$$3p + 1(1-p) = 5p.$$

Solving, we have $p = \frac{1}{3}$. Thus, the MSNE is $\sigma_1 = (\frac{1}{3}, 0, 0, \frac{2}{3})$ and $\sigma_2 = (0, 0, \frac{1}{2}, \frac{1}{2})$.

The same calculation procedure with the second matrix yields $\sigma_1 = (0, \frac{1}{3}, 0, \frac{2}{3})$ and $\sigma_2 = (\frac{1}{2}, 0, 0, \frac{1}{2})$.

6 and 7. On the tests marked with a "plus" or a "triangle", the payoff matrix is given by

| | | player 2 | | | | | |
|---|---|---|---|---|---|---|---|
| | | K | L | M | N | O | P |
| player 1 | AA | $2,0$ | $2,1$ | $\underline{3},\underline{2}$ | $0,0$ | $0,0$ | $0,0$ |
| | BB | $3,1$ | $\underline{4},\underline{4}$ | $1,2$ | $0,0$ | $0,0$ | $0,0$ |
| | CC | $\underline{5},4$ | $1,\underline{5}$ | $2,\underline{5}$ | $0,0$ | $0,0$ | $0,0$ |
| | DD | $0,0$ | $0,0$ | $0,0$ | $\underline{2},\underline{2}$ | $\underline{1},1$ | $2,1$ |
| | EE | $0,0$ | $0,0$ | $0,0$ | $1,7$ | $0,8$ | $1,\underline{9}$ |
| | FF | $0,0$ | $0,0$ | $0,0$ | $1,1$ | $0,\underline{9}$ | $\underline{7},0$ |

and on the tests marked with a "circle" or a "circle around plus", the payoff matrix is given by

| | | player 2 | | | | | |
|---|---|---|---|---|---|---|---|
| | | K | L | M | N | O | P |
| player 1 | AA | $2,1$ | $2,0$ | $\underline{3},\underline{2}$ | $0,0$ | $0,0$ | $0,0$ |
| | BB | $\underline{4},\underline{4}$ | $3,1$ | $1,2$ | $0,0$ | $0,0$ | $0,0$ |
| | CC | $1,\underline{5}$ | $\underline{5},4$ | $2,\underline{5}$ | $0,0$ | $0,0$ | $0,0$ |
| | DD | $0,0$ | $0,0$ | $0,0$ | $\underline{2},\underline{2}$ | $\underline{1},1$ | $2,1$ |
| | EE | $0,0$ | $0,0$ | $0,0$ | $1,7$ | $0,8$ | $1,\underline{9}$ |
| | FF | $0,0$ | $0,0$ | $0,0$ | $1,1$ | $0,\underline{9}$ | $\underline{7},0$ |

6. Best responses are underlined in the matrices. For the first matrix, the NE are (AA,M), (BB,L), and (DD,N). Thus, (CC,K) is not a NE. For the second matrix, the NE are (AA,M), (BB,K), and (DD,N). Thus, (CC,L) is not a NE.

7. To find the rationalizable strategies for the first matrix, we can dominate EE by a mixed strategy that plays DD with probability 0.99 and AA with probability 0.01. Having eliminated EE, we can dominate P by a mixed strategy

that plays N with probability 0.99 and M with probability 0.01. In the next round, we can dominate FF by a mixed strategy that plays DD with probability 0.99 and AA with probability 0.01. In the next round, we can dominate O by a mixed strategy that plays N with probability 0.99 and M with probability 0.01. Therefore, O is not rationalizable. (It turns out that the other choices are rationalizable.)

For the second matrix, we can dominate L by a mixed strategy that plays M with probability 0.99 and N with probability 0.01. Therefore, L is not rationalizable. (It turns out that the other choices are rationalizable.)

8. In all of the Cournot problems, inverse demand is of the form $p = A - q_1 - q_2$, where $A$ is either 20, 50, 100, or 200, depending on which version of the midterm you have. So I will solve the problem for general $A$ and then plug in the relevant value.

Setting up the payoff or profit function of firm 1, we have

$$u_1(q_1, q_2) = (A - q_1 - q_2)q_1 - (q_1)^2.$$

To find the best response to $q_2$, we maximize payoff by taking the derivative with respect to $q_1$, setting the expression equal to zero, and solving for $q_1$. The derivative is given by

$$A - 2q_1 - q_2 - 2q_1.$$

Setting it equal to zero and solving:

$$\begin{aligned} A - 2q_1 - q_2 - 2q_1 &= 0 \\ 4q_1 &= A - q_2 \\ q_1 &= \frac{A - q_2}{4}. \end{aligned}$$

The last equation is firm 1's best response function. By the symmetry of the problem, firm 2's best response function is

$$q_2 = \frac{A - q_1}{4} \text{ or } BR_2(q_1) = \frac{A - q_1}{4}.$$

To find the NE, simultaneously solve the two best response functions. We have

$$q_1 = \frac{A - q_2}{4} = \frac{A - \frac{A - q_1}{4}}{4}.$$

Multiplying the first term and the third term by 4 and setting them equal, then solving, we have

$$\begin{aligned} 4q_1 &= A - \frac{A - q_1}{4} \\ 16q_1 &= 4A - (A - q_1) \\ 16q_1 &= 3A + q_1 \\ q_1 &= \frac{A}{5}. \end{aligned}$$

The NE is $(\frac{A}{5}, \frac{A}{5})$. Substituting these quantities into the profit functions, we see that the equilibrium price is $\frac{3A}{5}$, and each firm makes profit of

$$\begin{aligned} & \frac{3A}{5}(\frac{A}{5}) - (\frac{A}{5})^2 \\ = \quad & \frac{2A^2}{25}. \end{aligned}$$

Recapping:

If $A = 20$, the best responses are $q_1 = \frac{20-q_2}{4}$ and $q_2 = \frac{20-q_1}{4}$, the NE is $(4, 4)$, and the profits for each firm are 32.

If $A = 50$, the best responses are $q_1 = \frac{50-q_2}{4}$ and $q_2 = \frac{50-q_1}{4}$, the NE is $(10, 10)$, and the profits for each firm are 200.

If $A = 100$, the best responses are $q_1 = \frac{100-q_2}{4}$ and $q_2 = \frac{100-q_1}{4}$, the NE is $(20, 20)$, and the profits for each firm are 800.

If $A = 200$, the best responses are $q_1 = \frac{200-q_2}{4}$ and $q_2 = \frac{200-q_1}{4}$, the NE is $(40, 40)$, and the profits for each firm are 3200.