

Normal Form Games

Here is the definition of a strategy: *A strategy is a complete contingent plan for a player in the game.*

For extensive form games, this means that a strategy must specify the action that a player would take at each of his information sets (including singleton information sets).

"Color" in a branch at each of her information sets in the game tree. The strategy is the entire picture, and changing just one of the branches being colored represents a different strategy.

For the example of an owner deciding whether to hire a worker, not hire the worker, or delegate the decision, possible strategies for the owner are: $\{D_h, D_n, S_h, S_n\}$.

Note that (i) it is a good idea to use a different label for each action, to avoid confusion in describing a strategy, and (ii) D_h and D_n are "almost" the same strategy, but they imply different expectations other players might have of what the owner would do if he somehow "made a mistake" and chose not to delegate.

Sometimes it is useful to abstract from the details of the game tree and the various actions that together comprise a strategy. We can think of the players as *simultaneously* coming up with plans on how to play. These plans can be implemented by an employee who is given instructions, by a computer that is programmed to play the game, or by the player himself.

This is the idea behind the normal form representation.

For the previous example, the owner chooses a strategy which is an element of the set $\{D_h, D_n, S_h, S_n\}$, and simultaneously, the manager chooses an element of his strategy set, $\{n, h\}$, and the worker chooses an element of her strategy set, $\{w, s\}$.

We will denote the number of players as n , and use the notation i to denote a particular player.

In a normal form game, each player i has a set of available strategies denoted by S_i . We denote the set of strategy profiles of all the players as $S = S_1 \times \dots \times S_n$.

We use the notation s_i to denote the choice of strategy by player i , where $s_i \in S_i$, and $s = (s_1, \dots, s_n)$ to denote a *strategy profile* or choice of strategies by all of the players, where $s \in S$.

We use the notation $-i$ to denote the players other than i . Thus, $S_{-i} = S_1 \times \dots \times S_{i-1} \times S_{i+1} \times \dots \times S_n$ and $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$. We can also write a strategy profile as $s = (s_i, s_{-i})$.

Example with 3 Players:

$$S_1 = \{a, b\}$$

$$S_2 = \{c, d\}$$

$$S_3 = \{e, f\}$$

Then $s = (a, c, f) \in S$ is a strategy profile and $s_{-2} = (a, f)$ is a strategy profile of players other than player 2.

The formal definition of a normal form game is:

A set of players, $\{1, \dots, n\}$

For each player, i , a set of available strategies, S_i

For each player, i , a payoff function, $u_i : S \rightarrow \mathbb{R}$, which maps a strategy profile into a real number corresponding to the utility received.

For the strategy profile, (s_1, \dots, s_n) , the payoff to player i is written as $u_i(s_1, \dots, s_n)$.

Two-player normal form games with finite strategy sets can be written as matrices, sometimes called *matrix form*.

Here are some common examples of normal form games:

Matching Pennies. This game is a zero-sum game, purely confrontational.

		player 2	
		heads	tails
player 1	heads	1, -1	-1, 1
	tails	-1, 1	1, -1

Prisoner's Dilemma. This is not a two-person zero sum game. Elements of competition and elements of gains from cooperation.

		player 2	
		cooperate	defect
player 1	cooperate	2, 2	0, 3
	defect	3, 0	1, 1

Battle of the Sexes. The two players want to be together, but each has his/her preferred activity.

		player 2	
		opera	wrestling match
player 1	opera	2, 1	0, 0
	wrestling match	0, 0	1, 2

Chicken. The person who swerves out of the way is the chicken, but that is better than crashing.

		player 2	
		passive	aggressive
player 1	passive	6, 6	2, 7
	aggressive	7, 2	0, 0

Pure Coordination Game. Which side of the street do you drive on?

		player 2	
		left	right
player 1	left	1, 1	0, 0
	right	0, 0	1, 1

"Pareto" Coordination Game.

		player 2	
		A	B
player 1	A	2, 2	0, 0
	B	0, 0	1, 1

Stag Hunt Game. The players must cooperate in order to catch the stag.

		player 2	
		hunt	pick berries
player 1	hunt	5, 5	0, 4
	pick berries	4, 0	4, 4

Interpretations of the Normal Form:

1. The game truly involves simultaneous play.
2. The game involves sequential choices of actions over time, but we are abstracting from the details of the dynamic structure.

For every extensive form game, there is a unique normal form representation if we label the strategies appropriately.

The reverse is not true. For a game represented in normal form, there may be several different extensive form representations (game trees).