

Rationalizability and Iterated Dominance

Assuming that people are rational means that they choose a strategy that is a best response to their beliefs about the other players' strategy profile. In the previous section, we saw that being rational (without restricting beliefs at all) is equivalent to not playing a dominated strategy.

Unfortunately, assuming rationality is only the starting point in providing a theory of how people choose a strategy. Some games have many undominated strategies!

Here we explore the implications of common knowledge of rationality.

Consider the following game, Figure 7.1 in the text:

		player 2		
		<i>X</i>	<i>Y</i>	<i>Z</i>
player 1	<i>A</i>	3, 3	0, 5	0, 4
	<i>B</i>	0, 0	3, 1	1, 2

Player 1 does not have any dominated strategies, so either A or B is rational, depending on his beliefs.

For player 2, X is dominated by Y (by Z as well), so player 2 will not play X if she is rational. If player 1 knows that player 2 is rational, then he knows that the probability that she plays X is zero. The game is equivalent to the simpler game where we eliminate the X column:

		player 2	
		<i>Y</i>	<i>Z</i>
player 1	<i>A</i>	0, 5	0, 4
	<i>B</i>	3, 1	1, 2

		player 2	
		<i>Y</i>	<i>Z</i>
player 1	<i>A</i>	0, 5	0, 4
	<i>B</i>	3, 1	1, 2

But we are not done yet. In the simpler game, player 1's strategy *A* is dominated by *B*. Therefore, player 1 will not play *A* if both players are rational, and if player 1 knows that player 2 is rational.

Thus, if both players are rational, and if player 1 knows that player 2 is rational, player 1 will choose *B*.

		player 2	
		<i>Y</i>	<i>Z</i>
player 1	<i>B</i>	3, 1	1, 2

If both players are rational, and if player 1 knows that player 2 is rational, and if player 2 knows that player 1 knows that player 2 is rational, then player 2 will choose *Z*.

For this game, the assumption of common knowledge of rationality allowed us to solve the game, by a process called *iterative elimination of dominated strategies*. Sometimes this is called *iterated dominance*.

Because the set of dominated strategies is the same as the set of strategies that are not a best response to any belief, iterated dominance is equivalent to the following process:

1. For each player, eliminate the strategies that cannot be rationalized as a best response to any belief. We are left with a smaller set of strategies.
2. For each player, eliminate the strategies that are not a best response to any belief that puts positive probability only on the strategies that have not previously been eliminated. We are left with an even smaller set of strategies.
3. Iterate this process until no more strategies can be eliminated. We are left with the set of rationalizable strategies for each player.

The process of iterated dominance does not always yield a unique outcome, or a unique rationalizable strategy for each player. When this happens, there is unavoidable *strategic uncertainty*.

This strategic uncertainty is most easily seen in coordination games like the Stag Hunt Game. Either strategy is rationalizable.

		player 2	
		hunt	pick berries
player 1	hunt	5, 5	0, 4
	pick berries	4, 0	4, 4

Notice that picking berries is the "safe" option, so that a coordination problem might prevent a perfectly rationalizable and more efficient outcome from occurring.

How do we resolve coordination problems?

–focal points (Disneyland, New York City)

–communication and agreement in advance

–institutions, rules, and cultural norms (drive on the right, walk on the right)

Solving a game through iterated dominance: A Location Game

Two ice-cream sellers, Ben and Jerry, must simultaneously decide where to locate along a beach. They each sell the same brand of ice cream and are required to set a price of exactly \$2.00, so the only strategic choice is where to locate. Assume that Ben and Jerry's cost per ice cream is \$1.00. The beach has 9 sections all in a row along a wide shoreline, and there are 100 potential customers in each section.

Each potential customer will purchase exactly one ice cream from either Ben or Jerry, depending on who is closest. If Ben and Jerry are equally close to a particular section, assume that 50 people from that section go to each seller. For example, if Ben locates in section 3 and Jerry locates in section 5, then everyone in sections 1, 2, and 3 go to Ben, everyone in sections 5, 6, 7, 8, and 9 go to Jerry, and half of section 4 goes to Ben and half to Jerry.

The strategy sets for Ben and Jerry are:

$$S_B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \text{ and}$$
$$S_J = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

If you want, you can fill out the 9×9 payoff matrix.

What are the rationalizable strategies?

First, Ben locating in section 1 is dominated by locating in section 2. To see this, if Jerry locates in section 1, Ben's profits are \$450 in section 1 and \$800 in section 2. If Jerry locates in section 2, Ben's profits are \$100 in section 1 and \$450 in section 2. If Jerry locates in section 3 or higher, Ben attracts an additional 50 customers by locating in section 2 rather than in section 1.

By similar reasoning, either Ben or Jerry locating in section 1 or in section 9 is a dominated strategy.

After eliminating section 1 and section 9 strategy choices for each player (but not the customers in those sections!), locating in section 2 or section 8 is dominated. For example, Ben locating in section 2 is dominated by locating in section 3. To see this, if Jerry locates in section 2, Ben's profits are \$450 in section 2 and \$700 in section 3. If Jerry locates in section 3, Ben's profits are \$200 in section 2 and \$450 in section 3. If Jerry locates in section 4 or higher, Ben attracts an additional 50 customers by locating in section 3 rather than in section 2.

Thus, after two rounds of iterative elimination of dominated strategies, we are left with $\{3, 4, 5, 6, 7\}$. After four rounds, we are left with the single rationalizable strategy for each player, $\{5\}$.