The Ohio State University
Department of Economics
Econ 501.02–Prof. James Peck
Homework #1 Answers

1. Suppose we have the utility function,

$$U = x^3 y^3.$$

(a) Find the function for the marginal rate of substitution.
(b) Show that this utility function satisfies the first 3 axioms of consumer preference.

Answer:
(a) The MRS is given by

$$\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{3x^2y^3}{3x^3y^2} = \frac{y}{x}.$$

(b) Suppose preferences are based on this utility function, and denote bundle A by (x^A, y^A), bundle B by (x^B, y^B), and bundle C by (x^C, y^C). Then preferences are complete, because either $(x^A y^A)^3 > (x^B y^B)^3$ (in which case $A^P B$), or $(x^A y^A)^3 < (x^B y^B)^3$ (in which case $B^P A$), or $(x^A y^A)^3 = (x^B y^B)^3$ (in which case $A^I B$). Preferences are reflexive, because $(x^A y^A)^3 = (x^A y^A)^3$. Preferences are transitive, because if $(x^A y^A)^3 > (x^B y^B)^3$ and $(x^B y^B)^3 > (x^C y^C)^3$ hold, then $(x^A y^A)^3 > (x^C y^C)^3$ must hold. In other words, the ">" relation on real numbers is complete, reflexive, and transitive.

2. Suppose we have the utility function,

$$U = xy + x + y.$$

(a) Find the function for the marginal rate of substitution.
(b) If prices are $p_x = \$2$ and $p_y = \$4$, and if income is $M = \$18$, find the utility maximizing consumption bundle.

Answer:
(a) The MRS is given by

$$\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{y+1}{x+1}.$$

(b) Set up the Lagrangean,

$$L = xy + x + y + \lambda[18 - 2x - 4y].$$

Setting the partial derivatives of L with respect to x, y, and λ equal to zero, we have the first order conditions

$$\begin{aligned} y+1+\lambda[-2] &= 0 \\ x+1+\lambda[-4] &= 0 \\ 18-2x-4y &= 0 \end{aligned}$$

To solve, we first eliminate λ by using the first two equations:

$$\lambda = \frac{y+1}{2} = \frac{x+1}{4},$$

which is the condition that the MRS equals the price ratio. This is simplified to

$$x = 2y + 1. \tag{1}$$

Plugging equation (1) into the third first-order condition above (budget equation), we get

$$18 - 2(2y+1) - 4y = 0,$$

which we solve for the demand for y:

$$y = 2. \tag{2}$$

Plugging (2) into (1), we solve for the demand for x:

$$x = 5.$$

3. Suppose we have the utility function,

$$U = xy.$$

(a) Derive the generalized demand function for x as a function of p_x, p_y, and M.
(b) Are the own-price ordinary demand functions downward sloping? Explain.
(c) Is good x normal or inferior? Explain.

Answer:
(a) Set up the Lagrangean,

$$L = xy + \lambda[M - p_x x - p_y y].$$

Setting the partial derivatives of L with respect to x, y, and λ equal to zero, we have the first order conditions

$$\begin{aligned} y+\lambda[-p_x] &= 0 \\ x+\lambda[-p_y] &= 0 \\ M-p_x x-p_y y &= 0 \end{aligned}$$

To solve, we first eliminate λ by using the first two equations:

$$\lambda = \frac{y}{p_x} = \frac{x}{p_y},$$

which is the condition that the MRS equals the price ratio. This is simplified to

$$y = \frac{p_x x}{p_y}. \tag{3}$$

Plugging equation (3) into the third first-order condition above (budget equation), we get

$$M - p_x x - p_y \frac{p_x x}{p_y} = 0,$$

which we solve for the generalized demand function for x:

$$x = \frac{M}{2p_x}. \tag{4}$$

(b) The own-price ordinary demand function is downward sloping. Differentiating (4) with respect to p_x, we get $\frac{-M}{2(p_x)^2}$, which is negative.

(c) To see that good x is a normal good, differentiate the generalized demand function with respect to income, M. Because this derivative is positive, $\frac{1}{2p_x} > 0$, increasing income increases the demand for good x.

4. Suppose there are 100 consumers, each with an income of \$900 and utility function

$$U = x^2 y.$$

Suppose that the price of good y is \$4. Derive the own-price market demand function for x.

Hint: First derive the demand function for one consumer. Since there are 100 identical consumers, the total market demand is 100 times what one consumer would demand.

Answer:

To find the market demand function, we first derive the ordinary demand function for one consumer. For one consumer, set up the Lagrangean

$$L = (x)^2 y + \lambda[900 - p_x x - 4y].$$

Setting the partial derivatives of L with respect to x, y, and λ equal to zero, we have the first order conditions

$$\begin{aligned} 2xy + \lambda[-p_x] &= 0 \\ (x)^2 + \lambda[-4] &= 0 \\ 900 - p_x x - 4y &= 0. \end{aligned}$$

Use the first two equations to eliminate λ

$$\lambda = \frac{2xy}{p_x} = \frac{(x)^2}{4}.$$

This is the condition that the MRS equals the price ratio, which can be simplified to

$$y = \frac{p_x x}{8}. \qquad (5)$$

Plugging (5) into the third first-order equation (budget equation), we get

$$900 - p_x x - 4(\frac{p_x x}{8}) = 0,$$

which can be solved for a single consumer's ordinary demand function for x

$$x = \frac{600}{p_x}. \qquad (6)$$

Equation (6) gives a single consumer's ordinary demand function. The ordinary market demand function is determined by adding up the demands of each consumer, as a function of the price. Since there are 100 consumers,

$$X = \frac{60,000}{p_x}.$$