The Ohio State University
Department of Economics
Econ 501.02 Prof. James Peck
Homework #3 Answers

1. Consider the following production function

$$x = \frac{5KL}{K+L}.$$

Suppose that input prices are given by $w = 4$ and $r = 1$.
(a) Find the conditional demands for K and L, as a function of x.
(b) What is the firm's long run average cost function?

Answer:
(a) First, we set up the Lagrangean for the cost minimization problem

$$Lagr. = 4L + K + \lambda[x - \frac{5KL}{K+L}].$$

The first order conditions are

$$\begin{aligned}
\frac{\partial Lagr.}{\partial L} &= 0 = 4 - \lambda\frac{5K^2}{(K+L)^2} \\
\frac{\partial Lagr.}{\partial K} &= 0 = 1 - \lambda\frac{5L^2}{(K+L)^2} \\
\frac{\partial Lagr.}{\partial \lambda} &= 0 = x - \frac{5KL}{K+L}.
\end{aligned}$$

Solving the first two equations for λ and setting them equal to each other, we have the condition that MRTS = w/r,

$$\frac{K^2}{L^2} = 4,$$

which can be written as

$$K = \sqrt{\frac{w}{r}}L = 2L. \tag{1}$$

Plugging (1) into the third condition yields

$$x = \frac{5(2L)L}{2L+L} = \frac{10L}{3}.$$

Solving for L, we have

$$L = \frac{3x}{10}.$$

From (1), we have

$$K = \frac{6x}{10}.$$

(b) The long run total cost function is

$$LRTC = wL^* + rK^* = 4(\frac{3x}{10}) + (\frac{6x}{10}) = \frac{18x}{10} = \frac{9x}{5}.$$

Long run *average* cost is therefore

$$LRAC = \frac{LRTC}{x} = \frac{9}{5}.$$

Notice that long run average cost is independent of x, because of constant returns to scale.

2. Suppose the firm has the production function of problem 1, $x = \frac{5KL}{K+L}$, and that input prices are given by $w = 4$ and $r = 1$. Also suppose capital is fixed at 4 units, $\overline{K} = 4$.

(a) Compute the firm's short run average total cost, average variable cost, and marginal cost functions.

(b) (This is challenging. Don't spend too much time on this.) First find the value for x that minimizes short run average total cost. Now substitute that value of x into the following functions, and show that they take on the same value: short run average total cost, short run marginal cost, and long run average total cost.

Answer:

Now we have a short run situtation in which capital is fixed at 4 units, $\overline{K} = 4$.

(a) If capital is fixed at 4, $\overline{K} = 4$, then the short run production function is given by

$$x = \frac{20L}{4+L}.$$

Solving for L, we have $4x + xL = 20L$, so

$$L = \frac{4x}{20-x}.$$

Therefore, the short run total cost and average total cost functions are

$$SRTC = 4(\frac{4x}{20-x}) + 4 \text{ and } SRATC = \frac{16}{20-x} + \frac{4}{x}.$$

The short run average variable cost function is

$$SRAVC = \frac{wL}{x} = \frac{16}{20-x}.$$

Taking the derivative of SRTC, and using the quotient rule and chain rule, we have the short run marginal cost function,

$$SRMC = \frac{(20-x)16 - 16x(-1)}{(20-x)^2} = \frac{320}{(20-x)^2}.$$

(b) First we must find the minimum SRATC, by solving

$$\frac{\partial SRATC}{\partial x} = 0 = 16(20-x)^{-2} - 4x^{-2}.$$

Simplifying, we have

$$\begin{aligned} \frac{16}{(20-x)^2} &= \frac{4}{x^2} \quad \text{or} \\ (20-x)^2 &= 4x^2 \end{aligned}$$

Taking the square root of both sides, we have

$$20 - x = 2x.$$

Simplifying, we have $x = \frac{20}{3}$. This is the x that minimizes SRATC. To find that minimum cost, we evaluate SRATC at $x = \frac{20}{3}$:

$$\begin{aligned} SRATC &= \frac{16}{20-x} + \frac{4}{x} = \frac{16}{40/3} + \frac{4}{20/3} = \frac{36}{20} = \frac{9}{5}. \\ SRMC &= \frac{320}{(20-x)^2} = \frac{320}{(20-\frac{20}{3})^2} = \frac{9}{5}. \end{aligned}$$

Thus, when evaluated at the x that minimizes SRATC, the following functions all take the value $\frac{9}{5}$: SRATC, SRMC, and LRAC (from problem 1).