The Ohio State University
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Econ 501.02 Prof. James Peck
Homework #4 Answers

1. Chapter 12, questions for discussion 3. Since you are already committed to the leasing contract, the equipment is a fixed input. Even though the price paid for the equipment can fluctuate, you are committed to paying for the equipment no matter how much output you produce. If the prime rate increases, your cost of capital increases, but the amount you must pay for the capital does not depend on how much output you produce. Therefore, the change in the rental rate should not affect your output decision, so you should not have reduced your production rate. (Note: my answer implicitly assumes that all of the firm's capital is covered by the leasing contract, and that it is not possible for the firm to "sublet" some of the equipment to another firm. Otherwise, the capital input would not be fixed.)

2. (a) The production function is

$$x = 32K^{1/4}L^{3/4},$$

and we have $w = 3, r = 1$, and $\overline{K} = 1$. The short run production function is

$$x = 32L^{3/4},$$

which gives us the labor requirement when we solve for L:

$$L = \left(\frac{x}{32}\right)^{4/3}.$$

The total cost and marginal cost functions are then

$$\begin{aligned} SRTC &= wL + r\overline{K} = 3\left(\frac{x}{32}\right)^{4/3} + 1, \\ SRMC &= \frac{4}{32}\left(\frac{x}{32}\right)^{1/3}. \end{aligned}$$

To get the short run supply function, set SRMC equal to the price:

$$\frac{4}{32}\left(\frac{x}{32}\right)^{1/3} = p_x. \tag{1}$$

To express the supply function in terms of output as a function of price, solve (1) for x:

$$x = 16384(p_x)^3$$

(b) To find the long run equilibrium price of good x, we must find the minimum long run average cost. The long run total cost function is found by solving the Lagrangean problem:

$$\begin{aligned} & \min 3L + K \\ \text{subject to } 32K^{1/4}L^{3/4} &= x \end{aligned}$$

Set up the Lagrangean,

$$Lagr. = 3L + K + \lambda[x - 32K^{1/4}L^{3/4}]$$

The first order conditions are

$$\begin{aligned} \frac{\partial Lagr.}{\partial L} &= 0 = 3 - \lambda 24K^{1/4}L^{-1/4} \\ \frac{\partial Lagr.}{\partial K} &= 0 = 1 - \lambda 8K^{-3/4}L^{3/4} \\ \frac{\partial Lagr.}{\partial \lambda} &= 0 = x - 32K^{1/4}L^{3/4} \end{aligned}$$

Solving the first two equations for λ, we have

$$\lambda = \frac{3}{24L^{-1/4}K^{1/4}} = \frac{1}{8L^{3/4}K^{-3/4}},$$

which can be simplified to

$$L = K. \tag{2}$$

Now plug (2) into the constraint, $x = 32K^{1/4}L^{3/4}$, which yields

$$x = 32K. \tag{3}$$

From (2) and (3), it follows that $L = K = \frac{x}{32}$, so the total cost of producing x is $3\frac{x}{32} + \frac{x}{32}$, so $LRTC(x) = \frac{x}{8}$. Long run average cost is constant at $\frac{1}{8}$, so the long run equilibrium price is $\frac{1}{8}$.

3.

$$\begin{aligned} X^d &= 2900 - 100p_x \\ X^s &= 200p_x - 400 \end{aligned}$$

(a) The equilibrium price is found by setting demand equal to supply.

$$\begin{aligned} 2900 - 100p_x &= 200p_x - 400, \text{ which implies} \\ 3300 &= 300p_x, \text{ or } \quad p_x = 11. \end{aligned}$$

Substituting the price into the demand (or supply) equation, the equilibrium quantity is 1800.

(b) With a \$1 per unit tax, marginal cost increases by 1, so the *inverse* supply curve shifts up by 1. The equation for the original inverse supply curve is found by solving for p_x

$$p_x = \frac{x + 400}{200}.$$

The new supply curve is

$$p_x = \frac{x + 400}{200} + 1 = \frac{x + 600}{200}.$$

Therefore, we have

$$X^s = 200p_x - 600.$$

The new equilibrium price solves

$$2900 - 100p_x = 200p_x - 600.$$

Solving for the price, we have

$$p_x = \frac{3500}{300} = 11.67.$$

The equilibrium quantity is $2900 - 100(11.67) = 1733$.

(c) The price paid by consumers (demand price) is $p_x = 11.67$, and the price received by firms after tax (supply price) is 10.67. Therefore, two thirds of the tax burden falls on consumers in the short run.

(d) In the long run, the tax is paid entirely by consumers, because firms receive zero profits before the tax and zero profits after the tax. Assuming we started at long run equilibrium, minimum LRAC equals 11, so the initial long run supply curve is flat at that price. After the \$1 tax, the new minimum LRAC equals 12, so the new long run supply curve is flat at that price. After the \$1 tax, the long run equilibrium price goes up by \$1. Since the demand curve has not changed, the effect of the tax on quantity is a reduction, as we move up the demand curve in the direction of higher price and lower quantity. At a price of 12, the new long run equilibrium quantity demanded is 1700.