Deriving the Total Cost Function

Since (9) and (10) tell us the amounts of K and L to choose in order to produce x units of output, we can derive the total cost of producing x, assuming the firm chooses its inputs optimally to minimize costs.

$$TC^* = wL^* + rK^* \tag{1}$$

For our example, (11) becomes

$$\begin{aligned}
TC^* &= w(\frac{2r}{w})^{1/3}A^{-1}x + r(\frac{2r}{w})^{-2/3}A^{-1}x \\
&= [2^{1/3} + 2^{-2/3}]w^{2/3}r^{1/3}A^{-1}x
\end{aligned}$$

Notice that total cost is proportional to x. This is a property of any constant-returns-to-scale production function. Since all inputs are variable for this calculation, TC^* is sometimes called the *long run total cost function*, LRTC.
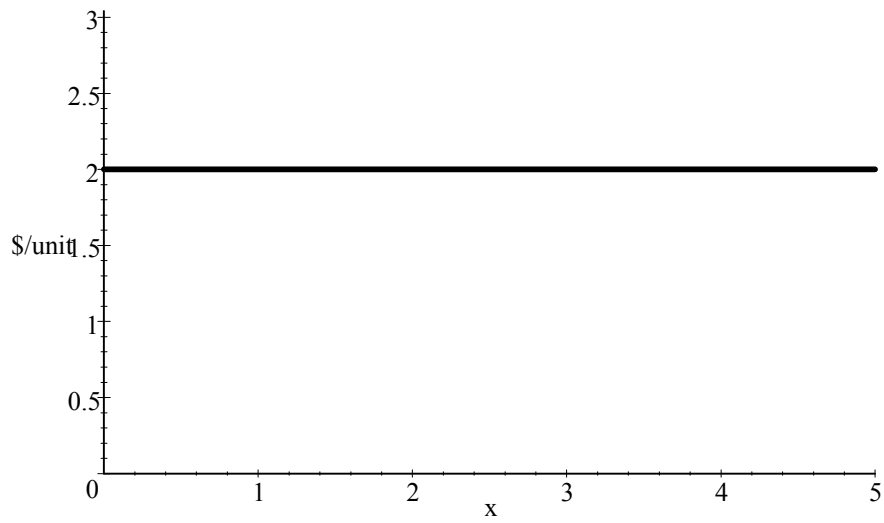
We can also define *long run average cost and long run marginal cost*:

$$LRAC = \frac{LRTC}{x} \quad \text{and} \quad LRMC = \frac{d(LRTC)}{dx}$$
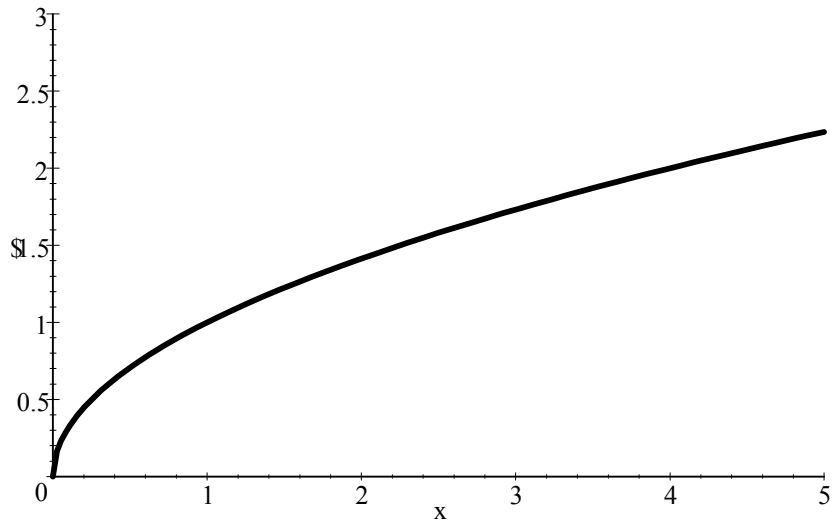
For constant returns to scale, these cost functions are constant, independent of x
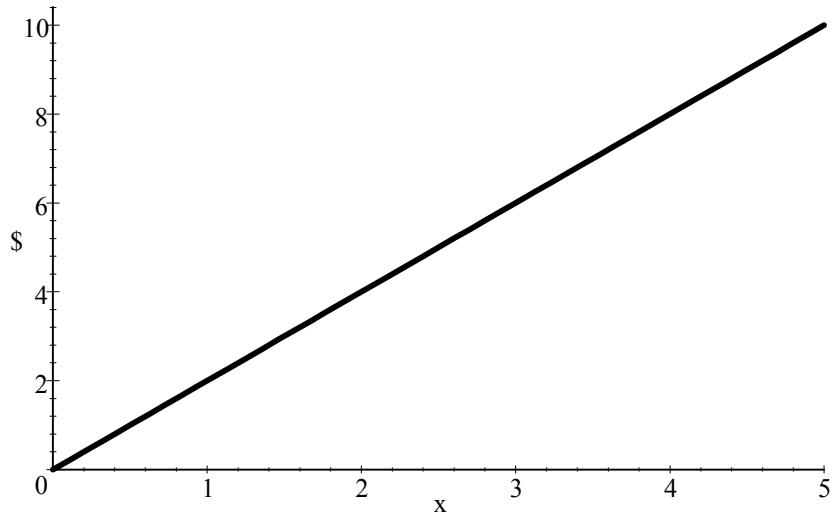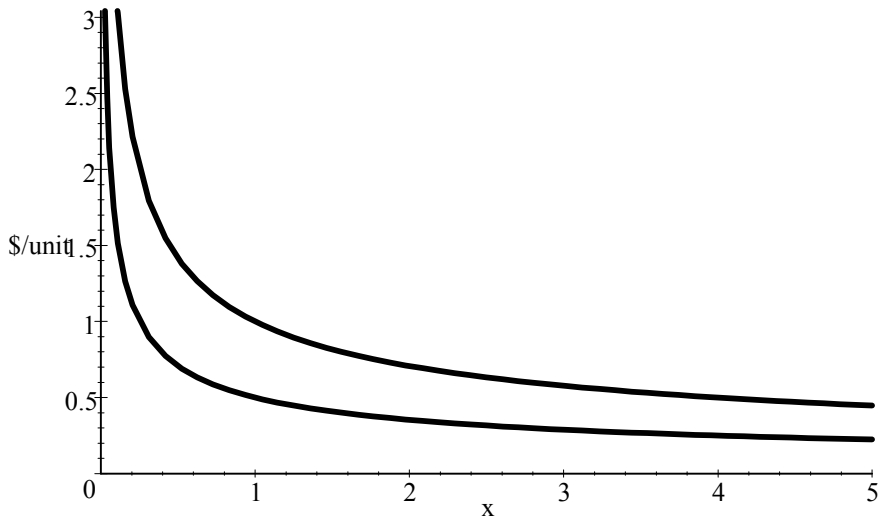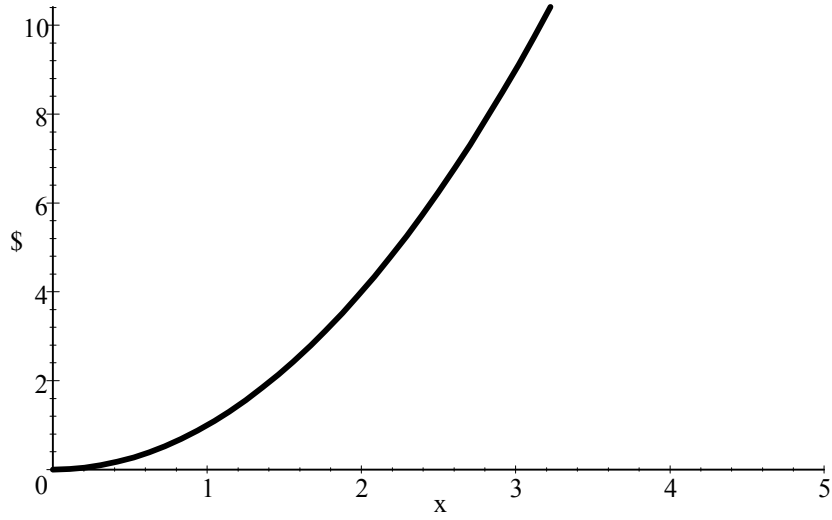
LRTC (constant returns)

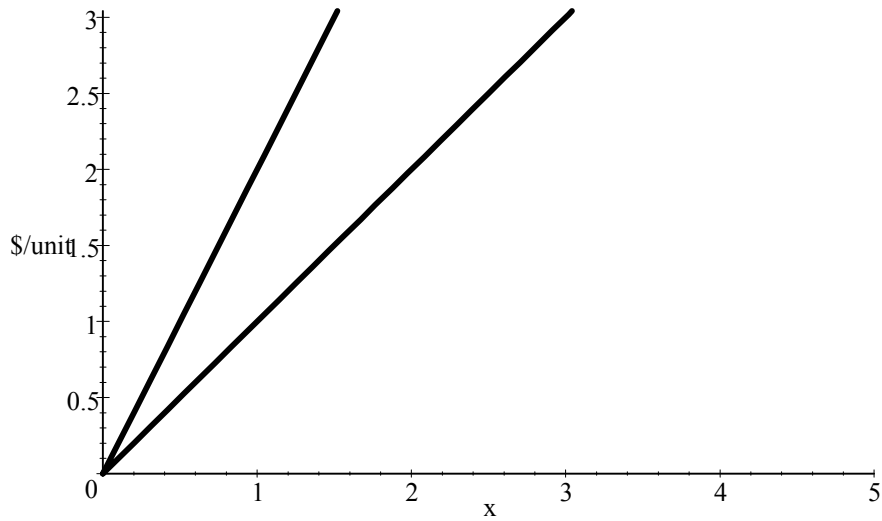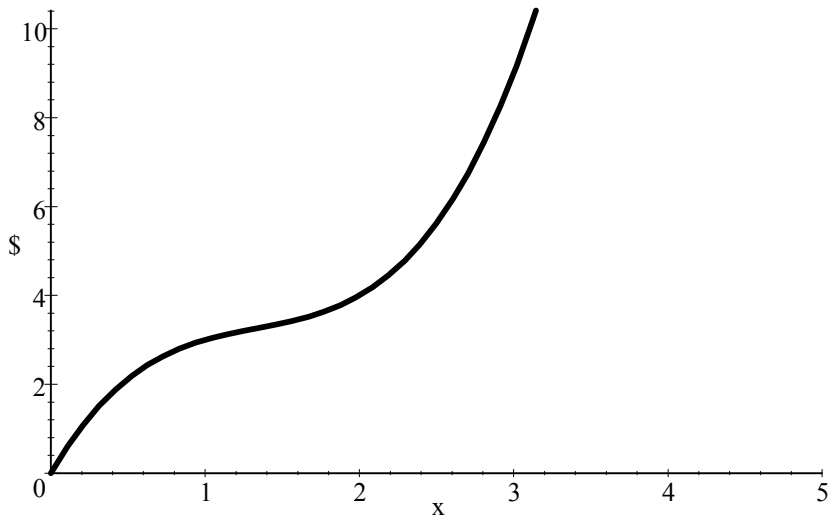LRAC and LRMC (constant returns)

LRTC (increasing returns)

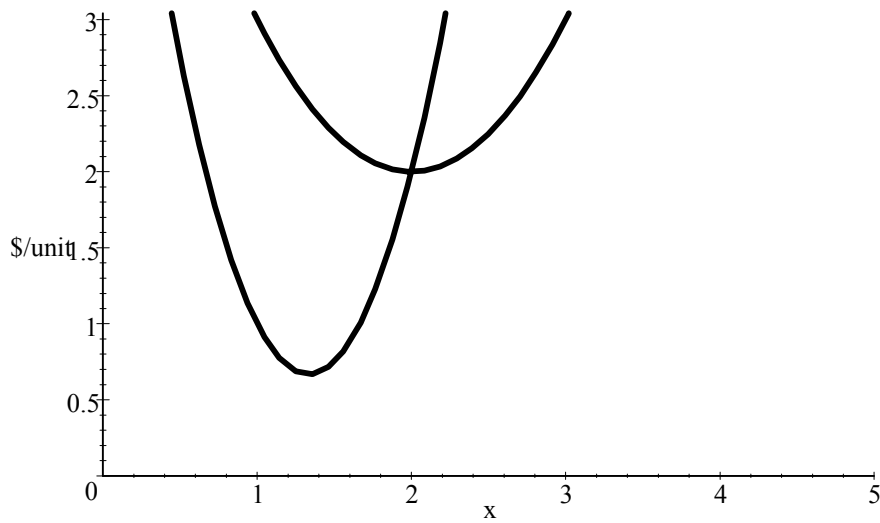LRAC and LRMC (increasing returns)

LRTC (decreasing returns)

LRAC and LRMC (decreasing returns)

LRTC (S-shaped)

LRAC and LRMC (U-shaped LRAC)

There are IRS for $x < 2$ and DRS for $x > 2$.

If the LRAC curve is falling, then at the margin, we are bringing down the average as we increase x. Thus, the marginal cost must be below average cost (in order to be bringing down the average).

If the LRAC curve is increasing, then marginal cost must be above average cost.

For a U-shaped LRAC curve, then at the minimum point, the curve is flat. Since one more unit is not changing the average, it must be that marginal cost is equal to average cost.

Returns to Scope and Joint Production (We will not treat this topic formally.) Sometimes there can be cost savings by expanding the set of products produced, rather than expanding the amount of a given good produced.

Examples include: (1) airlines, spoke and hub networks. By adding the Milwaukee to Chicago route, more passengers will want to fly the Chicago to New York route. Federal Express. (2) Gas station and convenience store. (3) Time-share condos, hotels. (4) Produce intermediate as well as final products.