

## Demand for Inputs

Since profits are

$$\pi = p_x f(K, L) - wL - rK,$$

the profit maximizing choice of L satisfies

$$\frac{\partial \pi}{\partial L} = 0 = p_x MP_L - w.$$

Thus, the firm hires labor until the amount of revenues generated by one more labor hour equals the hourly wage. Treating K as fixed, so  $MP_L$  depends only on L, the equation for the firm's labor demand curve is  $w = p_x MP_L$ .

Similarly, holding L fixed, the firm's demand for capital is  $r = p_x MP_K$ .

Demand for inputs are *derived* from demand for outputs.

## The Firm's Long Run Profit Maximization Problem

(or the short run profit maximization problem with 2 variable inputs)

In the long run, the firm chooses all inputs to maximize profits. Setting up profits as a function of L and K and optimizing (approach 1, discussed earlier) yields the first order conditions:

$$\begin{aligned}\frac{\partial \pi}{\partial L} &= 0 = p_x MP_L - w \\ \frac{\partial \pi}{\partial K} &= 0 = p_x MP_K - r,\end{aligned}$$

which can be solved simultaneously for L and K, then plugged into the production function to find x.

Approach 2 is to derive the long run marginal cost curve, and equate price to marginal cost.

example:  $f(K, L) = K^\alpha L^\alpha$

Approach 1 can work, but it is a mess for this problem. We will instead derive the LRMC curve.

First derive LRMC by setting up the Lagrangean expression for the cost minimization problem, and getting as first order conditions:

$$x = K^\alpha L^\alpha \quad \text{and} \quad (1)$$

$$\frac{MP_L}{MP_K} = \frac{\alpha K^\alpha L^{\alpha-1}}{\alpha K^{\alpha-1} L^\alpha} = \frac{w}{r},$$

which simplifies to

$$L = \frac{rK}{w}. \quad (2)$$

Now plug (2) into (1) to get

$$x = K^\alpha \left(\frac{rK}{w}\right)^\alpha = K^{2\alpha} \left(\frac{r}{w}\right)^\alpha.$$

Simplifying, we have

$$x^{1/\alpha} = \left(\frac{rK^2}{w}\right). \quad (3)$$

Solving (3) for K as a function of x, we have

$$K = \left(\frac{w}{r}\right)^{1/2} x^{1/2\alpha}. \quad (4)$$

From (4) and (2), we have

$$L = \left(\frac{r}{w}\right)^{1/2} x^{1/2\alpha}. \quad (5)$$

From (4) and (5), the LRTC function is

$$\begin{aligned} LRTC &= w\left(\frac{r}{w}\right)^{1/2} x^{1/2\alpha} + r\left(\frac{w}{r}\right)^{1/2} x^{1/2\alpha} \\ &= 2(wr)^{1/2} x^{1/2\alpha} \end{aligned} \quad (6)$$

Differentiating (6) with respect to  $x$ , we have the marginal cost function,

$$LRMC = \frac{(wr)^{1/2}}{\alpha} x^{\frac{1}{2\alpha}-1}. \quad (7)$$

Case 1:  $\alpha < \frac{1}{2}$ . This is the case of decreasing returns to scale, and LRMC is upward sloping. The formula for the supply function is

$$p_x = \frac{(wr)^{1/2}}{\alpha} x^{\frac{1}{2\alpha}-1}. \quad (8)$$

From (8), and given prices, we solve for  $x$ . Then  $K$  and  $L$  are found by plugging  $x$  into (4) and (5).

Case 2:  $\alpha = \frac{1}{2}$ . This is the case of constant returns to scale, and we have  $LRMC = 2(wr)^{1/2}$ .

The marginal cost curve is flat, and equal to LRAC, independent of  $x$ . In other words, the firm's supply curve is flat.

There is no supply at a price below  $2(wr)^{1/2}$ , there is infinite supply at a price above  $2(wr)^{1/2}$ , and the firm makes zero profits at any quantity of output at a price of exactly  $2(wr)^{1/2}$ .

$$\begin{aligned}\pi &= p_x x - wL - rK \\ &= p_x x - w\left(\frac{r}{w}\right)^{1/2} x - r\left(\frac{w}{r}\right)^{1/2} x \\ &= \left[ p_x - 2(wr)^{1/2} \right] x = 0.\end{aligned}$$

Case 3:  $\alpha > \frac{1}{2}$ . This is the case of increasing returns to scale, and LRMC is downward sloping. The firm is not profit maximizing by choosing  $x$  such that  $LRMC = p_x$ . In fact, no matter what  $p_x$  is, LRAC will fall below  $p_x$  for large enough  $x$ . Higher and higher  $x$  will only increase the profit per unit, so overall profits are infinite!

For example, let  $\alpha = 1$ .

Now  $LRAC = 2(wr)^{1/2}x^{1/2\alpha-1} = 2(wr)^{1/2}x^{-1/2}$ . As  $x$  approaches infinity, average costs fall to zero.

For any production function exhibiting increasing returns to scale, if the firm is willing to produce positive output, profits will be more than double by doubling the output. There is an inconsistency between *increasing returns to scale* and *perfect competition*. Some firm or firms will become so big that they influence the market price. Natural Monopoly.