Computing an Example (see Binger and Hoffman, p. 147-149): $u(x, y) = xy + x + y$

The consumer's utility maximization problem is

$$\begin{aligned} & \max xy + x + y \\ \text{subject to} \quad : & \quad p_x x + p_y y = M \end{aligned} \tag{1}$$

To solve (1), we set up the Lagrangean expression, $L = xy+x+y+\lambda[M - p_x x - p_y y]$. The first-order conditions are given by

$$x \quad : \quad y + 1 - \lambda p_x = 0 \tag{2}$$
$$y \quad : \quad x + 1 - \lambda p_y = 0 \tag{3}$$
$$\lambda \quad : \quad M - p_x x - p_y y = 0. \tag{4}$$

We can solve (2) and (3) to eliminate λ, yielding the equation for the income-consumption curve:

$$y = \frac{p_x}{p_y}(x+1) - 1. \tag{5}$$

Equation (5) is the condition, $MRS = p_x/p_y$.

By plugging (5) into the budget equation (4), we can solve for the generalized demand function for x. Then plug x^* into (5) to get y^*:

$$x^* = \frac{M + p_y - p_x}{2p_x} \text{ and } y^* = \frac{M + p_x - p_y}{2p_y}. \tag{6}$$

Equation (6) gives us the ordinary demand function, the Engel curve, or the cross-price demand function, depending on which variables we plug in numbers for.

$$x^* = \frac{M + p_y - p_x}{2p_x}$$

1. For fixed M and p_y, equation (6) gives us the ordinary demand function, x as a function of p_x. By taking the derivative, we have

$$\frac{\partial x^*}{\partial p_x} = -\frac{M + p_y}{2(p_x)^2}.$$

Since this expression is negative, demand is downward sloping. Note that (6) tells us exactly how the demand curve shifts when we change M or p_y.

2. For fixed p_x and p_y, equation (6) gives us the Engel curve, x as a function of M. By taking the derivative, we have

$$\frac{\partial x^*}{\partial M} = \frac{1}{2p_x}.$$

Since this expression is positive, x is a normal good.

$$x^* = \frac{M + p_y - p_x}{2p_x}$$

3. For fixed M and p_x, equation (6) gives us the cross-price demand function, x as a function of p_y. By taking the derivative, we have

$$\frac{\partial x^*}{\partial p_y} = \frac{1}{2p_x}.$$

Since this expression is positive, x is a gross substitute for y.

Note: you can use the generalized demand function for y to derive the ordinary demand function, the Engel curve, and the cross-price demand function for good y.

Homogeneity of Degree Zero: If we multiply prices and income by the same number, the consumer's utility maximization problem is unchanged, so optimal choices are unchanged.

$$\begin{aligned} & \max u(x,y) \\ \text{subject to} \quad : \quad & p_x x + p_y y = M \end{aligned}$$

One implication is that a "pure" inflation that multiplies all prices (including sources of income like wages, rents, etc.) by the same factor should have no effect on economic activity.

Another implication is that an econometrician trying to estimate demand should specify a function that is homogeneous of degree zero. Otherwise, the function is inconsistent with rational behavior.

Discussion Question: How reasonable is a model which requires consumers to spend all their income? What about saving?

Market Demand Functions: The market demand function specifies total demand in the market, as a function of the price. If the generalized demand function of consumer i is $x_i^*(p_x, p_y, M_i)$, then the generalized market demand function is

$$X^*(p_x, p_y, M_1, ..., M_i, ...M_n) = \sum_{i=1}^{n} x_i^*(p_x, p_y, M_i).$$

Things to note: (1) The "ordinary" market demand function, X^* as a function of p_x, holds constant p_y and the incomes of all consumers. When estimating demand functions, we do not want to include everyone's income in the regression, but we may want to include a measure of the distribution of income.

(2) We hold the price fixed and add the quantities: add the individual demand curves *horizontally*.

(3) Market demand is flatter than individual demands, because a given change in price results in a bigger change in quantity.

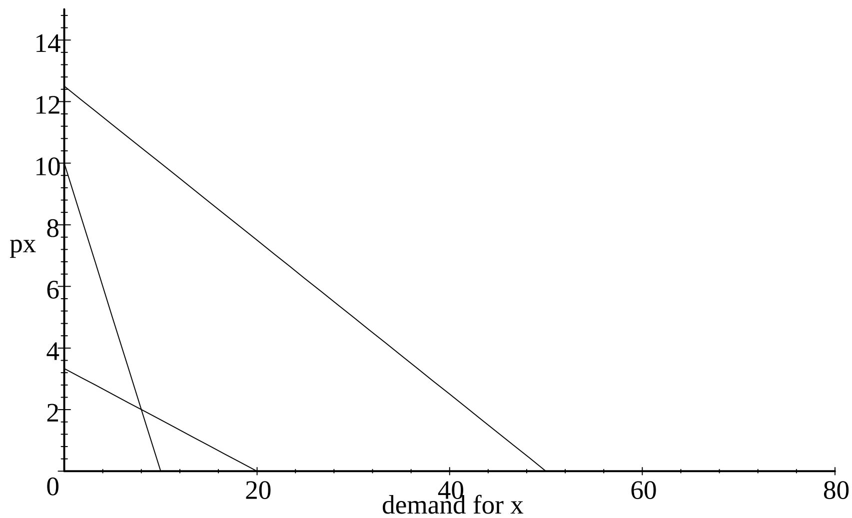
(4) When adding linear demand functions that intersect the price axis, remember that no individual demand can be negative.

Example (Binger and Hoffman, p. 154):

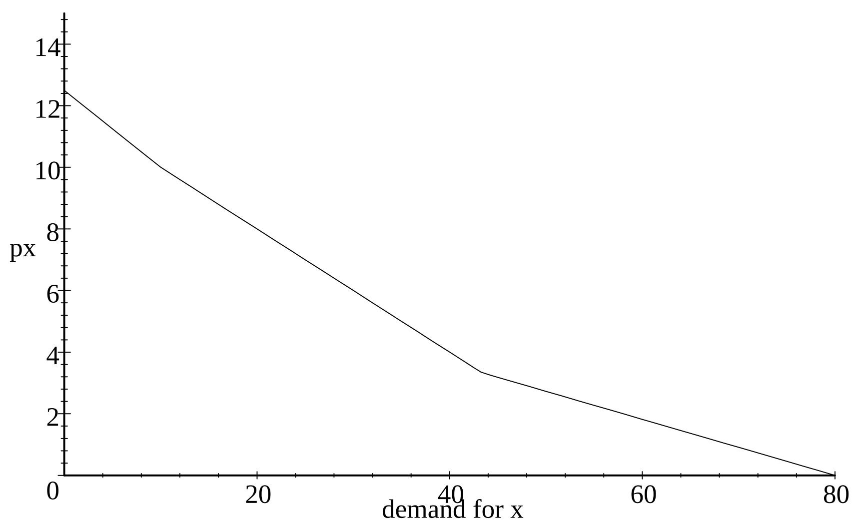
$$\begin{aligned} x_1 &= 10 - p_x \\ x_2 &= 20 - 6p_x \\ x_3 &= 50 - 4p_x \end{aligned}$$

Consumer 1 is "in the market" whenever $p_x \leq 10$, consumer 2 is "in the market" whenever $p_x \leq 3.33$, and consumer 3 is "in the market" whenever $p_x \leq 12.5$. Therefore, we have

$$\begin{aligned} X^* &= 80 - 11p_x \text{ if } 0 \leq p_x \leq 3.33 \\ X^* &= 60 - 5p_x \text{ if } 3.33 \leq p_x \leq 10 \\ X^* &= 50 - 4p_x \text{ if } 10 \leq p_x \leq 12.5 \\ X^* &= 0 \text{ if } 12.5 \leq p_x \end{aligned}$$



Individual Demands



Market Demand