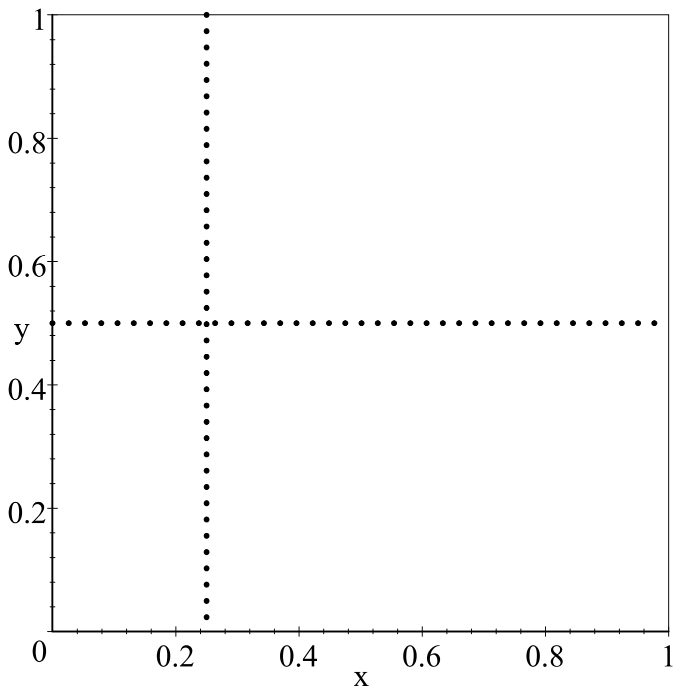
Efficiency and Trade

Previously, we treated prices and income as parameters, and saw how demand depended on these parameters. Now, we ask what prices might emerge as consumers make mutually beneficial trades.

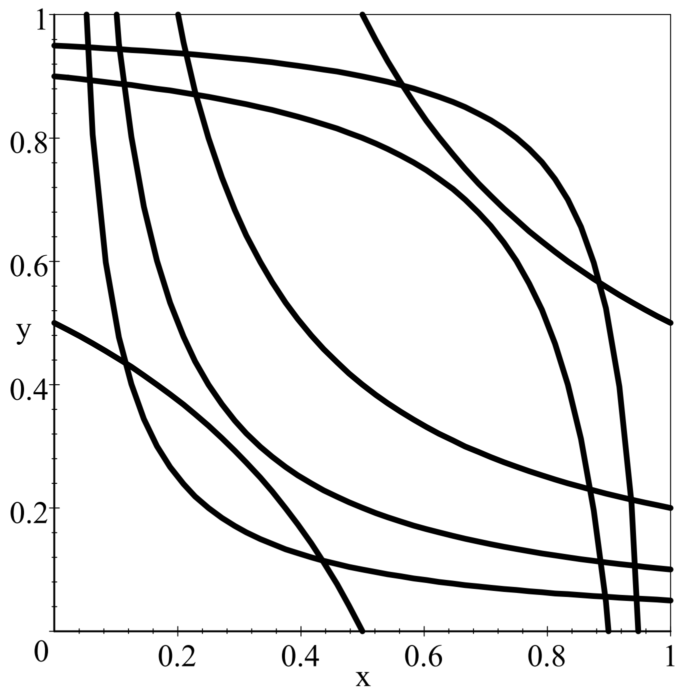
Suppose that there are two consumers, and fix the total amount of goods x and y to be allocated: $\overline{x}$ and $\overline{y}$.

The Edgeworth Box–Consider a rectangle of size $\overline{x} \times \overline{y}$. Think of consumer 1 as being located at the southwest corner and consumer 2 as being located at the northeast corner (upside down). Then any point in the box represents an allocation of the available goods across the two consumers.



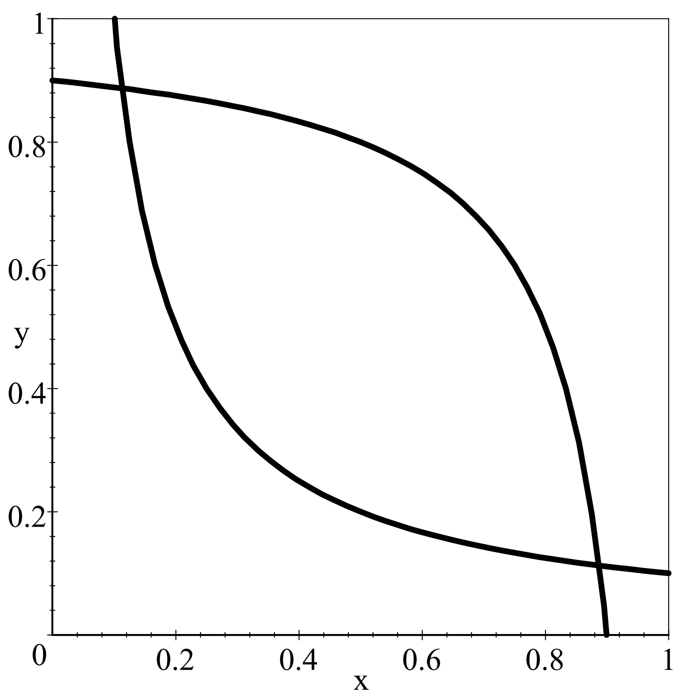
An Edgeworth Box

Preferred bundles for consumer 1 are to the northeast, and preferred bundles for consumer 2 are to the southwest.



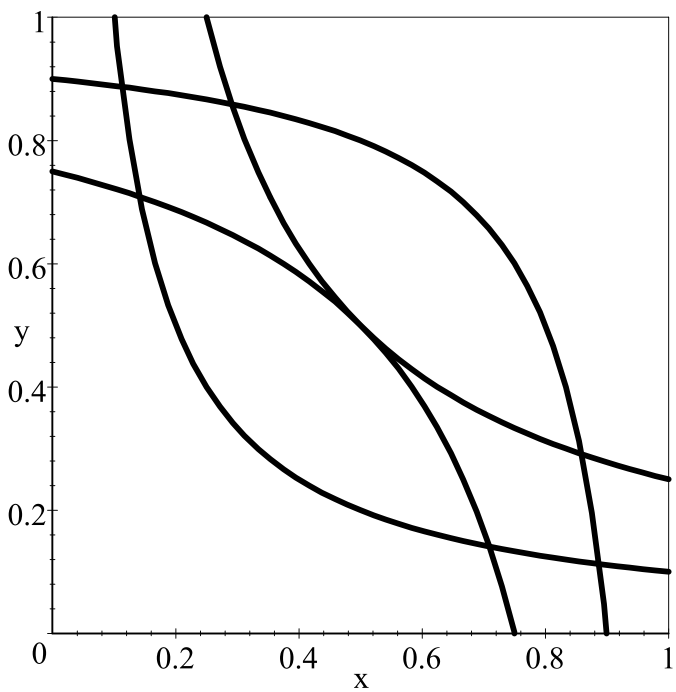
Indifference Curves in the Edgeworth Box

By finding the bundles that both consumers prefer to an “initial” bundle, we can determine the gains from trade.



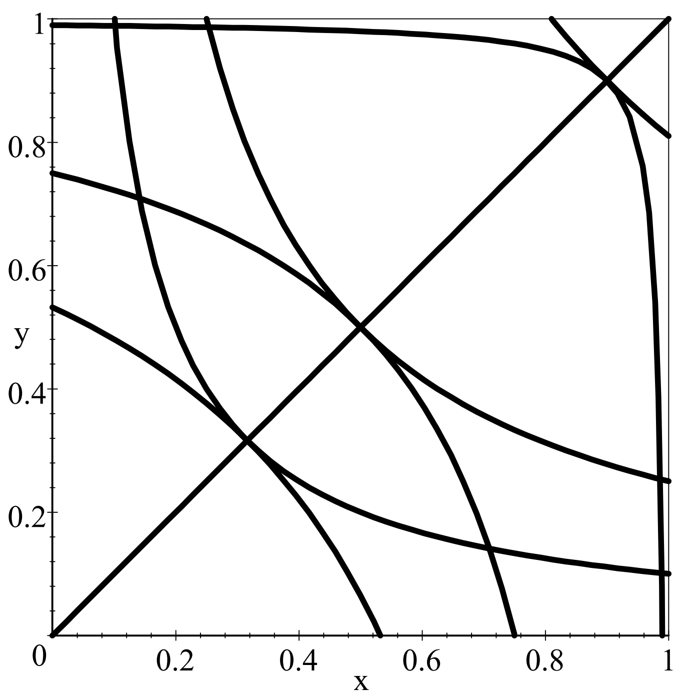
Gains From Trade

Pareto optimal allocations are ones for which there are no gains from trade. It is impossible to make one consumer better off without hurting the other consumer.



A Pareto Optimal Allocation

The set of Pareto Optimal Allocations is called the Contract Curve. Indifference curves are tangent, so marginal rates of substitution are equal.



The Contract Curve

Mathematical Derivation of the Contract Curve

Allocate resources to maximize consumer 1's utility, subject to the constraint that we are on a particular indifference curve of consumer 2.

$$
\begin{aligned}
& \max u_1(x_1, y_1) \\
\text{subject to } \overline{u}_2 &= u_2(x_2, y_2) \\
x_1 + x_2 &= \overline{x} \\
y_1 + y_2 &= \overline{y}.
\end{aligned}
$$

This can be solved with three multipliers, but it is simpler to use the resource constraints to eliminate x_2 and y_2.

$$
\begin{aligned}
& \max u_1(x_1, y_1) \qquad (1) \\
\text{subject to } \overline{u}_2 &= u_2(\overline{x} - x_1, \overline{y} - y_1)
\end{aligned}
$$

Set up the Lagrangean expression,

$$L = u_1(x_1, y_1) + \lambda[u_2(\overline{x} - x_1, \overline{y} - y_1) - \overline{u}_2].$$

The first order conditions are that the term in brackets is zero, and

$$\begin{aligned} \frac{\partial u_1}{\partial x_1} + \lambda \frac{\partial u_2}{\partial x_2}(-1) &= 0, \\ \frac{\partial u_1}{\partial y_1} + \lambda \frac{\partial u_2}{\partial y_2}(-1) &= 0. \end{aligned} \tag{2}$$

Solving by eliminating λ in (2), we have

$$\frac{\frac{\partial u_1}{\partial x_1}}{\frac{\partial u_1}{\partial y_1}} = \frac{\frac{\partial u_2}{\partial x_2}}{\frac{\partial u_2}{\partial y_2}},$$

which is the condition that marginal rates of substitution must be equal.

Trade on Markets

A market economy entails ownership of resources. The *initial endowment* of consumer 1 is denoted by $(\overline{x}_1, \overline{y}_1)$, and the initial endowment of consumer 2 is denoted by $(\overline{x}_2, \overline{y}_2)$.

Both consumers' initial endowments are represented by the same point in the Edgeworth Box, since

$$\overline{x}_1 + \overline{x}_2 = \overline{x} \quad \text{and} \quad \overline{y}_1 + \overline{y}_2 = \overline{y}.$$

Each consumer's income now depends on prices:

$$\begin{aligned} M_1 &= p_x\overline{x}_1 + p_y\overline{y}_1 \text{ and} \\ M_2 &= p_x\overline{x}_2 + p_y\overline{y}_2. \end{aligned}$$

What determines the prices?

The Auctioneer and Equilibrium

Market forces cause the price to fall when there is *excess supply*, and rise when there is *excess demand*. Eventually, things settle into *equilibrium*, where supply equals demand. We personify these forces into an imaginary "auctioneer."

1. Think of the auctioneer as calling out prices.

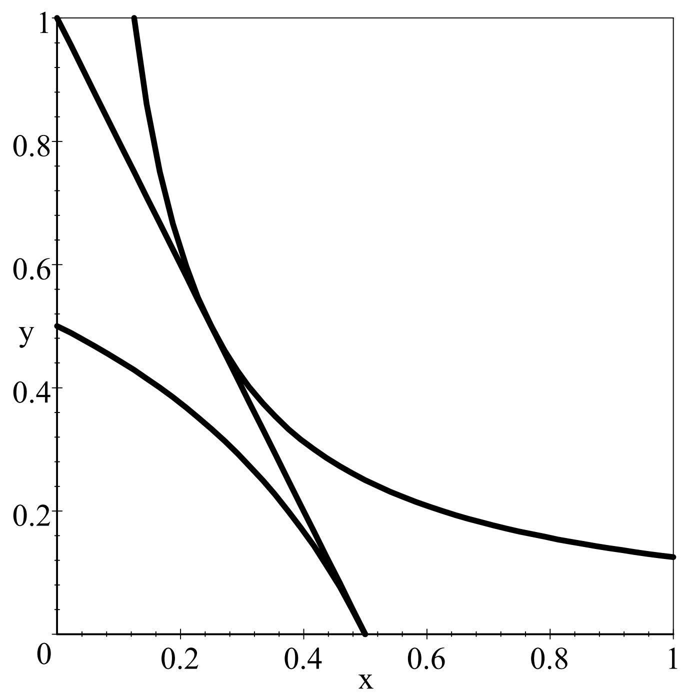
2. Consumers solve their utility maximization problems, and announce their demands.

3. The auctioneer then adjusts prices, raising them for commodities where demand exceeds supply, and lowering them for commodities where supply exceeds demand.

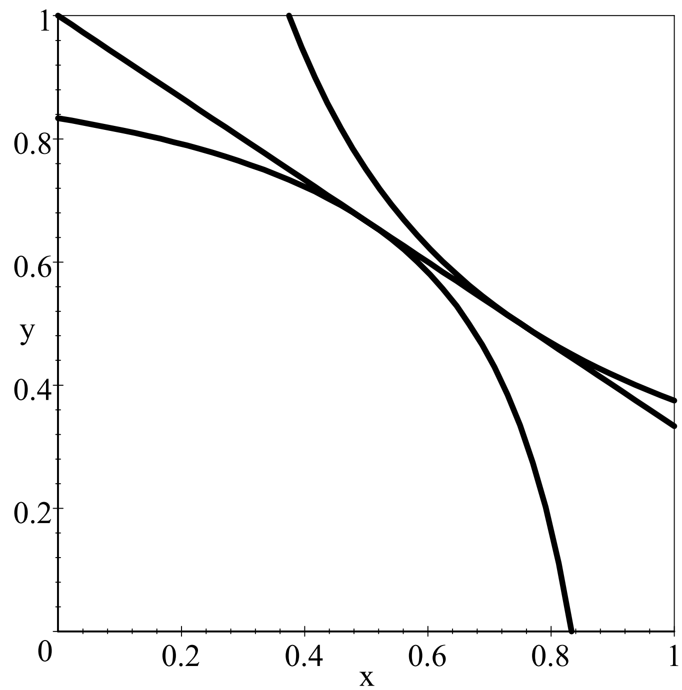
4. We then repeat steps 2 and 3 until we find prices where supply equals demand. Consumers receive their utility maximizing bundle, based on the *equilibrium* prices.

In the Edgeworth Box, the budget line for both consumers is the line with slope $-p_x/p_y$ and running through the initial endowment point.

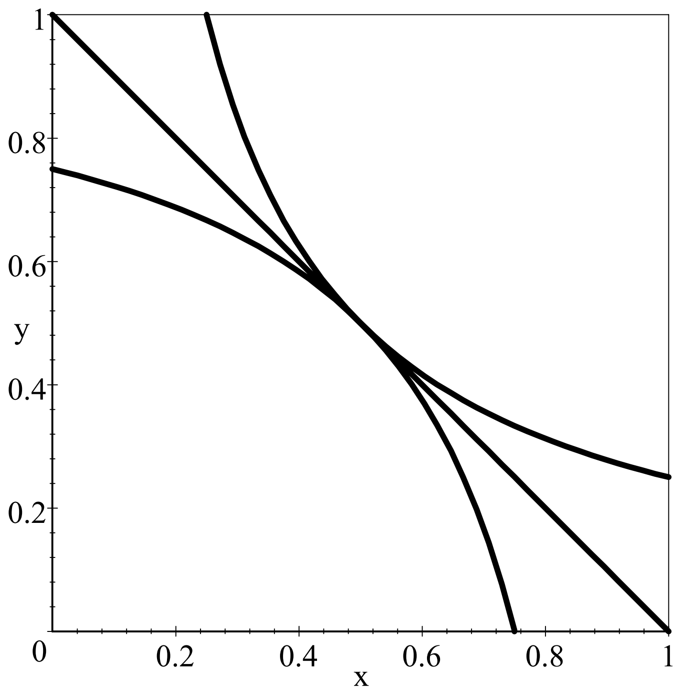
In the following diagrams, the initial endowment is in the northwest corner of the box, $(x, y) = (0, 1)$.



Excess demand for good y ($p_x/p_y = 2$)



Excess demand for good x ($p_x/p_y = \frac{2}{3}$)



Equilibrium ($p_x/p_y = 1$)

Consumer 1 is a net buyer of x and a net seller of y.
Consumer 2 is a net seller of x and a net buyer of y.

Definition: An *equilibrium* is a price, (p_x^*, p_y^*), and an allocation, $(x_1^*, y_1^*, x_2^*, y_2^*)$, such that

(i) For each consumer, i=1,2, (x_i^*, y_i^*) solves

$$\begin{aligned} & \max u_i(x_i, y_i) \\ \text{subject to} &: \\ p_x x_i + p_y y_i &= p_x \overline{x}_i + p_y \overline{y}_i \end{aligned}$$

(ii) $x_1^* + x_2^* = \overline{x}$ and $y_1^* + y_2^* = \overline{y}$.

Condition (i) reflects utility maximization, and condition (ii) reflects market clearing (demand = supply).

Equilibrium and Pareto Optimality

Utility maximization requires each consumer's indifference curve to be tangent to their budget line:

$$\frac{\frac{\partial u_1}{\partial x_1}}{\frac{\partial u_1}{\partial y_1}} = \frac{p_x}{p_y} = \frac{\frac{\partial u_2}{\partial x_2}}{\frac{\partial u_2}{\partial y_2}} \tag{3}$$

From (3), we see that the consumers' marginal rates of substitution are equal to each other.

First Fundamental Theorem of Welfare Economics: an equilibrium allocation is Pareto optimal.

This theorem extends to many consumers and many goods, and to the inclusion of production.

However, an efficient allocation is not necessarily fair. Is there a tradeoff between efficiency and fairness?

Suppose that we, as a society, could evaluate the "social welfare" of any allocation.

This social welfare function would prefer allocation A over allocation B if each individual preferred A to B. Thus, the socially optimal allocation will be Pareto optimal.

Also implicit in the social welfare function would be our notions of fairness (for instance, equal weights or importance for each consumer's utility).

In principle, we could calculate the socially optimal allocation, and assign each consumer his/her bundle. But do we trust Government to choose our consumption for us? How can the Government learn our utility functions and perform the calculation?

Second Fundamental Theorem of Welfare Economics: Any Pareto optimal allocation can be achieved as an equilibrium, by a suitable reassignment of the initial allocation.

We don't have to abandon markets to implement a fair and efficient allocation, just redistribute income and let markets work.

Privatization in Russia: give everyone 1 share of stock in every firm. Minimal informational requirements.

Don't take FFTWE and SFTWE too seriously:

1. We often make policy based on efficiency, and never do the redistributions (NAFTA).

2. Redistributions might not be feasible: tax someone $1,000,000 whether they become a doctor or an artist. An income tax is not a valid redistribution, because it is in response to choices on the labor market and capital market.

3. Imperfect competition or externalities make the equilibrium inefficient.

Solving for the Equilibrium: An Example

$u_1(x_1, y_1) = x_1 y_1$ and $(\overline{x}_1, \overline{y}_1) = (2, 1)$.

$u_2(x_2, y_2) = x_2 y_2$ and $(\overline{x}_2, \overline{y}_2) = (1, 2)$.

Step 1: normalize prices, so $p_y = 1$. We can do this because of homogeneity, only relative prices matter.

Step 2: solve for the demand functions.

For consumer 1, the Lagrangean expression is

$$L = x_1 y_1 + \lambda[2p_x + 1 - p_x x_1 - y_1]$$

The first order conditions are:

$$\frac{\partial L}{\partial x_1} = y_1 - \lambda p_x = 0 \tag{4}$$

$$\frac{\partial L}{\partial y_1} = x_1 - \lambda = 0 \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = 2p_x + 1 - p_x x_1 - y_1 = 0. \tag{6}$$

Solving (4)-(6), we first eliminate λ from (4) and (5), yielding the marginal rate of substitution equals the price ratio:

$$\frac{y_1}{x_1} = p_x. \tag{7}$$

Rearranging (7), we have

$$y_1 = p_x x_1 \tag{8}$$

Substituting (8) into (6), we have

$$2p_x + 1 - 2p_x x_1 = 0,$$

from which we solve for the demand for x:

$$x_1 = \frac{2p_x + 1}{2p_x}. \tag{9}$$

From (8) and (9), we have the demand for y:

$$y_1 = \frac{2p_x + 1}{2}. \tag{10}$$

For consumer 2, the Lagrangean expression is

$$L = x_2 y_2 + \lambda[p_x + 2 - p_x x_2 - y_2].$$

Solving the first order conditions, we have the marginal rate of substitution condition and the budget equation, from which we can solve for the demand functions:

$$x_2 = \frac{p_x + 2}{2p_x} \quad \text{and} \quad y_2 = \frac{p_x + 2}{2} \tag{11}$$

Step 3: Use market clearing to solve for p_x. Which market clearing condition do we use?

Market clearing for good x implies

$$x_1 + x_2 = \frac{2p_x + 1}{2p_x} + \frac{p_x + 2}{2p_x} = 3. \tag{12}$$

Solving (12) for p_x yields $p_x = 1$.

Market clearing for good y implies

$$y_1 + y_2 = \frac{2p_x + 1}{2} + \frac{p_x + 2}{2} = 3. \qquad (13)$$

Solving (13) for p_x yields $p_x = 1$.

You get the same answer both ways. Whenever supply equals demand for all markets except one, supply equals demand for the last market as well.

To get the equilibrium allocation, plug $p_x = 1$ into the demand functions:

$$\begin{aligned} x_1 &= \frac{2p_x + 1}{2p_x} = \frac{3}{2}, \quad x_2 = \frac{p_x + 2}{2p_x} = \frac{3}{2}, \\ y_1 &= \frac{2p_x + 1}{2} = \frac{3}{2}, \quad y_2 = \frac{p_x + 2}{2} = \frac{3}{2}. \end{aligned}$$

As a check, total consumption of good x is 3, and total consumption of good y is 3.