The Ohio State University
Department of Economics

Econ 501a
Spring 2004
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Midterm Answers

Part I: Short Answer.

1. **(10 points)** Suppose that a consumer's preferences over bundles of goods x and y are represented by the utility function, $u(x, y) = xy$. That is, one bundle is preferred over a second bundle if and only if the amount of good x multiplied by the amount of good y is greater in the first bundle than in the second bundle. *Briefly* show that these preferences satisfy the first three axioms of consumer rationality. That is, show that these preferences are complete, reflexive, and transitive (see the equation sheet).

Answer:
For any two bundles, (x_1, y_1) and (x_2, y_2), exactly one of the following conditions will be true:

$$\begin{aligned} x_1y_1 &> x_2y_2, \\ x_1y_1 &= x_2y_2, \quad \text{or} \\ x_1y_1 &< x_2y_2. \end{aligned}$$

Therefore, preferences are complete. Preferences are reflexive, because $x_1y_1 = x_1y_1$. Finally, if we have $x_1y_1 > x_2y_2$ and $x_2y_2 > x_3y_3$, then it follows that we have $x_1y_1 > x_3y_3$. Therefore, preferences are transitive.

2. **(10 points)** From January 1979 to January 1980, the price of gold more than doubled. As the price of gold rose, the quantity of gold demanded increased as well. Briefly explain why it is **not** reasonable to say that this is an example of the income effect swamping the substitution effect.

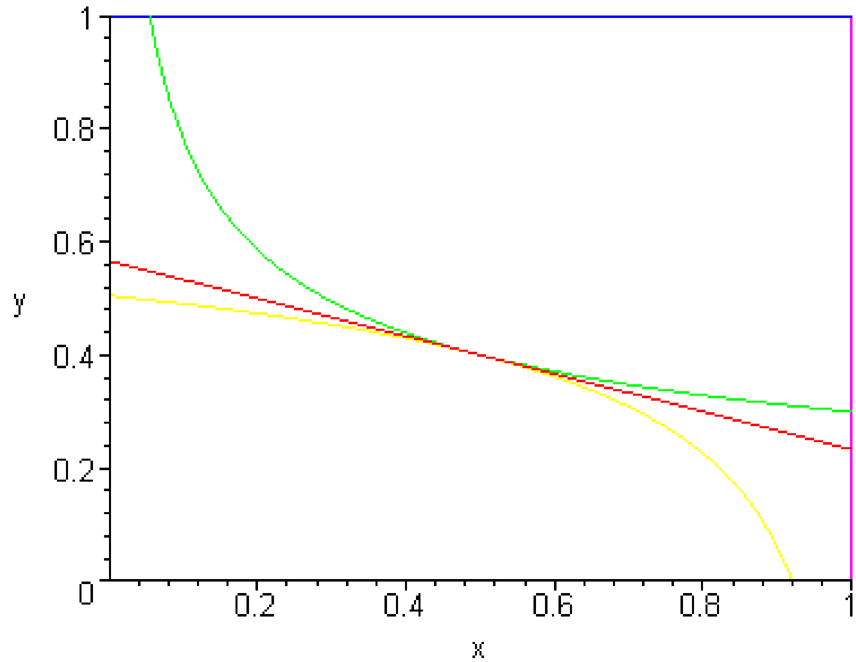
Answer:
The substitution effect of the price increase would be for the demand for gold to decrease. The income effect is the change in the demand for gold, due to the resulting reduction in purchasing power, holding relative prices constant. For normal goods, the income effect will reinforce the substitution effect, so that demand for gold should fall further. Unless you believe that gold is an inferior good, where poor people have more gold than rich people, the explanation cannot be due to the income effect swamping the substitution effect.

In case you are curious, what could be the explanation? The event causing the price change was probably not a shift in the supply curve, leading to a movement along the demand curve and a higher price. Rather, the demand curve was shifting out, moving along the supply curve to a higher price and a higher quantity. This was a time of increased inflationary expectations, and people thought that the price of gold was going to increase in the future. The increased attractiveness of gold as a hedge against inflation caused the demand curve to shift.

3. **(10 points)** Consider an Edgeworth Box economy with an initial allocation for the two consumers given by $(\overline{x}_1, \overline{y}_1, \overline{x}_2, \overline{y}_2)$. The initial allocation is represented by the point plotted in the Edgeworth Box, below. By drawing the relevant budget line and indifference curves, provide an example of an equilibrium with $p_x^* = 1$ and $p_y^* = 3$. [Try to be neat and as accurate as possible.]

Answer:

The budget line must run through the endowment point in the Edgeworth Box, $(.8, .3)$, and the slope of the budget line must be the negative of the price ratio, $-\frac{1}{3}$. This determines the budget line. To be an equilibrium, the allocation must be on the budget line, and each consumer's indifference curve through his/her bundle must be tanget to the budget line. The picture is drawn below, with the equilibrium allocation, $(x_1, y_1) = (.5, .4)$ and $(x_2, y_2) = (.5, .6)$. Other examples are possible, as long as the equilibrium allocation is on the budget line and the indifference curves are tangent.



4. (**10 points**) Show whether the following production function exhibits decreasing returns to scale, constant returns to scale, or increasing returns to scale:

$$f(K, L) = \frac{KL}{3(K + L)}$$

Answer:
From the formula, we have

$$f(\theta K, \theta L) = \frac{(\theta K)(\theta L)}{3(\theta K + \theta L)} = \frac{\theta^2 KL}{3\theta(K + L)} = \frac{\theta KL}{3(K + L)} = \theta f(K, L).$$

Therefore, we have constant returns to scale.

Part II: Longer Answer.

5. **(30 points)** A consumer has the utility function over goods x and y,

$$u(x, y) = x^3 y^2.$$

Let the price of good x be given by p_x, let the price of good y be given by p_y, and let income be given by M.

(a) Derive the consumer's generalized demand function for good x.
(b) Is good y normal or inferior? Explain precisely.

Answer:
(a) Set up the Lagrangean expression, $L = x^3y^2 + \lambda[M - p_x x - p_y y]$.
By differentiating L with respect to x, y, and λ, and setting the derivatives equal to zero, the resulting first order conditions are:

$$3x^2y^2 - \lambda p_x = 0, 2x^3y - \lambda p_y = 0, \text{ and } M - p_x x - p_y y = 0.$$

Solving the first two equations for λ and equating the expressions, we have

$$\lambda = \frac{3x^2y^2}{p_x} = \frac{2x^3y}{p_y}.$$

We can simplify and solve for y, yielding

$$y = \frac{2xp_x}{3p_y}. \tag{1}$$

Substituting this expression into the budget equation, we have

$$0 = M - p_x x - \frac{2xp_x}{3} = M - \frac{5p_x x}{3}.$$

Solving for x, we have the generalized demand function for x,

$$x = \frac{3M}{5p_x}. \tag{2}$$

(b) Substituting (2) into (1), and simplifying, we have the generalized demand function for y,

$$y = \frac{2M}{5p_y}.$$

From this expression, it is obvious that demand for y increases as M increases, so y is a normal good. To prove this, you can show that the derivative is positive:

$$\frac{\partial y}{\partial M} = \frac{2}{5p_y} > 0.$$

6. **(30 points)** The market for round trip airline travel from Columbus to Phoenix over the New Year's holiday consists of two types of customers. Let x_1 denote the demand for airline tickets by OSU students, and let x_2 denote the demand for airline tickets by OSU alumni. Let the price of a round trip ticket be denoted by p_x, and suppose that everyone who purchases a ticket pays

the same price. The demand functions of the two customer groups has been determined as follows

$$\begin{aligned} x_1 &= 12000 - 40p_x \\ x_2 &= 50000 - 100p_x \end{aligned}$$

(a) What is the total market demand for airline tickets, as a function of p_x?

(b) Suppose that the price-elasticity of demand by OSU students, at the price p_x, is -2. What is the price-elasticity of demand by OSU alumni?

(c) At what price is the total revenue (for the entire market) maximized?

Answer:

(a) The price at which OSU students demand zero solves $12000 - 40p_x = 0$, which occurs at $p_x = 300$. The price at which OSU alumni demand zero solves $50000 - 100p_x = 0$, which occurs at $p_x = 500$. Therefore, the market demand function is given by

$$\begin{aligned} X &= 62000 - 140p_x \qquad \text{if } p_x < 300, \\ X &= 50000 - 100p_x \qquad \text{if } 300 \leq p_x < 500, \\ X &= 0 \qquad \text{if } p_x \geq 500. \end{aligned}$$

(b) From the formula for the price-elasticity of demand, we have

$$\varepsilon^d = \frac{dx_1}{dp_x}\left(\frac{p_x}{x_1}\right) = -40\left(\frac{p_x}{x_1}\right) = -40\left(\frac{p_x}{12000 - 40p_x}\right).$$

Setting the last expression above equal to the elasticity of -2, we can solve for the price, $p_x = 200$. Now that we know the price, we can evaluate the price-elasticity of demand for the OSU alumni at that price.

$$\varepsilon^d = \frac{dx_2}{dp_x}\left(\frac{p_x}{x_2}\right) = -100\left(\frac{p_x}{50000 - 100p_x}\right) = -100\left(\frac{200}{50000 - 100(200)}\right) = -\frac{2}{3}.$$

(c) Total revenue is maximized at the point on the demand curve where the price-elasticity of demand is -1. Using the market demand function, we have

$$\varepsilon^d = \frac{dX}{dp_x}\left(\frac{p_x}{X}\right) = -140\left(\frac{p_x}{62000 - 140p_x}\right).$$

Setting the last expression above equal to -1, we can solve for the price,

$$p_x = \frac{62000}{280} \simeq 221.$$

Since this price is below 300, we do not have to worry about the possibility that the revenue maximizing price drives the OSU students out of the market.

Another way to solve this problem is to derive an expression for total revenue. Solving $X = 62000 - 140p_x$ for p_x, we have the inverse demand function, $p_x = (62000 - X)/140$. Therefore, total revenue is

$$TR = \frac{(62000 - X)X}{140}.$$

Setting the derivative (which is marginal revenue) equal to zero, and solving for X, we have $X = 31000$. Plugging this quantity into the inverse demand function, we have the price, $p_x \simeq 221$.