

The Ohio State University
Department of Economics

Econ 501.02
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Midterm Questions and Answers

Part I: Short Answer.

1. **(10 points)** Suppose good x is high quality navel oranges and good y is medium quality navel oranges. Call the prices in California (where the oranges are grown) p_x and p_y , and assume that the high quality oranges are more expensive than the medium quality oranges, $p_x > p_y$. Also assume that the prices in New York reflect the shipping cost, s , so the price of high quality oranges is $p_x + s$ and the price of medium quality oranges is $p_y + s$.

Based on the substitution effect, explain why the typical New York consumer chooses a higher ratio of high quality oranges to medium quality oranges than the typical California consumer.

Answer: The substitution effect is the change in demand due to a price change, holding utility constant. For this problem, we can see how a consumer's choices change when we move from the California prices to the New York prices. The shipping cost serves to bring the price ratio closer to one, so that the relative price of higher quality oranges to medium quality oranges has gone down. Simple algebra shows that $p_x > p_y$ implies

$$\frac{p_x}{p_y} > \frac{p_x + s}{p_y + s}$$

(try plugging in numbers). According to the substitution effect, the consumer should substitute towards the good whose price has gone down, so the New York consumer would increase the ratio of high quality oranges to medium quality oranges.

Note: The problem is complicated by the fact that there are other goods besides oranges. Thus, it is not surprising that the New York consumer would buy fewer of both types of oranges. However, the ratio of higher quality oranges to medium quality oranges should still be higher in New York, at least due to the substitution effect, because we move to a flatter portion of the indifference curve.

2. **(10 points)** True or False, and explain. In an Edgeworth Box economy, suppose the two consumers have the same utility function, and they are each allocated half of the economy's endowment of good x and half of the economy's endowment of good y . Then the allocation is Pareto optimal.

Answer: True. Since the two consumers have the same utility function and the same consumption bundle (equal to half of the economy's endowment of each good), their marginal rates of substitution must be equal. Therefore, the allocation is Pareto optimal. More formally, each consumer's MRS is the ratio of partial derivatives of the utility function evaluated at the same point, given by

$$\frac{\frac{\partial u(\bar{x}/2, \bar{y}/2)}{\partial x}}{\frac{\partial u(\bar{x}/2, \bar{y}/2)}{\partial y}}.$$

3. **(10 points)** True or False, and explain. Suppose a technology is described by the production function $x = f(K, L)$. If the two inputs are perfect substitutes, then the technology must exhibit constant returns to scale.

Answer: False. If the two inputs are perfect substitutes, then the isoquants must be straight lines. The returns to scale depend on the output associated with each of the isoquants, which is an entirely different matter. Here is a counterexample:

$$f(K, L) = (K + L)^2.$$

The marginal rate of technical substitution is equal to 1 everywhere, so we have perfect substitutes. However, if we double both inputs, output is four times higher, so we have increasing returns to scale.

4. **(10 points)** For the production function

$$f(K, L) = KL^{1/2},$$

what is the marginal rate of technical substitution at the input combination, $(L, K) = (4, 5)$?

Answer: The marginal rate of technical substitution is

$$MRTS = \frac{MP_L}{MP_K} = \frac{\frac{1}{2}KL^{-1/2}}{L^{1/2}} = \frac{K}{2L}.$$

Evaluated at $(L, K) = (4, 5)$, we have $MRTS = \frac{5}{8}$.

Part II: Longer Answer.

5. **(30 points)** A consumer has the utility function over goods x and y ,

$$u(x, y) = xy + x + y.$$

Let the price of good x be given by p_x , let the price of good y be given by p_y , and let income be given by M .

- (a) Derive the consumer's generalized demand function for good x .
 (b) Is good x a gross substitute or a gross complement for good y ? Explain precisely.
 (c) If we have $p_x = 10$, $p_y = 2$, and $M = 16$, compute the income elasticity of demand for good x .

Answer:

- (a) Set up the Lagrangean expression,

$$L = xy + x + y + \lambda[M - p_x x - p_y y].$$

By differentiating L with respect to x , y , and λ , and setting the derivatives equal to zero, the resulting first order conditions are:

$$\begin{aligned} y + 1 - \lambda p_x &= 0, \\ x + 1 - \lambda p_y &= 0, \text{ and} \\ M - p_x x - p_y y &= 0. \end{aligned}$$

Solving the first two equations for λ , we have

$$\lambda = \frac{y + 1}{p_x} = \frac{x + 1}{p_y}. \quad (1)$$

Cross multiplying (1), we have

$$\begin{aligned} p_y y + p_y &= p_x x + p_x \text{ or} \\ p_y y &= p_x x + p_x - p_y. \end{aligned} \quad (2)$$

Now substitute the right side of (2) into the budget equation:

$$p_x x + (p_x x + p_x - p_y) = M$$

which yields

$$\begin{aligned} 2p_x x &= M - p_x + p_y \text{ or} \\ x^* &= \frac{M - p_x + p_y}{2p_x} \end{aligned} \quad (3)$$

- (b) Good x is a gross substitute for good y if the demand for good x increases when the price of good y increases. Evaluating the derivative of (3), we have

$$\frac{\partial x^*}{\partial p_y} = \frac{1}{2p_x},$$

which is positive. Therefore, good x is a gross substitute for good y .

(c) The formula for the income elasticity of demand is given by

$$\varepsilon_I^d = \frac{dx}{dM} \left(\frac{M}{x} \right).$$

From (3), we compute

$$\frac{\partial x^*}{\partial M} = \frac{1}{2p_x},$$

so the elasticity is given by the formula

$$\varepsilon_I^d = \frac{1}{2p_x} \left(\frac{M}{\frac{M-p_x+p_y}{2p_x}} \right) = \frac{M}{M-p_x+p_y}.$$

Evaluated at $p_x = 10$, $p_y = 2$, and $M = 16$, we have an income elasticity of 2.

6. **(30 points)** Gateway Cinema has two types of customers, students and non-students. Let $x_{student}$ denote the demand for a particular movie screening by students, and let $x_{non-student}$ denote the demand for the same movie screening by non-students. Let the price of the movie screening be denoted by p_x , and suppose that everyone who purchases a ticket pays the same price. The demand functions of the two customer groups has been determined as follows

$$\begin{aligned} x_{student} &= 240 - 20p_x \\ x_{non-student} &= 400 - 20p_x. \end{aligned}$$

(a) What price maximizes the total revenue received from ticket sales to students?

(b) At what price is the total revenue for the entire market (students and non-students) maximized?

Answer:

(a) Student demand as a function of the price is $x_{student} = 240 - 20p_x$, from which we can solve for the inverse demand (price as a function of quantity demanded by students),

$$p_x = \frac{240 - x_{student}}{20}. \quad (4)$$

Therefore, total revenue from students, as a function of the quantity sold to students, is

$$\left(\frac{240 - x_{student}}{20} \right) x_{student}. \quad (5)$$

The quantity at which revenue from students is maximized is found by setting the derivative of (5) equal to zero and solving for $x_{student}$. Marginal revenue is

$$\frac{240 - 2x_{student}}{20},$$

which is zero at $x_{student} = 120$. To find the price at which revenue from students is maximized, substitute this quantity into (4), yielding $p_x = 6$.

(b) To find the price that maximizes revenue from the entire market, we need to find the equation for the market demand curve. As long as the price is not so high that the expression for $x_{student}$ or $x_{non-student}$ is negative, market demand is given by

$$x = 640 - 40p_x. \quad (6)$$

Solving (6) for the price gives us the inverse market demand,

$$p_x = \frac{640 - x}{40}. \quad (7)$$

From (7), total revenue as a function of the total quantity sold in the entire market, is given by

$$TR = \left(\frac{640 - x}{40} \right) x$$

and marginal revenue is the derivative of total revenue,

$$MR = \frac{640 - 2x}{40}.$$

Thus, total market revenue is maximized when marginal revenue is zero, at $x = 320$. Substituting a quantity of 320 into (7) gives us the price at which market revenues are maximized, $p_x = 8$.