The Ohio State University
Department of Economics

Econ 501.02
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Midterm Questions and Answers

Part I: Short Answer.

1. **(10 points)** Suppose a focus group leader working for Coca Cola asks a consumer to rank her utility from consuming various "bundles" of coke and cheeseburgers, on a scale from 1 to 100. Which of the following statements will be more useful to the sellers of coke? Explain in terms of cardinal vs. ordinal utility.

(a) Starting at (12 ounces of coke, 8 ounce cheeseburger), an additional ounce of coke increases her ranking from 50 to 55.
(b) The ranking of (12 ounces of coke, 8 ounce cheeseburger) is the same as the ranking of (10 ounces of coke, 12 ounce cheeseburger).

Answer:
Statement (b) will be more useful. Statement (a) does not have any meaning because the cardinal properties of the utility function do not affect any economic choices. What matters is whether one bundle is preferred to another, not by how much, and we already know that more is preferred to less. Statement (b) tells us something about how this consumer values coke relative to a cheeseburger, by giving us two points on the same indifference curve. This consumer is willing to trade 2 ounces of coke for 4 ounces of cheeseburger, which might give coke a little bit of information about how much it can charge for its product.

2. **(10 points)** True or False, and explain. The income effect always operates in the opposite direction as the substitution effect.

Answer:
The statement is false. The substitution effect is always negative, so an increase in p_x leads to a reduction in demand for x, holding utility constant. For the "textbook" case in which income is fixed at M, the income effect goes in the same direction as the substitution effect for normal goods. Because purchasing power has fallen, the effect of moving to a lower indifference curve results in a reduction in the demand for x, holding the price ratio constant.

If income is generated by endowments of goods as in the Reagan tax cut case, and if you are a net seller of good x, then the income effect goes in the same direction as the substitution effect for inferior goods. For example, if good x is leisure, then an increase in p_x (your wage) results in an increase in purchasing power and a move to a higher indifference curve. The income effect leads to a reduction in leisure consumption if leisure is inferior.

3. **(10 points)** In an Edgeworth Box economy with two consumers (1 and 2) and two goods (x and y), suppose that allocation A, $(x_1^A, y_1^A, x_2^A, y_2^A)$, and allocation B, $(x_1^B, y_1^B, x_2^B, y_2^B)$, are both Pareto optimal allocations. Explain why whenever consumer 1 prefers allocation A, then consumer 2 must prefer allocation B.

Answer:

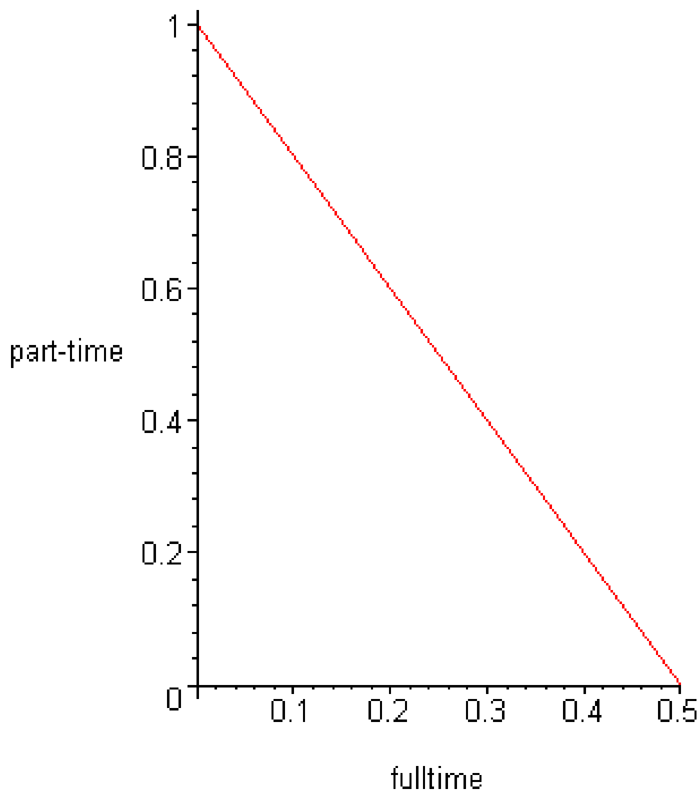
The easiest way to see this is the following. If hypothetically consumer 2 did not prefer allocation B, then by moving from B to A, we would make consumer 1 strictly better off without hurting consumer 2. That would mean that allocation B is not Pareto optimal, but we are told that B is Pareto optimal. Thus, our hypothetical case is impossible, and consumer 2 must prefer allocation B.

Another way to see this is by drawing an Edgeworth Box diagram depicting allocation B and the tangent indifference curves of consumer 1 and consumer 2. Because allocation A is preferred by consumer 1, that allocation must lie above consumer 1's indifference curve. However, all of those points lie on a lower indifference curve for consumer 2.

4. (**10 points**) A firm has a technology that uses two inputs, part-time workers and full-time workers. Suppose that the firm can always trade 2 part-time workers for 1 full-time worker, and still keep output constant. Below, carefully draw a graph of a typical isoquant for this firm, and make sure to label the axes.

Answer:

If the full-time worker input is on the x-axis and the part-time worker input is on the y-axis, the isoquant is a straight line with slope -2, reflecting the fact that the two inputs are perfect substitutes and one full-time worker is equivalent to 2 part-time workers.



Part II: Longer Answer.

5. **(30 points)** A consumer has the utility function over goods x and y,

$$u(x, y) = \sqrt{x} + y.$$

Let the price of good x be given by p_x, let the price of good y be given by p_y, and let income be given by M.

(a) Derive the consumer's generalized demand function for good x.
(b) Is good y a normal good or an inferior good? Explain precisely.

(c) If we have $p_x = 1$, $p_y = 4$, and $M = 1000$, compute the utility maximizing bundle of goods x and y.

Answer:
The consumer's utility maximization problem is

$$\begin{aligned} & \max \sqrt{x} + y \\ \text{subject to} \quad : \quad & p_x x + p_y y = M \end{aligned}$$

To solve, we set up the Lagrangean expression, $L = \sqrt{x} + y + \lambda[M - p_x x - p_y y]$. The first-order conditions are given by

$$x \quad : \quad \frac{1}{2}x^{-1/2} - \lambda p_x = 0 \tag{1}$$

$$y \quad : \quad 1 - \lambda p_y = 0 \tag{2}$$

$$\lambda \quad : \quad M - p_x x - p_y y = 0. \tag{3}$$

We can solve (1) and (2) to eliminate λ, yielding

$$\lambda = \frac{x^{-1/2}}{2p_x} = \frac{1}{p_y} \quad \text{or}$$

$$x^{-1/2} = \frac{2p_x}{p_y} \tag{4}$$

Taking both sides of (4) to the negative 2 power, we have the generalized demand function for good x (answer to part (a)):

$$x^* = \frac{(p_y)^2}{4(p_x)^2}. \tag{5}$$

By plugging (5) into the budget equation (3), we can solve for the generalized demand function for y. We have

$$M - p_x \frac{(p_y)^2}{4(p_x)^2} - p_y y = 0,$$

which can be simplified and solved for the generalized demand function,

$$y^* = \frac{M}{p_y} - \frac{p_y}{4p_x}. \tag{6}$$

From (6), we see that demand for good y increases when M increases, so y is a normal good (answer to part (b)).

To answer part (c), plug the values, $p_x = 1$, $p_y = 4$, and $M = 1000$, into (5) and (6), yielding $x = 4$ and $y = 249$.

6. **(30 points)** The market demand for good x is given by the equation,

$$X^d = 100,000 - 50,000p_x + 4M,$$

where X^d is the market demand, p_x is the price of good x, and M is per capita income. If the (own-price) elasticity of demand is -1 and the income elasticity of demand is 1, solve for the values of X^d, p_x, and M.

Answer:
The own-price elasticity of demand for this problem is

$$\varepsilon^d = \frac{dX^d}{dp_x}\left(\frac{p_x}{X^d}\right) = -50,000\left(\frac{p_x}{X^d}\right)$$

Since the elasticity is -1, we have

$$\begin{aligned} -50,000\left(\frac{p_x}{X^d}\right) &= -1 \text{ or} \\ X^d &= 50,000p_x \end{aligned} \tag{7}$$

Since the income elasticity of demand is 1, we have

$$\begin{aligned} \varepsilon_I^d &= \frac{dX^d}{dM}\left(\frac{M}{X^d}\right) = 4\left(\frac{M}{X^d}\right) = 1 \quad \text{or} \\ X^d &= 4M. \end{aligned} \tag{8}$$

We have 3 equations, (7), (8), and the demand equation, and we have three unknowns, X^d, p_x, and M. To solve, first plug (8) into the demand equation,

$$4M = 100,000 - 50,000p_x + 4M,$$

so M cancels and we can solve for $p_x = 2$. Now plug the price into (7), yielding $X^d = 100,000$. Finally, substitute $X^d = 100,000$ into (8), and we can solve for $M = 25,000$.