

The Ohio State University
Department of Economics

Econ 501.02
Spring 2007
Prof. James Peck

Midterm Questions and Answers

Part I: Short Answer.

1. **(10 points)** True or False, and explain. Every utility function uniquely determines the ordinal preferences, but for any ordinal preferences, there are many utility functions that represent those preferences.

Answer: This statement is true. Every utility function uniquely determines the ordinal preferences. The utility function always tells us which of two bundles is preferred, or whether they are indifferent, by comparing the utilities of the two bundles. On the other hand, there are many utility functions that represent the same preferences. For example, if the function $u(x, y)$ represents a consumer's preferences, then the function $Au(x, y) + B$ represents the same preferences, for any positive constants A and B .

2. **(10 points)** Suppose a consumer has income, $M = 100$, and faces prices, $p_x = 2$ and $p_y = 2$. Using a neat, carefully drawn and labeled diagram, show the substitution and income effects resulting from a **decrease** in p_x to 1, while M and p_y remain constant. Make sure that you indicate the magnitudes of the income and substitution effects.

Answer: This is like the textbook case gone over in class, except p_x decreases rather than increases. The new budget line is now flatter and outside the original budget line. Let point A be the tangency point of the original budget line and an indifference curve, let point B be the tangency point of the original indifference curve and the line with the slope of the new budget line, $-1/2$, and let point C be the tangency point of the new budget line and an indifference curve. Then the substitution effect is the increase in the x coordinate as we move from point A to point B, and the income effect is the change in the x coordinate as we move from point B to point C. The substitution effect is always positive for a price decrease, and the income effect will also be positive if x is a normal good. I will draw the graph in class.

3. **(10 points)** True or False, and explain. In an Edgeworth Box economy (with 2 goods and 2 consumers), the center of the Edgeworth Box must be a Pareto optimal allocation.

Answer: This statement is false. The easiest way to show this is to provide a counterexample, by drawing an Edgeworth Box diagram with an indifference curve for each consumer intersecting at the center of the Edgeworth Box. Since

the indifference curves intersect, there must be some allocations that make both consumers better off, so the center of the Edgeworth Box is not Pareto optimal.

4. **(10 points)** A firm's marginal rate of technical substitution captures the firm's willingness to trade capital for labor, but **not** the degree to which capital and labor are substitutes in the production process.

Answer: This statement is true. The MRTS is the absolute value of the slope of the isoquant, so it measures the amount of capital that the firm is willing to give up for one unit of labor, in order to maintain the same level of production. This is not the same thing as measuring the degree to which capital and labor are substitutes or complements. That has to do with how quickly the MRTS changes as we move along the isoquant, not the slope itself. To see this, think of two technologies. The first technology has straight line isoquants with slope -10 , and the second technology has straight line isoquants with slope $-1/10$. In both cases, capital and labor are perfect substitutes, even though the MRTS takes on very different values.

Part II: Longer Answer.

5. **(30 points)** A consumer has the utility function over goods x and y ,

$$u(x, y) = 12x^2y^4.$$

Let the price of good x be given by p_x , let the price of good y be given by p_y , and let income be given by M .

(a) What is the slope of the consumer's indifference curve at the consumption bundle, $(1,1)$?

(b) Derive the consumer's generalized demand function for goods x and y .

(c) If we have $p_x = 2$, $p_y = 2$, and $M = 24$, compute the utility maximizing consumption bundle.

Answer: (a) The marginal rate of substitution is the absolute value of the slope of the indifference curve. The MRS can be calculated as

$$MRS = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{24xy^4}{48x^2y^3} = \frac{y}{2x}.$$

Evaluated at the point $(1,1)$ by plugging $x = 1$ and $y = 1$ into the above formula, the MRS is $1/2$. Therefore, the slope of the indifference curve is $-1/2$.

(b) Set up the Lagrangean expression, $L = 12x^2y^4 + \lambda[M - p_x x - p_y y]$.

By differentiating L with respect to x , y , and λ , and setting the derivatives equal to zero, the resulting first order conditions are:

$$24xy^4 - \lambda p_x = 0, 48x^2y^3 - \lambda p_y = 0, \text{ and } M - p_x x - p_y y = 0.$$

Solving the first two equations for λ and equating the expressions, we have

$$\lambda = \frac{24xy^4}{p_x} = \frac{48x^2y^3}{p_y}.$$

We can simplify and solve for y , yielding

$$y = \frac{2xp_x}{p_y}. \quad (1)$$

Substituting this expression into the budget equation, we have

$$0 = M - p_x x - \frac{2xp_x}{p_y} p_y = M - 3p_x x.$$

Solving for x , we have the generalized demand function for x ,

$$x = \frac{M}{3p_x}. \quad (2)$$

Plugging (2) into (1), we get the generalized demand for y ,

$$y = \frac{2\left(\frac{M}{3p_x}\right)p_x}{p_y} = \frac{2M}{3p_y}.$$

(c) Simply substitute $p_x = 2$, $p_y = 2$, and $M = 24$ into the generalized demand functions from part (b), to get $x = 4$ and $y = 8$.

6. **(30 points)** The market for tickets to this year's superbowl consists of two groups of consumers. Let x_1 denote the demand for tickets by fans of one of the participating teams, and let x_2 denote the demand for tickets by rich people who enjoy the spectacle. Let the price of a superbowl ticket be denoted by p_x , and **suppose that everyone who purchases a ticket pays the same price**. The demand functions of the two consumer groups has been determined as follows

$$\begin{aligned} x_1 &= 120 - \frac{p_x}{2} \\ x_2 &= 60 - \frac{p_x}{6} \end{aligned}$$

(a) *What is the market demand function for superbowl tickets, as a function of p_x ? (Remember that demand cannot be negative for either consumer group.)*

(b) *Suppose that, at the ticket price chosen by the NFL, the price-elasticity of demand for consumers in group 1 is -2 . What is the price-elasticity of demand for consumers in group 2?*

Answer: (a) Consumers in group 1 demand a positive quantity ($x_1 > 0$) whenever $p_x < 240$, and consumers in group 2 demand a positive quantity

($x_2 > 0$) whenever $p_x < 360$. Therefore, the market demand function is given by

$$\begin{aligned} X &= 180 - \frac{p_x}{2} - \frac{p_x}{6} = 180 - \frac{2p_x}{3} && \text{for } p_x < 240 \\ X &= 60 - \frac{p_x}{6} && \text{for } 240 \leq p_x < 360 \\ X &= 0 && \text{for } 360 \leq p_x. \end{aligned}$$

(b) Since the price elasticity of demand for consumers in group 1 is -2 , we can figure out the price, as follows.

$$\varepsilon_1^d = \frac{dx_1}{dp_x} \left(\frac{p_x}{x_1} \right) = \left(-\frac{1}{2} \right) \left(\frac{p_x}{120 - \frac{p_x}{2}} \right).$$

Setting this expression equal to its value, -2 , cross-multiplying, and simplifying, we have

$$\begin{aligned} p_x &= 480 - 2p_x \\ p_x &= 160. \end{aligned}$$

Now that we know the price, we can compute the elasticity for those in group 2, as follows

$$\varepsilon_1^d = \frac{dx_2}{dp_x} \left(\frac{p_x}{x_2} \right) = \left(-\frac{1}{6} \right) \left(\frac{p_x}{60 - \frac{p_x}{6}} \right) = \left(-\frac{1}{6} \right) \left(\frac{160}{60 - \frac{160}{6}} \right) = -\frac{4}{5}.$$