The Ohio State University
Department of Economics

Econ 501.02
Winter 2007
Prof. James Peck

Midterm Questions and Answers

Part I: Short Answer.

1. **(10 points)** True or False, and explain. If a consumer's preferences for goods x and y can be represented by the utility function, $u(x,y) = xy$, then the preferences can also be represented by the utility function, $u(x,y) = 10xy$.

Answer: There are many ways to see that the answer is True. The coefficient of 10 does not affect the underlying preferences, because the indifference curves are the same, only the labelling is different. For example, the indifference curve corresponding to utility of 2 with the first utility function is the indifference curve corresponding to utility of 20 with the second utility function. Another way to see that the preferences are the same is to notice that the marginal rate of substitution at any point (x, y) is y/x with either utility function.

Finally, whenever (x^A, y^A) is preferred to (x^B, y^B) with the first utility function, so $x^A y^A > x^B y^B$ holds, then (x^A, y^A) is preferred to (x^B, y^B) with the second utility function, because $10x^A y^A > 10x^B y^B$ holds, and vice versa.

2. **(10 points)** True or False, and explain. If the price elasticity of demand for Honda Accords in the eastern half of the U.S. is -2, and if the price elasticity of demand for Honda Accords in the western half of the U.S. is also -2, then the price elasticity of demand for the entire U.S. market must be -2.

Answer: The answer I had in mind was True. Intuitively, if a 1% increase in price causes demand to fall by 2% in the East and 2% in the West, then overall demand must fall by exactly 2%. To show this formally, we know

$$\frac{dX^E}{dp_x}\frac{p_x}{X^E} = -2 = \frac{dX^W}{dp_x}\frac{p_x}{X^W},$$

so we can write

$$\begin{aligned} \frac{dX^E}{dp_x} &= \frac{-2X^E}{p_x} \\ \frac{dX^W}{dp_x} &= \frac{-2X^W}{p_x}. \end{aligned} \tag{1}$$

The elasiticity in the U.S. market is given by

$$\frac{dX^{US}}{dp_x}\frac{p_x}{X^{US}} = \left[\frac{dX^E}{dp_x} + \frac{dX^W}{dp_x}\right]\left[\frac{p_x}{X^E + X^W}\right].$$

From (1), the elasiticity in the U.S. market is

$$\left[\frac{-2X^E}{p_x} + \frac{-2X^W}{p_x}\right]\left[\frac{p_x}{X^E + X^W}\right] = -2.$$

The above argument implicitly assumes that the price in the U.S. market is the same in the eastern half as in the western half. Otherwise, the answer could be false.

3. **(10 points)** When the price of good x increases, explain how the magnitude of the income effect is related to the amount of good x that the consumer would have purchased without the price increase.

Answer: When the price of good x increases, the consumer's purchasing power is more severely reduced if the consumer was planning on purchasing a lot of good x, vs. planning on purchasing a little of good x. Thus, given the degree to which good x is a normal good, the income effect is stronger if the consumer would have purchased a lot of the good without the price increase. For example, the income effect of a gasoline price increase, say from \$2.00 per gallon to \$2.50 per gallon, would be greater for someone who drives 300 miles every week than for someone who does not own a car and only drives a rental car for 300 miles during one week per year.

4. (**10 points**) Consider an Edgeworth Box economy, with two consumers and a total economy-wide endowment of one unit of good x and one unit of good y. Let A be the allocation in which consumer 1's consumption is $(x_1, y_1) = (0.7, 0.3)$ and consumer 2's consumption is $(x_2, y_2) = (0.3, 0.7)$. Let B be the allocation in which each consumer consumes 0.5 units of each good.

In an Edgeworth Box diagram, draw an indifference curve for consumer 1 and for consumer 2 that safisfies the following properties:

(i) Allocation A is Pareto optimal.
(ii) Consumer 1 prefers (his bundle in) allocation A to allocation B.
(iii) Consumer 2 prefers (her bundle in) allocation B to allocation A.

Make sure you label the axes, the points corresponding to A and B, and which indifference curve is for which consumer.

Answer: In the following diagram, good x is on the x-axis and good y is on the y-axis, and consumer 1's origin is at the point (0,0), and consumer 2's origin is at the point (1,1). You should plot the point corresponding to allocation A, $(x, y) = (0.7, 0.3)$ as measured from consumer 1's origin, and the point corresponding to allocation B, $(x, y) = (0.5, 0.5)$ as measured from consumer 1's origin. Consumer 1's (red) indifference curve and consumer 2's (green) indifference curve are tangent at allocation A, so allocation A is Pareto optimal. Because $(0.5, 0.5)$ is below consumer 1's indifference curve, consumer 1 prefers allocation A to allocation B. Because $(0.5, 0.5)$ is on an indifference

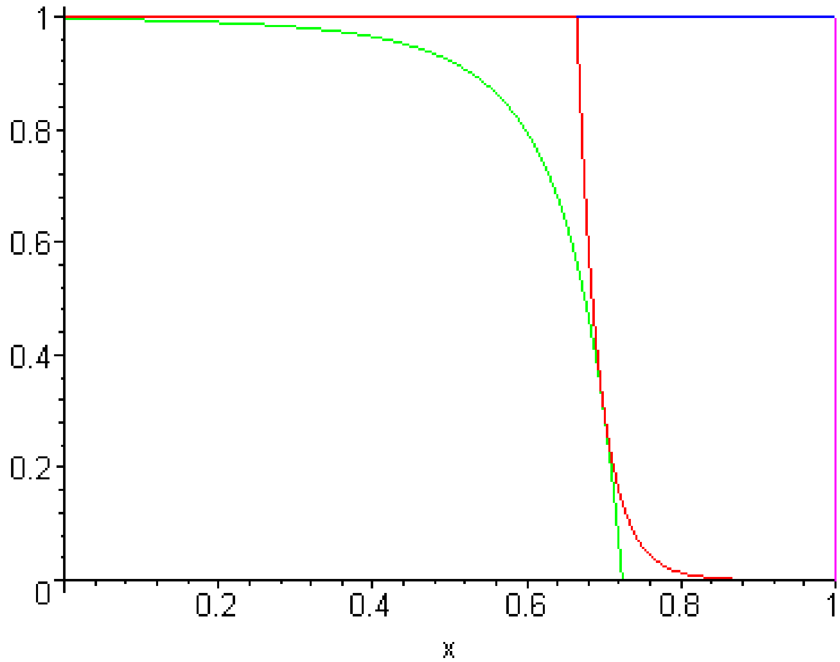

Figure 1:

curve for consumer 2 that is farther from consumer 2's origin than the green indifference curve, consumer 2 prefers allocation B to allocation A.

Part II: Longer Answer.

5. **(30 points)** A consumer has the utility function over goods x and y,

$$u(x, y) = y - \frac{1}{x}.$$

Let the price of good x be given by p_x, let the price of good y be given by p_y, and let income be given by M.

(a) Derive the consumer's generalized demand function for goods x and y.
(b) If we have $p_x = 1$, $p_y = 4$, and $M = 10$, compute the utility maximizing consumption bundle.
(c) Compute the price elasticity of demand for good x.

Answer: (a) Set up the Lagrangean expression,

$$L = y - \frac{1}{x} + \lambda[M - p_x x - p_y y].$$

By differentiating L with respect to x, y, and λ, and setting the derivatives equal to zero, the resulting first order conditions are:

$$\begin{aligned} \frac{1}{x^2} - \lambda p_x &= 0, \\ 1 - \lambda p_y &= 0, \text{ and} \\ M - p_x x - p_y y &= 0. \end{aligned}$$

Solving the first two equations for λ, we have

$$\lambda = \frac{1}{p_x x^2} = \frac{1}{p_y}. \tag{2}$$

Solving (2) for x, we have

$$\begin{aligned} x^2 &= \frac{p_y}{p_x} \quad \text{or} \\ x &= \sqrt{\frac{p_y}{p_x}} \end{aligned} \tag{3}$$

Equation (3) is the generalized demand function for good x. (Note: Usually when we eliminate the multiplier, we get an expression for x in terms of y, which has to be substituted into the budget equation to get the generalized demand. Here, since utility is linear in y, the marginal rate of substitution does not depend on consumption of good y, so the generalized demand for good x can be found without using the budget equation.)

To get the generalized demand for good y, substitute (3) into the budget equation:

$$M - p_x \sqrt{\frac{p_y}{p_x}} - p_y y = 0,$$

which can be solved for y to yield

$$y = \frac{M - p_x \sqrt{\frac{p_y}{p_x}}}{p_y} = \frac{M - \sqrt{p_x p_y}}{p_y}. \tag{4}$$

(b) Substitute $p_x = 1$, $p_y = 4$, and $M = 10$ into (3) and (4), yielding $x = 2$ and $y = 2$.

(c) The price elasticity of demand for good x is given by the formula

$$\varepsilon^d = \frac{dx}{dp_x}\left(\frac{p_x}{x}\right).$$

Differentiating (3) with respect to p_x gives

$$\frac{dx}{dp_x} = \sqrt{p_y}\left[-\frac{1}{2}p_x^{-3/2}\right]$$

so the elasticity for good x is given by

$$\varepsilon^d = \sqrt{p_y}\left[-\frac{1}{2}p_x^{-3/2}\right]\left[\frac{p_x}{\sqrt{\frac{p_y}{p_x}}}\right],$$

which simplifies to $\varepsilon^d = -\frac{1}{2}$. The elasticity is $-\frac{1}{2}$ for all values of prices and income.

Note: It is OK that the term involving good x in the utility function is being subtracted. Since x is in the denominator, the "more is better" axiom is satisfied.

6. **(30 points)** Oklahoma Optics produces high-tech optical instruments, which we will refer to as good x, using capital and labor. The production function is given by

$$f(K, L) = KL.$$

(a) Does this production function exhibit increasing returns to scale, constant returns to scale, or decreasing returns to scale? Explain your answer.
(b) Compute the generalized conditional demands for L and K.
(c) If Oklahoma Optics is obligated to supply 1000 units of good x, and input prices for capital and labor are $r = 1$ and $w = 10$, how much capital and how much labor should the firm hire in order to minimize costs?

Answer: (a) The production function exhibits increasing returns to scale. To see this,

$$f(\theta K, \theta L) = (\theta K)(\theta L) = \theta^2 KL = \theta^2 F(K, L) > \theta F(K, L).$$

(b) We need to solve the following Lagrangean problem

$$Lagr. = wL + rK + \lambda[x - KL].$$

The first order conditions are

$$\begin{aligned}
\frac{\partial Lagr.}{\partial L} &= 0 = w - \lambda K \\
\frac{\partial Lagr.}{\partial K} &= 0 = r - \lambda L \\
\frac{\partial Lagr.}{\partial \lambda} &= 0 = x - KL.
\end{aligned}$$

Solving the first two equations for λ and setting them equal to each other, we have the condition that MRTS = w/r,

$$\lambda = \frac{w}{K} = \frac{r}{L}$$

which can be written as

$$L = \frac{rK}{w}. \tag{5}$$

Substituting (5) into the output constraint, we have

$$x = \left[\frac{rK}{w}\right] K,$$

which can be solved for the generalized conditional demand for capital,

$$K = \sqrt{\frac{wx}{r}}. \tag{6}$$

Substituting (6) into (5), we have the generalized conditional demand for labor,

$$L = \frac{r\sqrt{\frac{wx}{r}}}{w} = \sqrt{\frac{rx}{w}}. \tag{7}$$

(c) Substituting $x = 1000$, $r = 1$, and $w = 10$ into (6) and (7), we have $K = 100$ and $L = 10$.