The Ohio State University
Department of Economics

Econ 501a
Prof. James Peck

Midterm Exam Questions and Answers

Part I: Short Answer.

1. **(10 points)** Suppose that there are two goods, x and y, and that x is an inferior good. Show as precisely as you can why good y must be a normal good.

Answer:

Since prices remain constant, when there is more income and less of good x being purchased, the amount of money left over to purchase good y is higher. Therefore, more of good y is demanded as income increases, so y is a normal good. From the budget equation,

$$p_x x + p_y y = M,$$

we see that if M goes up and x goes down, y must go up. Another way to see this is to differentiate the budget equation with respect to M,

$$p_x \frac{\partial x}{\partial M} + p_y \frac{\partial y}{\partial M} = 1.$$

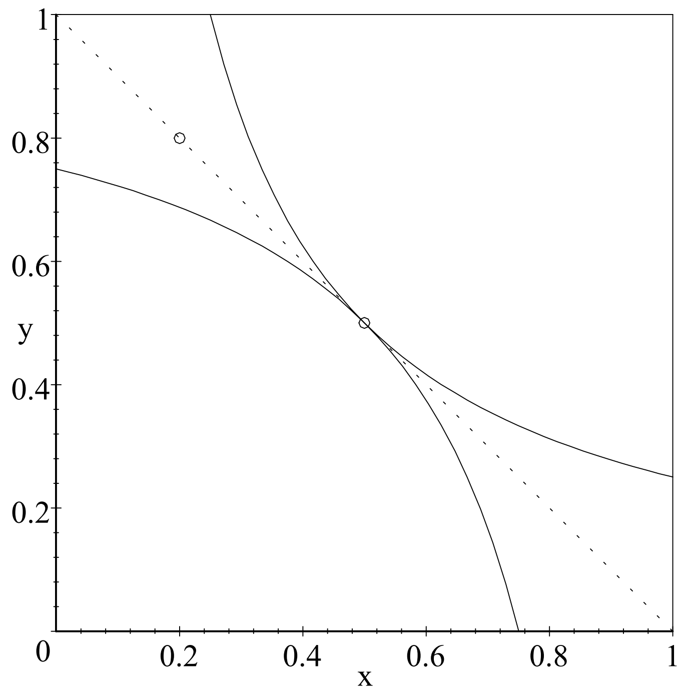
Since the first term is negative, the second term must be positive, so y is a normal good, $\frac{\partial y}{\partial M} > 0$.

2. **(10 points)** True or false, and explain: The income effect on the demand for gasoline, when the price of gasoline increases from \$1.50 to \$1.60, is likely to be greater for someone who drives a lot than for someone who does not drive very much. (Give a verbal explanation. You do not have to graph this.)

Answer:

This statement is TRUE. The income effect is based on the fact that a price increase lowers your overall purchasing power and puts you on a lower indifference curve. Other things equal, the person who consumes more gasoline will feel that his purchasing power has fallen by more than the person who does not consume very much gasoline. In the extreme case of someone who does not drive at all, the income effect would be zero. Income effects tend to be small for most goods, those that constitute a small fraction of our consumption spending.

3. **(10 points)** In the Edgeworth Box diagram, below, find an initial endowment point for which (x^*, y^*) is an equilibrium allocation. Clearly label the initial endowment point you are choosing, and briefly explain why (x^*, y^*) is an equilibrium allocation for that endowment.



Answer:

The point (.2,.8), on the dotted line in the Edgeworth box above, is an initial endowment point for which (x^*, y^*) is an equilibrium allocation. The price ratio is 1, so the budget line corresponding to the equilibrium prices is the dotted line. Because both consumers have their indifference curves tangent to the budget line, we can see that both conditions for an equilibrium are satisfied. Each consumer maximizes utility by choosing (x^*, y^*), and total demand for each good equals the amount of that good available (the market clearing condition). Any point on the dotted line, including the point (x^*, y^*) itself, is a correct answer, as long as you explain why (x^*, y^*) would be an equilibrium allocation.

Part II: Longer Answer.

4. **(20 points)** The Buckeye Corporation sells OSU t-shirts in Columbus and in Cincinnati. Marketing research has indicated that the demand for t-shirts in Columbus, x_{col}, is given by the function

$$x_{col} = 4800 - 200p,$$

and that the demand for t-shirts in Cincinnati, x_{cinc}, is given by the function

$$x_{cinc} = 3200 - 100p,$$

where p refers to the price of t-shirts. Assume that the same price must be charged in both cities.

(a) What is Buckeye's statewide demand function, when we aggregate the demands in the two cities into a single, statewide market?

(b) If the price elasticity of demand in the Columbus market is -1 (unitary elasticity), how many t-shirts are demanded in Columbus, and how many t-shirts are demanded in Cincinnati?

Answer:

To get the statewide demand function, add up the total quantity demanded at each price. Notice that x_{col} is positive for all prices below 24, and x_{cinc} is positive for all prices below 32. If the price is between 24 and 32, then Buckeye is just selling in the Cincinnati market. Therefore, the demand function is:

$$\begin{aligned} x_{ohio} &= 8000 - 300p \quad \text{if} \quad 0 \leq p \leq 24 \\ x_{ohio} &= 3200 - 100p \quad \text{if} \quad 24 < p \leq 32 \\ x_{ohio} &= 0 \quad \text{if} \quad 32 < p. \end{aligned}$$

The price elasticity of demand for good x is given by

$$\varepsilon^d = \frac{dx}{dp_x}\left(\frac{p_x}{x}\right).$$

From the Columbus demand function, $x_{col} = 4800 - 200p$, we can calculate $\frac{dx}{dp_x} = -200$. Solving the demand function for p as a function of x_{col}, we have the inverse demand function,

$$p = \frac{4800 - x_{col}}{200}.$$

Therefore, the elasticity can be written

$$\varepsilon^d = -200\left(\frac{\frac{4800 - x_{col}}{200}}{x_{col}}\right) = -\frac{4800 - x_{col}}{x_{col}}. \tag{1}$$

Since the elasticity is -1, we set the right side of (1) equal to -1, which can be solved to get $x_{col} = 2400$, which is the number of t-shirts demanded in Columbus. To get the number of t-shirts demanded in Cincinnati, we have to know what the price is. Plugging $x_{col} = 2400$ into the inverse demand function for Columbus, we have

$$p = \frac{4800 - 2400}{200} = 12.$$

Once we know the price is 12, we get $x_{cinc} = 3200 - 100p = 2000$.

5. **(20 points)** A consumer has the utility function over goods x and y,

$$u(x, y) = xy^{1/2}.$$

Suppose this consumer has an income of 12, $M = 12$. Let the price of good x be 4, $p_x = 4$, and let the price of good y be 1, $p_y = 1$. *Calculate the bundle of goods x and y that maximizes this consumer's utility, subject to her budget constraint.*

Answer:

To solve the utility maximization problem, set up the Lagrangean expression using the specified values for prices and income,

$$L = xy^{1/2} + \lambda[12 - 4x - y].$$

Differentiating with respect to x, y, and λ, we have the first order conditions,

$$\begin{aligned} y^{1/2} - 4\lambda &= 0, & (2) \\ \frac{1}{2}xy^{-1/2} - \lambda &= 0, & (3) \\ 12 - 4x - y &= 0. & (4) \end{aligned}$$

Solving (2) and (3) for λ and then setting them equal to each other, we have the condition that the marginal rate of substitution equals the price ratio:

$$\lambda = \frac{y^{1/2}}{4} = \frac{1}{2}xy^{-1/2},$$

which can be simplified to

$$y = 2x. \qquad (5)$$

Plugging (5) into (4), we can solve for the demand for x:

$$12 - 4x - 2x = 0, \quad \text{or} \quad x = 2.$$

Plugging x=2 into equation (5), we have $y = 4$. Thus, the answer is, $(x, y) = (2, 4)$.

6. **(30 points)** Xavier's Xylophone Repair uses capital and labor to provide xylophone repair services, according to the production function,

$$x = KL^{1/2},$$

where x is the quantity of xylophones repaired, K is the quantity of capital used, and L is the quantity of labor used.

(a) Does this production function exhibit decreasing returns to scale, constant returns to scale, or increasing returns to scale? Explain.

(b) Does this production function exhibit diminishing marginal returns to labor? Explain.

(Problem 6, parts c and d, continued on next page.)

(c) If the price of labor is 1, w=1, and the price of capital is 2, r=2, derive the long run total cost function (that is, total cost as a function of output, x).

(d) Suppose that we are in a short run situation in which the capital input is fixed at 10, $\overline{K} = 10$. If the price of labor is 1, w=1, and the price of capital is 2, r=2, derive the short run ***average*** *total cost function.*

Answer:

(a) If we multiply all inputs by θ, the output is $\theta K(\theta L)^{1/2} = \theta^{3/2}KL^{1/2} = \theta^{3/2}x$. Thus, output is greater than θ times the original output, so we have increasing returns to scale.

(b) The marginal product of labor is

$$MP_L = \frac{1}{2}KL^{-1/2}.$$

Therefore, we have diminishing marginal returns to labor, since MP_L falls as we increase L:

$$\frac{dMP_L}{dL} = -\frac{1}{4}KL^{-3/2} < 0.$$

(c) To derive the long run total cost function, we first solve for the conditional demands for capital and labor, by minimizing cost subject to an output constraint. The Lagrangean expression is

$$Lagr. = L + 2K + \lambda[x - KL^{1/2}].$$

Differentiating Lagr. with respect to L, K, and λ, we have the first order conditions

$$1 - \lambda(\frac{1}{2})KL^{-1/2} = 0, \quad (6)$$

$$2 - \lambda L^{1/2} = 0, \quad (7)$$

$$x - KL^{1/2} = 0. \quad (8)$$

Solving (6) and (7) for λ and setting them equal to each other, we have the condition that the MRTS equals the ratio of input prices

$$\lambda = 2K^{-1}L^{1/2} = 2L^{-1/2},$$

which can be simplified to

$$L = K. \tag{9}$$

Plugging (9) into (8), we have $x - K(K)^{1/2} = 0$, which simplifies to $x = K^{3/2}$. Solving for K in terms of x, we have the conditional demand for K

$$K = x^{2/3}. \tag{10}$$

From (9) and (10), we have the demand for L: $L = x^{2/3}$. The total cost function is found by evaluating the cost of the inputs used to produce x:

$$LRTC = wL + rK = x^{2/3} + 2x^{2/3} = 3x^{2/3}.$$

(d) If capital is fixed at 10, the production function can be written in terms of L only:

$$x = 10L^{1/2}.$$

Solving for the labor required to produce x, we have: $L^{1/2} = x/10$. Squaring both sides, we have

$$L = \frac{x^2}{100}. \tag{11}$$

The short run total cost function is the cost of the fixed and variable inputs:

$$SRTC = wL + rK = \frac{x^2}{100} + 20.$$

The short run **average** total cost function is

$$SRATC = \frac{SRTC}{x} = \frac{x}{100} + \frac{20}{x}.$$