

The Ohio State University  
Department of Economics

Econ 501a  
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Midterm Questions and Answers

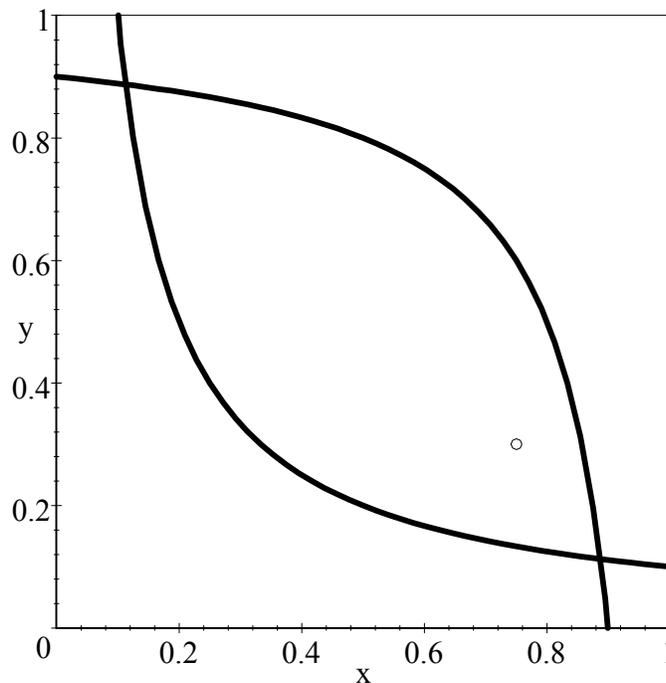
Part I: Short Answer.

1. **(10 points)** True or false, and explain: Suppose that good  $x$  is a normal good. If a consumer's income increases, then that will cause a movement to a new point on her ordinary demand curve, with a lower  $p_x$  and a higher quantity of  $x$ .

Answer: False. It was specified that income changed, not  $p_x$ . Rather than moving to a new point on her ordinary demand curve, the increase in income will shift the demand curve. There will be a horizontal movement from one ordinary demand curve (corresponding to the original income) to another ordinary demand curve (corresponding to the higher income), keeping  $p_x$  constant. Because the good is normal, the demand curve will shift to the right.

2. **(10 points)** True or false, and explain: Consider an Edgeworth Box economy with an initial allocation for the two consumers given by  $(\bar{x}_1, \bar{y}_1, \bar{x}_2, \bar{y}_2)$ . The initial allocation is represented by the point plotted in the Edgeworth Box, below. If there is another feasible allocation that both consumers prefer to the initial allocation, then that allocation is not necessarily Pareto optimal.

Answer: True. Just because the consumers can find a new allocation to make themselves better off, it does not mean that there cannot be further gains from a second trade. In the diagram below, suppose that the initial allocation is where the two indifference curves intersect (near the southeast corner of the box). The utility functions underlying this example are tangent along the 45 degree line, so any Pareto optimal allocation must be on the diagonal of the box (the segment connecting (0,0) and (1,1)). It is easy to see that the point plotted as a circle makes both consumers better off than their initial endowments, but it is not Pareto optimal.



Gains From Trade

3. (10 points) True or false, and explain: If a production function exhibits constant returns to scale, then it cannot also exhibit diminishing marginal returns to labor.

Answer: False. In fact, if a production function exhibits constant returns to scale, it **must** also exhibit diminishing marginal returns to labor. For example, suppose 1 typist on 1 computer produces 1 unit of output. If we have 2 typists on 2 computers, we have double the output. This is constant returns to scale. However, although the first typist produces one unit of output, if we add a second typist without adding a computer, then the output does not go up very much, if at all. This is an extreme case of diminishing  $MP_L$ . To show this mathematically, refer to the Cobb-Douglas production function,  $x = K^\alpha L^\beta$ . With constant returns to scale, we have  $\alpha + \beta = 1$ , so in particular  $\beta$  must be less than 1. We can calculate  $MP_L = \beta K^\alpha L^{\beta-1}$ . Diminishing marginal returns to labor set in if  $\frac{\partial MP_L}{\partial L}$  is negative.

$$\frac{\partial MP_L}{\partial L} = (\beta - 1)\beta K^\alpha L^{\beta-2} < 0.$$

4. (10 points) Suppose that a firm is currently hiring a bundle of capital and labor, and it is determined that the marginal product of labor is 2,  $MP_L =$

2, and the marginal product of capital is 4,  $MP_K = 4$ . The prices of labor and capital are given by  $w = 8$  and  $r = 8$ . Carefully explain how the firm could produce more output with the same expenditure on inputs.

Answer: The marginal rate of technical substitution is not equal to the input price ratio, so the firm is not choosing its inputs efficiently. The marginal cost of producing output using labor is  $w/MP_L = 4$ , and the marginal cost of producing output using capital is  $r/MP_K = 2$ . Because capital is more cost effective, we can increase output without increasing costs by moving up the isocost line, trading labor for capital. Increase K by 1 and reduce L by 1. The increase in output is  $4-2=2$ .

Part II: Longer Answer.

5. **(30 points)** A consumer has the utility function over goods x and y,

$$u(x, y) = \sqrt{x} + \sqrt{y}.$$

Let the price of good x be given by  $p_x$ , let the price of good y be given by  $p_y$ , and let income be given by  $M$ .

(a) Derive the consumer's generalized demand function for good x.

(b) Are goods x and y gross substitutes, gross complements, or neither?

Explain precisely.

Answer: Set up the Lagrangean,

$$L = \sqrt{x} + \sqrt{y} + \lambda[M - p_x x - p_y y].$$

Differentiating with respect to x, y, and  $\lambda$ , we have

$$\frac{1}{2}x^{-1/2} - \lambda p_x = 0 \tag{1}$$

$$\frac{1}{2}y^{-1/2} - \lambda p_y = 0 \tag{2}$$

$$M - p_x x - p_y y = 0. \tag{3}$$

From (1) and (2), we have

$$\lambda = \frac{\frac{1}{2}x^{-1/2}}{p_x} = \frac{\frac{1}{2}y^{-1/2}}{p_y}$$

After simplifying, squaring both sides, and solving for y in terms of x, we have

$$y = \frac{(p_x)^2 x}{(p_y)^2}. \tag{4}$$

Plugging (4) into the budget constraint, (3), we have

$$M - p_x x - p_y \frac{(p_x)^2 x}{(p_y)^2} = 0$$

Simplifying, putting the x terms on the right side, and factoring out x, we have

$$M = x \left[ p_x + \frac{(p_x)^2}{(p_y)} \right]. \quad (5)$$

Solving (5) for x, we have the generalized demand function

$$x = \frac{M}{\left[ p_x + \frac{(p_x)^2}{(p_y)} \right]} = \frac{M p_y}{p_x (p_x + p_y)} \quad (6)$$

(b) To determine whether x and y are gross substitutes or gross complements, differentiate x with respect to  $p_y$  to see whether demand goes up or down when the other price changes. From (6), and using the “division” rule, we have

$$\frac{\partial x}{\partial p_y} = \frac{p_x (p_x + p_y) M - M p_y p_x}{[p_x (p_x + p_y)]^2} = \frac{(p_x)^2 M}{[p_x (p_x + p_y)]^2} > 0.$$

Because demand for x increases when the price of y increases, we have gross substitutes.

6. **(30 points)** Marketing research indicates that three types of customers purchase Dr. Miles’ Magical Elixir. Let  $x_1$  denote the number of customers purchasing the elixir in order to prevent baldness. Let  $x_2$  denote the number of customers purchasing the elixir in order to cure insomnia. Let  $x_3$  denote the number of customers purchasing the elixir in order to enhance their memory. Let the price of a six ounce bottle, in dollars, be denoted by  $p_x$ . The demand functions of the three customer groups has been determined as follows

$$\begin{aligned} x_1 &= 100 - 4p_x \\ x_2 &= 50 - p_x \\ x_3 &= 30 - p_x. \end{aligned}$$

- (a) What is the total demand for Dr. Miles’ Magical Elixir, as a function of  $p_x$ ?
- (b) What is the price-elasticity of demand (of the entire market) when we have  $p_x = 20$ ?
- (c) At what price is the total revenue (for the entire market) maximized?

Answer: (a) To find the market demand, we add the total quantities demanded by the three groups, given the price. Taking into account the fact

that group 1 demands zero at prices above 25, group 2 demands zero at prices above 50, and group 3 demands zero at prices above 30, we have

$$\begin{aligned}x &= 180 - 6p_x && \text{if } p_x < 25, \\x &= 80 - 2p_x && \text{if } 25 < p_x < 30, \\x &= 50 - p_x && \text{if } 30 < p_x < 50, \\x &= 0 && \text{if } p_x > 50.\end{aligned}$$

(b) When the price is 20, demand is given by the equation  $x = 180 - 6p_x$ . The price elasticity of demand is

$$\frac{\partial x}{\partial p_x} \frac{p_x}{x} = -6 \frac{p_x}{180 - 6p_x} = -6 \frac{20}{60} = -2. \quad (7)$$

(c) First of all, the revenue maximizing price will certainly be below 25, because that is the price that maximizes revenue for the big spending group 3 consumers. Thus, the relevant demand function is  $x = 180 - 6p_x$ . The easiest way to solve this is to find the price at which the price elasticity of demand is  $-1$ . From (7), we solve

$$-6 \frac{p_x}{180 - 6p_x} = -1$$

yielding  $p_x = 15$ . Alternatively, we can calculate the inverse demand function,  $p_x = 30 - \frac{x}{6}$ . The total revenue function is then  $TR = (30 - \frac{x}{6})x = 30x - x^2/6$ . Differentiating, we have the marginal revenue function,  $MR = 30 - \frac{x}{3}$ . Setting marginal revenue equal to zero and solving, we have  $x=30$ . Substituting back into the inverse demand function, we find the price,  $p_x = 15$ .