

The Ohio State University
Department of Economics
Econ 501.02–Prof. James Peck
Equations for the Midterm

Axiom 1: Preferences are complete: for any two bundles, A and B, exactly one of the following is true. (1) A is preferred to B, $A^P B$, (2) B is preferred to A, $B^P A$, or (3) A is indifferent to B, $A^I B$.

Axiom 2: Preferences are reflexive: $A^I A$.

Axiom 3: Preferences are transitive: $A^P B$ and $B^P C \Rightarrow A^P C$.

Axiom 4: Preferences are continuous. If $A^P B$, and if C is sufficiently close to B , then $A^P C$.

Axioms 1-4 allow preferences to be represented graphically by *indifference curves*, and by a utility function, u . $A^P B$ if and only if $u(x^A, y^A) > u(x^B, y^B)$.

Axiom 5: More is better. Starting with the bundle $A = (x^A, y^A)$, then increasing any of the goods in A yields a new bundle that is preferred to A .

Axiom 6: All indifference curves exhibit diminishing *marginal rates of substitution*. The MRS is the absolute value of the slope of the indifference curve. The indifference curve flattens as you move along the indifference curve to the right.

The marginal rate of substitution equals the ratio of marginal utilities:

$$MRS_{yx} = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}}.$$

The utility maximization problem is

$$\begin{aligned} & \max u(x, y) \\ \text{subject to} & : p_x x + p_y y = M \end{aligned}$$

The Lagrangean approach transforms a constrained optimization problem into an unconstrained problem of choosing x , y , and the Lagrange multiplier, λ , to maximize

$$L = u(x, y) + \lambda[M - p_x x - p_y y].$$

By differentiating L with respect to x , y , and λ , and setting the derivatives equal to zero, the resulting first order conditions are:

$$\frac{\partial u}{\partial x} - \lambda p_x = 0, \frac{\partial u}{\partial y} - \lambda p_y = 0, \text{ and } M - p_x x - p_y y = 0.$$

$x^*(p_x, p_y, M)$ and $y^*(p_x, p_y, M)$ are known as the *generalized demand functions*.

The demand for, say, good y as a function of income, holding prices constant, is called the *Engel Curve*. This is related to the *income-consumption curve*, the set of consumption bundles chosen as income varies, holding prices constant.

The set of consumption bundles chosen as, say, p_x varies, holding M and p_y constant, is called the *price-consumption curve*.

The demand for x, as a function of p_x , holding M and p_y constant, is called the *ordinary demand function*.

If the ordinary demand function of consumer i is $x_i^*(p_x, p_y, M_i)$, then the generalized market demand function is

$$X^*(p_x, p_y, M_1, \dots, M_i, \dots, M_n) = \sum_{i=1}^n x_i^*(p_x, p_y, M_i).$$

The (own-price) elasticity of demand, from one point on the demand curve to another, is

$$\varepsilon^d = \frac{\% \Delta x}{\% \Delta p_x} = \frac{\Delta x/x}{\Delta p_x/p_x}.$$

By convention, we take x and p_x to be the averages, $(x_1 + x_2)/2$ and $(p_{x,1} + p_{x,2})/2$.

The own-price elasticity of demand at a single point is:

$$\varepsilon^d = \frac{dx}{dp_x} \left(\frac{p_x}{x} \right).$$

If $\varepsilon^d < -1$, we say that demand is *elastic* (demand changes a lot or “stretches” in response to a price change).

If $\varepsilon^d = -1$, we say that demand has *unitary elasticity*.

If $\varepsilon^d > -1$, we say that demand is *inelastic*.

For any demand curve, we can write the inverse demand function as $p_x(x)$. Then the total revenue received, as a function of x, is $p_x(x)x$.

Marginal revenue is defined to be the derivative of total revenue, given by

$$MR(x) = p_x + x \frac{dp_x}{dx} = p_x \left[1 + \frac{1}{\varepsilon^d} \right].$$

The Income Elasticity of Demand

$$\varepsilon_I^d = \frac{dx}{dM} \left(\frac{M}{x} \right)$$

If $\varepsilon_I^d > 0$, x is a normal good.

If $\varepsilon_I^d < 0$, x is an inferior good. (used cars)

If $\varepsilon_I^d > 1$, x is a luxury good.

The Cross Price Elasticity of Demand

$$\varepsilon_c^d = \frac{dx}{dp_y} \left(\frac{p_y}{x} \right)$$

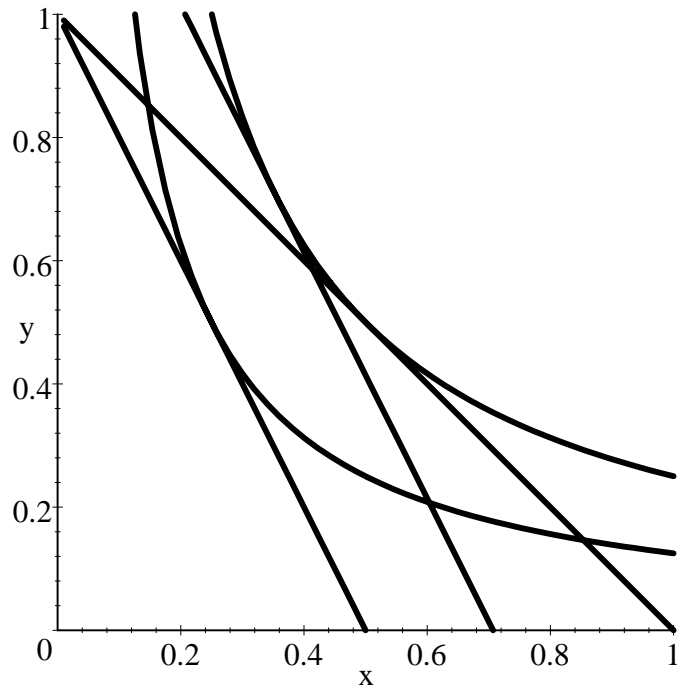
If $\varepsilon_c^d > 0$, x is a gross substitute for y.

If $\varepsilon_c^d < 0$, x is a gross complement for y.

Substitution Effect—the change in demand resulting from a change in the price ratio, leaving utility unchanged.

Income Effect—the change in demand resulting from the change in purchasing power (movement from the initial indifference curve to the final indifference curve), leaving the price ratio unchanged.

total effect = substitution effect + income effect

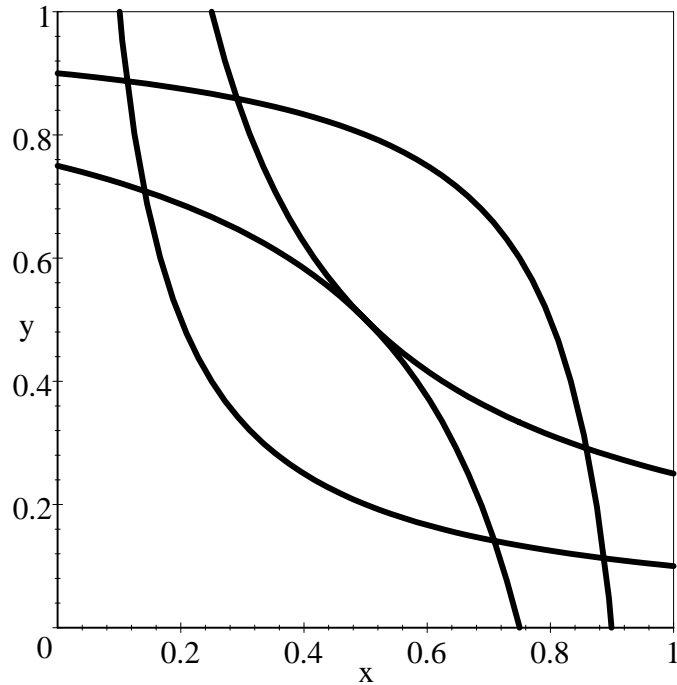


Income and Substitution Effects (normal good)

Suppose that there are two consumers, and fix the total amount of goods x and y to be allocated: \bar{x} and \bar{y} .

The Edgeworth Box—Consider a rectangle of size $\bar{x} \times \bar{y}$. Think of consumer 1 as being located at the southwest corner and consumer 2 as being located at the northeast corner (upside down). Then any point in the box represents an allocation of the available goods across the two consumers.

Pareto optimal allocations are ones for which there are no gains from trade. It is impossible to make one consumer better off without hurting the other consumer.



A Pareto Optimal Allocation

The set of Pareto Optimal Allocations is called the Contract Curve. Indifference curves are tangent, so marginal rates of substitution are equal.

$$\frac{\frac{\partial u_1}{\partial x_1}}{\frac{\partial u_1}{\partial y_1}} = \frac{\frac{\partial u_2}{\partial x_2}}{\frac{\partial u_2}{\partial y_2}},$$

The *initial endowment* of consumer 1 is denoted by (\bar{x}_1, \bar{y}_1) , and the initial endowment of consumer 2 is denoted by (\bar{x}_2, \bar{y}_2) .

Both consumers' initial endowments are represented by the same point in the Edgeworth Box, since

$$\bar{x}_1 + \bar{x}_2 = \bar{x} \quad \text{and} \quad \bar{y}_1 + \bar{y}_2 = \bar{y}.$$

Each consumer's income now depends on prices:

$$\begin{aligned} M_1 &= p_x \bar{x}_1 + p_y \bar{y}_1 \quad \text{and} \\ M_2 &= p_x \bar{x}_2 + p_y \bar{y}_2. \end{aligned}$$

Definition: An *equilibrium* is a price, (p_x^*, p_y^*) , and an allocation, $(x_1^*, y_1^*, x_2^*, y_2^*)$, such that

(i) For each consumer, $i=1,2$, (x_i^*, y_i^*) solves

$$\begin{aligned} & \max u_i(x_i, y_i) \\ \text{subject to} & \quad : \\ p_x x_i + p_y y_i & = p_x \bar{x}_i + p_y \bar{y}_i \end{aligned}$$

(ii) $x_1^* + x_2^* = \bar{x}$ and $y_1^* + y_2^* = \bar{y}$.

Condition (i) reflects utility maximization, and condition (ii) reflects market clearing (demand = supply).

Utility maximization requires each consumer's indifference curve to be tangent to their budget line:

$$\frac{\frac{\partial u_1}{\partial x_1}}{\frac{\partial u_1}{\partial y_1}} = \frac{p_x}{p_y} = \frac{\frac{\partial u_2}{\partial x_2}}{\frac{\partial u_2}{\partial y_2}}$$

First Fundamental Theorem of Welfare Economics: an equilibrium allocation is Pareto optimal.

Second Fundamental Theorem of Welfare Economics: Any Pareto optimal allocation can be achieved as an equilibrium, by a suitable reassignment of the initial allocation.

The *production function* specifies the most output that can be produced with a given combination of inputs, based on the technology available to the firm.

$$x = f(K, L)$$

Technological efficiency occurs if the firm is on its "production frontier." That is, it is impossible to achieve more output with the same inputs. We typically assume that marginal products are positive

$$\frac{\partial f(K, L)}{\partial K} > 0 \quad \text{and} \quad \frac{\partial f(K, L)}{\partial L} > 0.$$

A *production isoquant* is a curve describing the set of capital-labor combinations yielding the same output, according to the production function.

The *marginal rate of technical substitution* is defined to be the negative of the slope of the isoquant

$$MRTS = - \frac{dK}{dL} \Big|_{f(K,L)=\text{constant}}$$

The MRTS is the rate at which the firm would be willing to give up capital in exchange for labor. We assume diminishing MRTS.

In the *long run*, all inputs are variable, so the firm can choose any combination of capital and labor. In the *short run*, at least one input is fixed and cannot be varied.

Returns to Scale

For long run decisions, we may be interested in what happens as we vary all of the inputs simultaneously.

The production function exhibits *decreasing returns to scale* if, for $\theta > 1$, we have

$$f(\theta K, \theta L) < \theta f(K, L).$$

The production function exhibits *constant returns to scale* if, for $\theta > 1$, we have

$$f(\theta K, \theta L) = \theta f(K, L).$$

The production function exhibits *increasing returns to scale* if, for $\theta > 1$, we have

$$f(\theta K, \theta L) > \theta f(K, L).$$

Hold all but one of the inputs fixed (say, fix $K = \bar{K}$). The *total product of labor* is given by the function, $x = f(L; \bar{K})$. The average product of labor is defined as

$$AP_L = \frac{f(L; \bar{K})}{L}.$$

The marginal product of labor is defined as

$$MP_L = \frac{\partial f(L; \bar{K})}{\partial L}.$$

Cobb-Douglas example: $x = K^\alpha L^\beta$

$$\begin{aligned} AP_L &= \frac{K^\alpha L^\beta}{L} = K^\alpha L^{\beta-1} \\ MP_L &= \frac{\partial K^\alpha L^\beta}{\partial L} = \beta K^\alpha L^{\beta-1} \\ AP_K &= \frac{K^\alpha L^\beta}{K} = K^{\alpha-1} L^\beta \\ MP_K &= \frac{\partial K^\alpha L^\beta}{\partial K} = \alpha K^{\alpha-1} L^\beta \end{aligned}$$

$$MRTS = \frac{MP_L}{MP_K}.$$

Diminishing marginal returns (to labor) occur when the marginal product (of labor) eventually falls as L increases.

$$\frac{\partial MP_L}{\partial L} < 0$$

Cost minimization subject to an output constraint:

$$\begin{aligned} & \min wL + rK \\ \text{subject to } & f(K, L) = x \end{aligned} \tag{1}$$

Set up the Lagrangean,

$$Lagr. = wL + rK + \lambda[x - f(K, L)]$$

The first order conditions are

$$\begin{aligned} \frac{\partial Lagr.}{\partial L} &= 0 = w - \lambda \frac{\partial f}{\partial L} \\ \frac{\partial Lagr.}{\partial K} &= 0 = r - \lambda \frac{\partial f}{\partial K} \\ \frac{\partial Lagr.}{\partial \lambda} &= 0 = x - f(K, L) \end{aligned}$$

Solving, we have

$$\lambda = \frac{w}{MP_L} = \frac{r}{MP_K}. \tag{2}$$

This is the condition that the MRTS equals the input price ratio. It is also the condition that the marginal cost of producing output using any variable input must be the same. To solve for the generalized (conditional) input demand functions, $L^*(w, r, x)$ and $K^*(w, r, x)$, solve (1) and (2) for K and L.