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**Extra Practice Problems Answers (for final)**

Watson, Chapter 15, Exercise 1(part a).

Looking at the final subgame, player 1 must choose F, leading to payoff (2,3). Therefore, player 2 faces a choice between a payoff of 3 and 4, so she chooses C, leading to payoff (1,4). Working backward to player 1's first information set, he faces a choice between 1 and 2, so he chooses A.

The solution is (AF,C).

Watson, Chapter 15, Exercise 2(part b).

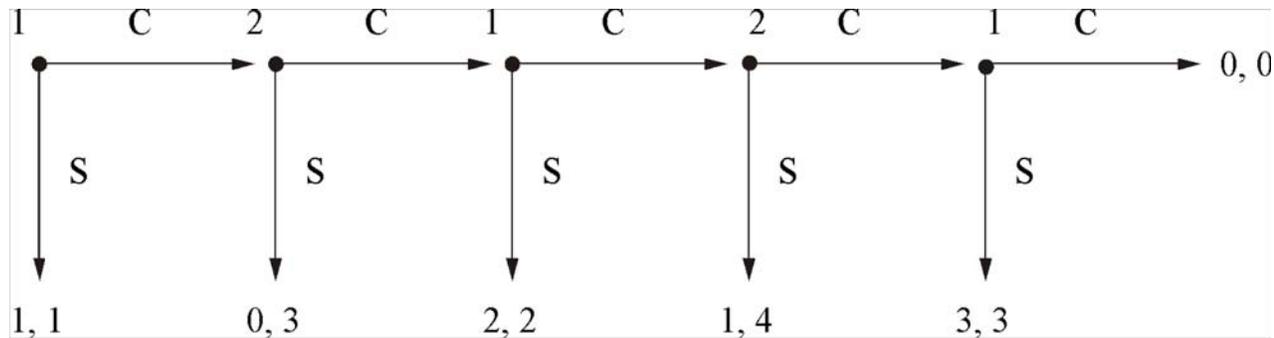
The payoff matrix with best responses underlined is:

		player 2			
		AC	AD	BC	BD
player 1	UE	2, 3	<u>2</u> , 3	5, <u>4</u>	<u>5</u> , <u>4</u>
	UF	2, 3	<u>2</u> , 3	5, <u>4</u>	<u>5</u> , <u>4</u>
	DE	<u>6</u> , <u>2</u>	0, <u>2</u>	<u>6</u> , <u>2</u>	0, <u>2</u>
	DF	2, <u>6</u>	0, 2	2, <u>6</u>	0, 2

From the matrix, we see that the NE are: (UE, BD), (UF, BD), (DE, AC), and (DE, BC). To be subgame perfect, player 1 must be choosing E, which rules out (UF, BD). Also, player 2 must be choosing B, which rules out (DE, AC). Therefore, the subgame perfect Nash equilibria are: (UE, BD) and (DE, BC).

Watson, Chapter 15, Exercise 5.

(a) Here is the game tree:



(b) Working backward, it is easy to see that in round 5 player 1 will choose S. Thus, in round 4 player 2 will choose S. Continuing in this fashion, we find that, in equilibrium, each player will choose S any time he is on the move.

(c) For any finite  $k$ , the backward induction outcome is that player 1 chooses S in the first round and each player receives one dollar.

Watson, Chapter 16, Exercise 4(parts a and b only).

(a) Firm 2's payoff function is

$$u_2(q_1, q_2) = (1000 - 3q_1 - 3q_2)q_2 - 100q_2 - F.$$

We find firm 2's optimal quantity as a function of  $q_1$  by differentiating with respect to  $q_2$ , setting the expression equal to zero, and solving for  $q_2$ . We have

$$\begin{aligned} 1000 - 3q_1 - 6q_2 - 100 &= 0 \\ q_2 &= \frac{900 - 3q_1}{6} = 150 - \frac{q_1}{2}. \end{aligned}$$

Note: this calculation presumes that the resulting output is nonnegative; just set  $q_2 = 0$  if  $q_1 > 300$ .

(b) Setting  $F = 0$ , player 1's payoff as a function of the two quantities is

$$u_1(q_1, q_2) = (1000 - 3q_1 - 3q_2)q_1 - 100q_1.$$

Since there are no fixed costs and the two marginal costs are the same, we know that in the subgame perfect NE, firm 2 will be producing positive output, so we have

$$q_2 = 150 - \frac{q_1}{2}. \tag{1}$$

Substituting this dependence of firm 2's output on firm 1's output into the payoff function, firm 1's best response maximizes

$$(1000 - 3q_1 - 3[150 - \frac{q_1}{2}])q_1 - 100q_1.$$

Simplifying, we have

$$(450 - \frac{3q_1}{2})q_1.$$

Differentiating, setting the expression equal to zero, and solving, we have

$$\begin{aligned} 450 - 3q_1 &= 0 \\ q_1 &= 150. \end{aligned}$$

This is firm 1's equilibrium strategy and output. Firm 2's equilibrium strategy is given by equation (1), but its equilibrium output is found by substituting

$q_1 = 150$  into (1), yielding  $q_2 = 75$ . The demand equation gives the price,  $p = 325$ , and payoffs 33750 for firm 1 and 16875 for firm 2.

Watson, Chapter 22, Exercise 1.

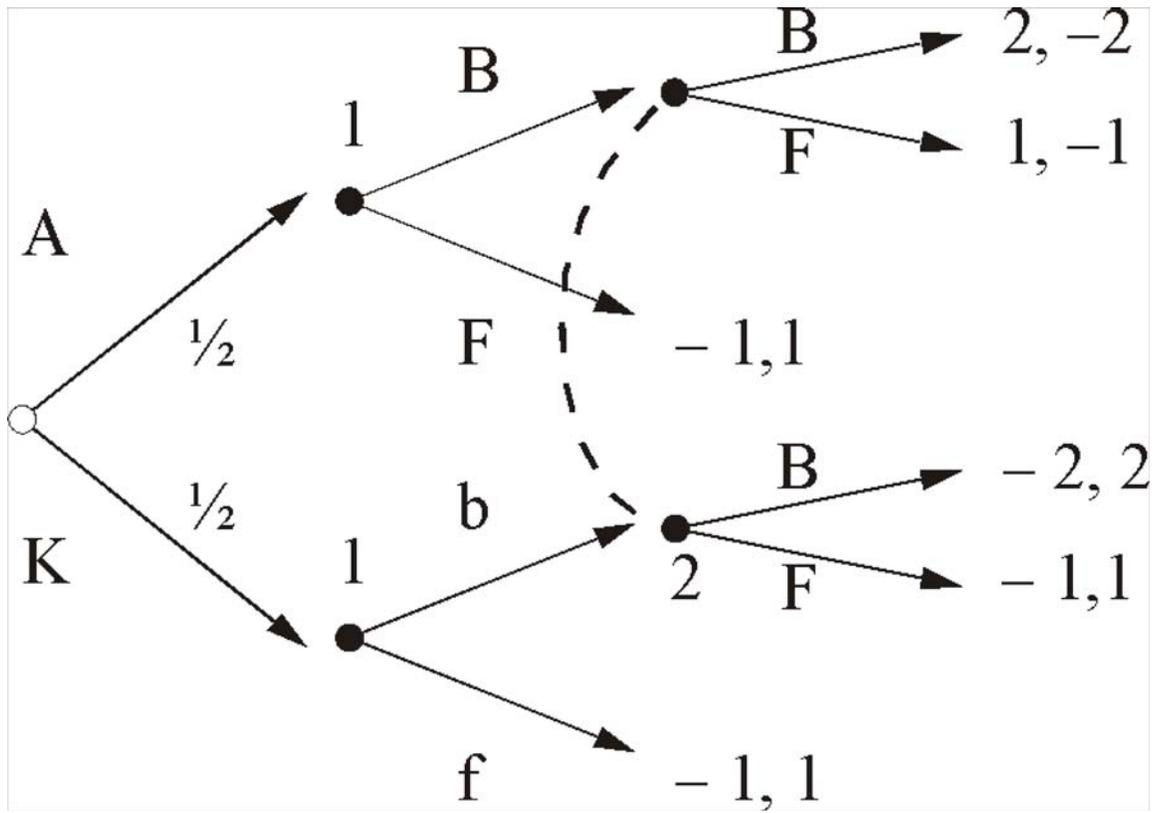
(U, L) can be supported as follows. If player 2 defects ((U,M) is played) in the first period, then the players coordinate on (C, R) in the second period. If player 1 defects ((C, L) is played) in the first period, then the players play (D, M) in the second period. Otherwise, the players play (D,R) in the second period.

Watson, Chapter 22, Exercise 4.

In period 2, subgame perfection requires play of the only Nash equilibrium of the stage game. Since there is only one Nash equilibrium of the stage game, the play in period 1 cannot affect the play in period 2, so incentives in period 1 are based on the one-shot game. Thus, the only subgame perfect equilibrium is play of the Nash equilibrium of the stage game in both periods. For any finite  $T$ , the logic from the two period case applies, and the answer does not change.

Watson, Chapter 24, Exercise 1.

Here is the game in extensive form.



Here is the game in Bayesian normal form.

		2	
		B	F
1	Bb	0,0	1, -1
	Bf	$\frac{1}{2}, \frac{1}{2}$	0,0
	Fb	$-\frac{3}{2}, \frac{3}{2}$	0,0
	Ff	-1,1	-1,1

Watson, Chapter 24, Exercise 3.

Here is the game in Bayesian normal form:

		2	
		U	D
1	LL'	2, 0	2, 0
	LR'	1, 0	3, 1
	RL'	1, 2	3, 0
	RR'	0, 2	4, 1

Watson, Chapter 26, Exercise 8.

Note: This is a tough one, so do not worry if you do not come up with the answer on your own.

We will look for a symmetric equilibrium where each player  $i$  bids if and only if his number satisfies

$$x_i \geq \alpha,$$

for some number  $\alpha$  that we must compute. For this to be an equilibrium, a player with a number exactly equal to  $\alpha$  must be indifferent between bidding and folding. Then higher numbers have a better chance of winning and strictly prefer to bid, and lower numbers are willing to fold.

If  $x_i = \alpha$ , player  $i$  receives a payoff of 1 whenever his opponent has a number below  $\alpha$  (and folds), and receives a payoff of  $-2$  whenever his opponent has a number above  $\alpha$  (in which case player  $i$  has the lower number). Because of the uniform distribution, the probability that the other player's number is below  $\alpha$  is  $\alpha$ . Therefore, player  $i$ 's payoff from bidding is

$$\alpha \cdot 1 + (1 - \alpha) \cdot (-2) = 3\alpha - 2.$$

Setting this payoff equal to the payoff from folding, we have

$$\begin{aligned} 3\alpha - 2 &= -1 \\ \alpha &= \frac{1}{3}. \end{aligned}$$

Watson, Chapter 28, Exercise 2.

(a) In a separating equilibrium, player 1's types make different choices, so he either chooses the strategy  $AB'$  or the strategy  $BA'$ . If player 1's strategy is  $AB'$ , player 2's sequentially rational choice is  $Y$ , leaving player 1L with a payoff of zero, so  $B'$  is not sequentially rational. This is inconsistent with equilibrium.

If player 1's strategy is  $BA'$ , player 2's sequentially rational choice is  $X$ . However, player 1L receives a payoff of 4 by playing  $A'$ , but would receive 6 by playing  $B'$ . Thus,  $A'$  is not sequentially rational. This is inconsistent with equilibrium, so there are no separating equilibria.

(b) In a pooling equilibrium, player 1's types both choose the same action, so possible pooling strategies are  $AA'$  or  $BB'$ . There cannot be an equilibrium where player 1 plays  $BB'$ . To see this, notice that player 2's beliefs (from Bayes' rule) are that either type is equally likely, so her sequentially rational best response is  $Y$ . But then type 1L is not being sequentially rational by choosing  $B'$ .

There is a pooling PBE with strategy profile  $(AA', Y)$ . Player 2's beliefs (that player 1 is type H) can be any  $q$  satisfying  $q \leq \frac{3}{5}$ . To see this, first note that player 1's strategy is sequentially rational for both types (given player 2's

strategy), since the alternative is to choose  $B$  or  $B'$  and receive 0. Next, note that player 2's strategy is sequentially rational given her beliefs, because her conditional expected payoff from  $Y$  is greater than her payoff from  $X$ :

$$\begin{aligned}
 q \cdot 0 + (1 - q) \cdot 6 &\geq q \cdot 4 + (1 - q) \cdot 0 \\
 6 - 6q &\geq 4q \\
 6 &\geq 10q \\
 q &\leq \frac{3}{5}.
 \end{aligned}$$

Finally, player 2's beliefs are consistent because her information set occurs with probability zero (given player 1's strategy), so Bayes' rule does not apply and any beliefs are consistent.

(c) Here is the normal form matrix

		2	
		X	Y
1	AA'	4,3	4,3
	AB'	5,2	2,5
	BA'	5,3	2,1
	BB'	6,2	0,3