

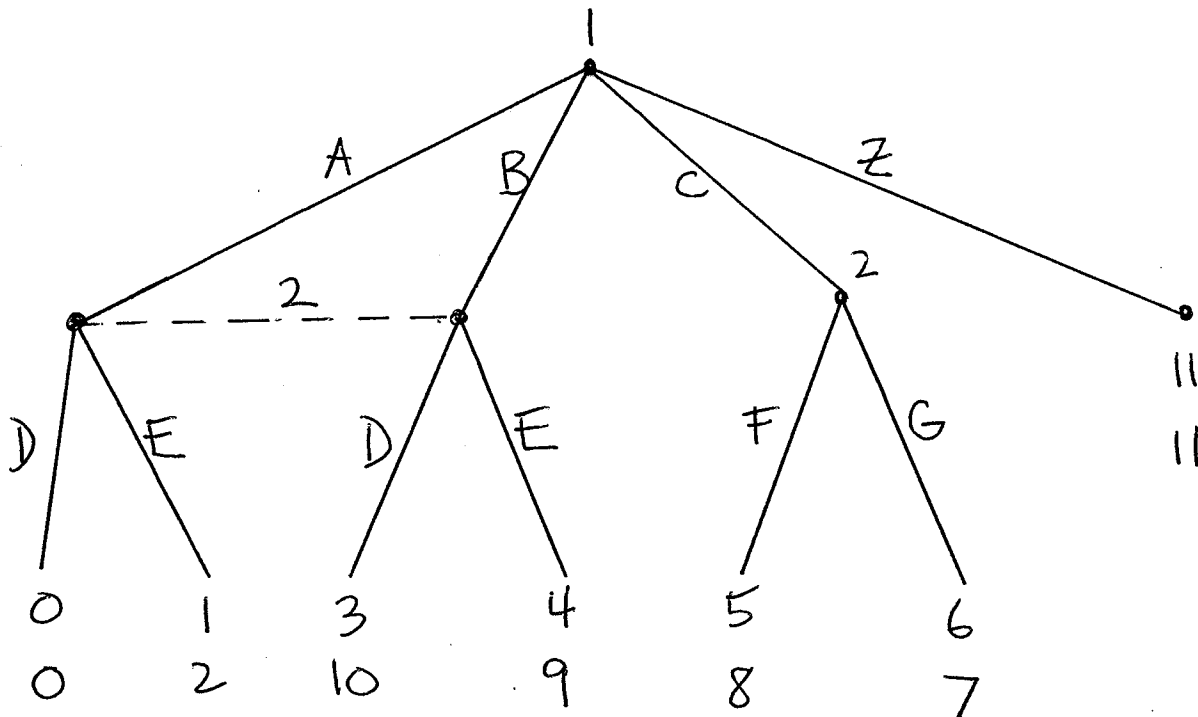
The Ohio State University  
 Department of Economics  
 Midterm Answers

Econ 601  
 Fall 2011  
 Prof. James Peck

1. (20 points) Convert the following extensive form game into normal form, by drawing the payoff matrix, labeling the strategies corresponding to the rows and columns, and filling in the payoffs.

Answer:

		player 2			
		DF	DG	EF	EG
player 1	A	0,0	0,0	1,2	1,2
	B	3,10	3,10	4,9	4,9
	C	5,8	6,7	5,8	6,7
	Z	11,11	11,11	11,11	11,11



2. (20 points) Consider the following game in matrix form.

		player 2		
		L	C	R
player 1	U	5, 5	5, 0	0, 2
	M	4, 1	3, 5	7, 2
	D	3, 8	2, 6	3, 10

Determine the rationalizable strategies for each player by iteratively eliminating dominated strategies. Show your work, with explanations so that I can follow what you are doing. Hint: some strategies might be dominated by a mixture.

**Answer:** First, notice that player 1's strategy D is dominated by M, so we can eliminate D, yielding the reduced game

		player 2		
		L	C	R
player 1	U	5, 5	5, 0	0, 2
	M	4, 1	3, 5	7, 2

Next, in the reduced game player 2's strategy R is dominated by the mixed strategy assigning probability one half to L and C,  $\sigma_2 = (\frac{1}{2}, \frac{1}{2}, 0)$ . To see this, if player 1 plays U, player 2's expected payoff under  $\sigma_2$  is  $\frac{5}{2}$  and her expected payoff under R is 2; if player 1 plays M, player 2's expected payoff under  $\sigma_2$  is 3 and her expected payoff under R is 2. Thus, we can eliminate R, yielding the reduced game

		player 2	
		L	C
player 1	U	5, 5	5, 0
	M	4, 1	3, 5

In the reduced game, player 1's strategy M is dominated by U, so the only rationalizable strategy for player 1 is U. In the reduced game with only strategy U for player 1, player 2's strategy C is dominated by L. Thus the rationalizable strategies are  $\{U\}$  for player 1 and  $\{L\}$  for player 2.

3. (15 points) Find all of the pure-strategy Nash equilibria of the following game, and indicate your answer below.

		player 2					
		U	V	W	X	Y	Z
player 1	A	<u>25</u> , 6	<u>7</u> , 7	0, 10	12, <u>36</u>	8, 8	<u>9</u> , 13
	B	12, 8	5, 3	12, 6	1, 1	<u>9</u> , <u>10</u>	<u>9</u> , <u>10</u>
	C	20, 11	6, 8	15, 12	<u>14</u> , <u>14</u>	6, 0	6, 12
	D	16, <u>16</u>	2, 13	<u>17</u> , <u>16</u>	9, 7	4, 3	2, 1

**Answer:** The best responses to each of player 2's strategies are found by going down each column and underlining the highest payoff for player 1. The best responses to each of player 1's strategies are found by going across each row and underlining the highest payoff for player 2. The Nash equilibria are:  $(B, Y)$ ,  $(B, Z)$ ,  $(C, X)$ , and  $(D, W)$ .

4. (25 points) Two firms are playing a game of Cournot (quantity) competition. Denoting the quantity chosen by firm 1 as  $q_1$  and the quantity chosen by firm 2 as  $q_2$ , the market price is given by the demand equation

$$p = 140 - 2q_1 - 2q_2.$$

Each firm has a production cost of 20 per unit of output. Each firm's payoff is defined to be its profit.

(a) (10 points) Find the Nash equilibrium strategy profile, and show your work.

(b) (5 points) In the Nash equilibrium, what is the market price and what are the profits of each firm?

(c) (10 points) If both firms have their cost per unit increased to 50 (maybe the government imposes a tax), what are the profits of each firm in the Nash equilibrium of the new game?

Answer: (a) The payoff to firm 1 is given by its profit

$$\begin{aligned}\pi_1 &= (140 - 2q_1 - 2q_2)q_1 - 20q_1 \\ &= 120q_1 - 2(q_1)^2 - 2q_2q_1\end{aligned}$$

To find firm 1's best response to firm 2's output, set the derivative with respect to  $q_1$  equal to zero and solve for  $q_1$ . We have

$$\begin{aligned}120 - 4q_1 - 2q_2 &= 0 \\ q_1 &= 30 - \frac{q_2}{2}.\end{aligned}\tag{1}$$

Similarly, the payoff to firm 2 is given by  $\pi_2 = (140 - 2q_1 - 2q_2)q_2 - 20q_2$ . Differentiating with respect to  $q_2$ , setting the expression equal to zero, and solving for  $q_2$  yields firm 2's best response function

$$q_2 = 30 - \frac{q_1}{2}.\tag{2}$$

The Nash equilibrium is found by simultaneously solving (1) and (2). Substituting (2) into (1) and simplifying yields

$$\begin{aligned}q_1 &= 30 - \frac{30 - \frac{q_1}{2}}{2} \\ 2q_1 &= 60 - 30 + \frac{q_1}{2} \\ 3q_1 &= 60 \\ q_1 &= 20\end{aligned}$$

Substituting  $q_1 = 20$  into (2) gives  $q_2 = 20$ , so the NE is (20, 20).

(b) Since each firm produces 20, the price is 60 (that is,  $140 - 2(20) - 2(20) = 60$ ). Each firm makes revenue of 1200, has cost of 400, so each firm's profit is 800.

(c) If each firm's cost is 50 per unit, we have a new game and can go through the same steps to find the NE. Instead of dividing 120 by 4 to get the number 30 in best response functions as we did in part (a), now we divide 90 by 4, giving us the best response functions

$$q_1 = 22.5 - \frac{q_2}{2}$$

$$q_2 = 22.5 - \frac{q_1}{2}$$

Simultaneously solving these two equations yields the new Nash equilibrium in which each firm produces 15. Thus, the price is 80, each firm's revenue is 1200, and each firm's cost is 750. Each firm then makes a profit of 450.

5. (20 points) A crime is witnessed by Abner and Billy, and they must simultaneously decide whether to report the crime to the police. Each person would like the police to be called, but prefers that the other person makes the call. Specifically, for each player, he receives a payoff of 2 if he reports the crime (no matter what the other player does), a payoff of 0 if neither player reports the crime, and a payoff of 3 if he does not report the crime but the other player reports it.

*Draw, label, and fill out the payoff matrix for this game. Compute the mixed strategy Nash equilibrium of this game.*

**Answer:** The matrix for the game is

		Billy	
		report	don't report
Abner	report	2, 2	2, 3
	don't report	3, 2	0, 0

Because of the symmetry of the game, it is clear that in the mixed strategy Nash equilibrium, each player will be mixing with the same probabilities  $(p, 1 - p)$ . If Billy reports with probability  $p$  and does not report with probability  $1 - p$ , then Abner's expected payoff from reporting is 2, and his expected payoff from not reporting is  $3p + 0(1 - p)$ . Setting  $2 = 3p$ , we have  $p = \frac{2}{3}$ . The MSNE is  $((\frac{2}{3}, \frac{1}{3}), (\frac{2}{3}, \frac{1}{3}))$ .