

The Ohio State University
 Department of Economics
 Midterm Answers

Econ 601
 Winter 2011
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1. (20 points) Convert the following extensive form game into normal form, by drawing the payoff matrix, labeling the strategies corresponding to the rows and columns, and filling in the payoffs.

Answer:

		player 2			
		DF	DG	EF	EG
player 1	AH	0, 0	0, 0	6, 2	6, 2
	AI	2, 6	2, 6	6, 2	6, 2
	BH	4, 4	4, 4	4, 4	4, 4
	BI	4, 4	4, 4	4, 4	4, 4
	CH	5, 1	3, 3	5, 1	3, 3
	CI	5, 1	3, 3	5, 1	3, 3

2. (20 points) Two firms are playing a game of Cournot (quantity) competition. Denoting the quantity chosen by firm 1 as q_1 and the quantity chosen by firm 2 as q_2 , the market price is given by the demand equation

$$p = 120 - 3q_1 - 3q_2.$$

Each firm has a production cost of 30 per unit of output. Each firm's payoff is defined to be its profit.

(a) (5 points) Express the payoff of firm 1 as a function of the strategy profile, (q_1, q_2) .

(b) (15 points) Suppose that firm 1 believes that firm 2 will select a quantity of zero with probability one half, and that firm 2 will select a quantity of 16 with probability one half. Compute firm 1's best response to this belief.

Answer: (a)

$$\begin{aligned} u_1(q_1, q_2) &= (120 - 3q_1 - 3q_2)q_1 - 30q_1 \\ &= (90 - 3q_1 - 3q_2)q_1 \end{aligned}$$

(b) As a function of its beliefs, firm 1's expected payoff is given by

$$\begin{aligned} u_1(q_1, \theta_2) &= \frac{1}{2}[(90 - 3q_1 - 3 \cdot 0)q_1] + \frac{1}{2}[(90 - 3q_1 - 3 \cdot 16)q_1] \\ &= 66q_1 - 3(q_1)^2. \end{aligned}$$

To compute firm 1's best response, differentiate the payoff function with respect to q_1 , set the expression equal to zero, and solve for q_1 .

$$\begin{aligned} \frac{\partial u_1(q_1, \theta_2)}{\partial q_1} &= 66 - 6q_1 = 0 \\ q_1 &= 11. \end{aligned}$$

3. (15 points) Find all of the pure-strategy Nash equilibria of the following game, and indicate your answer below.

		player 2					
		U	V	W	X	Y	Z
player 1	A	7, 9	0, 7	3, $\widehat{11}$	4, 8	8, 1	8, 5
	B	$\widehat{10}$, 2	$\widehat{6}$, 4	$\widehat{9}$, $\widehat{9}$	$\widehat{7}$, 3	$\widehat{12}$, 0	$\widehat{11}$, 7
	C	$\widehat{10}$, 6	$\widehat{6}$, $\widehat{8}$	$\widehat{9}$, $\widehat{8}$	6, 2	$\widehat{12}$, $\widehat{8}$	4, 3
	D	3, $\widehat{12}$	4, 5	6, $\widehat{12}$	0, 6	9, 7	$\widehat{11}$, 1

Answer: Best responses are indicated by "hats" around the corresponding payoffs in the matrix above. We see that there are four cells in which both payoffs have hats, indicating that each player is best-responding to the other player's strategy. The NE are: (B, W) (C, V) (C, W) and (C, Y) .

4. (20 points) Four investors are considering an investment opportunity. Investor 1 has initial holdings of \$1000, investor 2 has initial holdings of \$2000, investor 3 has initial holdings of \$3000, and investor 4 has initial holdings of \$4000. Each investor decides whether to "join" or "not join." An investor choosing "not join" receives a payoff equal to his/her initial holdings. An investor choosing "join" contributes her initial holdings into a fund, and the total fund is divided equally among all those who have joined. For example, if investor 1 chooses "not join" and the other investors choose "join," then investor 1 receives a payoff of 1000 and each of the other investors receive a payoff of $(2000 + 3000 + 4000)/3 = 3000$.

Find a pure-strategy Nash equilibrium of this game, and explain why the strategy profile is a Nash equilibrium. Make sure that I can follow your notation and your explanation.

Answer: Investor 4 has a weakly dominant strategy, NJ (not join). He/she is indifferent only if the other three players choose NJ, but that cannot happen in a NE. (Whenever investor 4 joins, investor 1 will surely join.) Thus, in any NE, investor 4 chooses NJ. Given that investor 4 chooses NJ, similar reasoning shows that investor 3 must choose NJ in any NE. The only two NE are: (NJ, NJ, NJ, NJ) and (J, NJ, NJ, NJ) .

To show that (NJ, NJ, NJ, NJ) is a NE, each player is best responding to the strategy profile of the other players, because his/her payoff from NJ is the same as his/her payoff from J.

To show that (J, NJ, NJ, NJ) is a NE, each player is best responding to the strategy profile of the other players. Investor 1 is best responding because his/her payoff is 1000 for J and NJ. Other investors would receive a lower payoff if they chose J instead of NJ, so NJ is a best response.

5. (25 points) Consider the following game in matrix form.

		player 2		
		L	C	R
player 1	U	5, 6	2, 3	1, 5
	M	8, 2	1, 5	4, 1
	D	0, 3	6, 1	7, 2

(a) Determine the rationalizable strategies for each player by iteratively eliminating dominated strategies. Show your work. Hint: some strategies might be dominated by a mixture.

(b) Compute the mixed-strategy Nash equilibrium of this game.

Answer: (a) For player 2, R is dominated by L. For player 1, a mixture of M and D dominates U. To see this, consider the mixed strategy $(0, p, 1 - p)$. When player 2 chooses L, the mixture gives player 1 a higher expected payoff when

$$8p + 0(1 - p) > 5, \text{ or}$$

$$p > \frac{5}{8}.$$

When player 2 chooses C, the mixture gives player 1 a higher expected payoff when

$$1p + 6(1 - p) > 2, \text{ or}$$

$$p < \frac{4}{5}.$$

If p is between these two bounds (say, two thirds), then the mixture dominates U. The rationalizable strategies for player 1 are $\{M, D\}$, and the rationalizable strategies for player 2 are $\{L, C\}$.

(b) We will look for a mixed strategy NE, with player 1 choosing a strategy of the form $(0, p, 1 - p)$ and player 2 choosing a strategy of the form $(q, 1 - q, 0)$. Thus, player 1 must be indifferent between M and D. Setting the expected payoffs equal to each other, we have

$$\begin{aligned}8q + (1 - q) &= 6(1 - q) \\ q &= \frac{5}{13}.\end{aligned}$$

Similarly, for player 2 to be indifferent between L and C, we have

$$\begin{aligned}2p + 3(1 - p) &= 5p + (1 - p) \\ p &= \frac{2}{5}.\end{aligned}$$

The MSNE is $((0, \frac{2}{5}, \frac{3}{5}), (\frac{5}{13}, \frac{8}{13}, 0))$.