Department of Economics
The Ohio State University
Final Exam Answers–Econ 805
Questions 2,3, and 4

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2. (20 points)
Suppose that an economy with two commodities has the excess demand function given by

$$z(p^1, p^2) = (-1, \frac{p^1}{p^2})$$

for $p^1 \geq 0$ and $p^2 > 0$. [If we have $p^2 = 0$, then excess demand is not well-defined.]

(a) Is this excess demand function consistent with Walras' law? That is, does $p \cdot z(p) = 0$ for all prices for which excess demand is well-defined?
(b) Could this excess demand function have been derived from each consumer maximizing utility subject to a budget constraint, where each utility function is continuous, strictly monotonic, and strictly quasi-concave, and where all initial endowments are strictly positive? Explain carefully.

Answer:
(a) Yes, this is consistent with Walras' Law. Evaluating $p \cdot z(p)$, we have

$$p^1(-1) + p^2(\frac{p^1}{p^2}) = 0.$$

(b) This is impossible. Under all these assumptions, our existence theorem guarantees that a CE exists. Therefore, there is a price vector such that $z(p) \leq 0$. However, $z^2(p) \leq 0$ can only hold if $p^1 = 0$, which violates strict monotonicity. (If a price was zero, there would necessarily be excess demand for that good.)

3. (20 points)
Consider the following economy, with two consumers and two commodities. The economy's aggregate resources are given by the vector, $(1, 1)$. Consumer 1 has the utility function,

$$u_1(x_1) = 2\log(x_1^1) + \log(x_1^2),$$

and consumer 2 has the utility function,

$$u_2(x_2) = \log(x_2^1) + 2\log(x_2^2).$$

(a) Prove that there is no specification of endowments $(\omega_1^1, \omega_1^2, \omega_2^1, \omega_2^2)$ such that $(x_1^1, x_1^2, x_2^1, x_2^2) = (.5, .5, .5, .5)$ is a competitive equilibrium allocation. Be sure to specify which theorems you use in order to prove this result.

*(b) Find **all** initial allocations $(\omega_1^1, \omega_1^2, \omega_2^1, \omega_2^2)$ for which $(x_1^1, x_1^2, x_2^1, x_2^2) = (\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3})$ is a competitive equilibrium allocation. [Note: since anything not allocated to consumer 1 is allocated to consumer 2, you can characterize all such allocations by specifying ω_1^2 as a function of ω_1^1.]*

Answer:

(a) Suppose $(x_1^1, x_1^2, x_2^1, x_2^2) = (.5, .5, .5, .5)$ is a competitive equilibrium allocation for some endowments. The FFTWE implies that this allocation is Pareto optimal, and our theorem characterizing Pareto optimality when utility is differentiable establishes that the marginal rates of substitution (at the PO allocation) must be equal across the two consumers. This contradicts the fact that consumer 1's MRS=2 and consumer 2's MRS=$\frac{1}{2}$.

(b) At the allocation $(x_1^1, x_1^2, x_2^1, x_2^2) = (\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3})$, the two consumers have equal marginal rates of substitution, so our characterization theorem implies that the allocation is Pareto optimal. The SFTWE guarantees that this allocation is a CE allocation for the initial allocation $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3})$. Normalizing $p^2 = 1$, the supporting price ratio must equal consumer 1's marginal rate of substitution,

$$\frac{2x_1^2}{x_1^1},$$

which equals 1 at $(x_1^1, x_1^2) = (\frac{2}{3}, \frac{1}{3})$. Therefore, we have $p^1 = 1$. Any initial endowment vector on the same budget line yields the same solution to the utility maximization problems of the two consumers, so the set of endowments for which $(x_1^1, x_1^2, x_2^1, x_2^2) = (\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3})$ is a CE allocation must satisfy

$$p^1 x_1^1 + x_1^2 = p^1 \omega_1^1 + \omega_1^2.$$

Plugging the price and consumption bundle into the above equation, we have

$$1 = \omega_1^1 + \omega_1^2.$$

4. (30 points)

In the following economy, there are three consumers and two commodities. Consumer 1 has the endowment vector, $\omega_1 = (2, 1)$, and the utility function,

$$u_1(x_1) = \log(x_1^1) + \log(x_1^2).$$

Consumer 2 has the endowment vector, $\omega_2 = (1, 2)$, and the utility function,

$$u_2(x_2) = \log(x_2^1) + \log(x_2^2).$$

Consumer 3 has the endowment vector, $\omega_3 = (1, 1)$, and the utility function,

$$u_3(x_3) = \log(x_3^1) + A\log(x_3^2),$$

where A is a positive constant.

(a) Define a competitive equilibrium for this economy.

(b) Normalizing the price of commodity 2 to be one, $p^2 = 1$, calculate the competitive equilibrium price vector, $(p^1, 1)$, as a function of the parameter, A.

(c) For what values of A will consumer 2 be a net purchaser of commodity 2 at the competitive equilibrium?

Answer:

(a) A CE is a price vector, (p^{*1}, p^{*2}), and an allocation, $(x_1^{*1}, x_1^{*2}, x_2^{*1}, x_2^{*2}, x_3^{*1}, x_3^{*2})$, such that

(x_1^{*1}, x_1^{*2}) solves

$$\begin{aligned} & \max \log(x_1^1) + \log(x_1^2) \\ & \text{subject to} \\ p^{*1}x_1^1 + p^{*2}x_1^2 \leq {} & 2p^{*1} + p^{*2} \\ x_1 \geq {} & 0, \end{aligned}$$

(x_2^{*1}, x_2^{*2}) solves

$$\begin{aligned} & \max \log(x_2^1) + \log(x_2^2) \\ & \text{subject to} \\ p^{*1}x_2^1 + p^{*2}x_2^2 \leq {} & p^{*1} + 2p^{*2} \\ x_2 \geq {} & 0, \end{aligned}$$

(x_3^{*1}, x_3^{*2}) solves

$$\begin{aligned} & \max \log(x_3^1) + A\log(x_3^2) \\ & \text{subject to} \\ p^{*1}x_3^1 + p^{*2}x_3^2 \leq {} & p^{*1} + p^{*2} \\ x_3 \geq {} & 0, \end{aligned}$$

and markets clear:

$$\begin{aligned} x_1^{*1} + x_2^{*1} + x_3^{*1} & \leq 4 \\ x_1^{*2} + x_2^{*2} + x_3^{*2} & \leq 4. \end{aligned}$$

(b) By monotonicity, budget inequalities and market clearing inequalities all hold as equalities. With the normalization, consumer 1's demand is characterized by

$$\begin{aligned} \frac{x_1^2}{x_1^1} &= p^{*1}, \\ p^{*1}x_1^1 + x_1^2 &= 2p^{*1} + 1. \end{aligned}$$

Solving for the demand functions, we have

$$\begin{aligned} x_1^1 &= \frac{2p^{*1} + 1}{2p^{*1}}, \\ x_1^2 &= \frac{2p^{*1} + 1}{2}. \end{aligned}$$

Consumer 2's demand is characterized by

$$\begin{aligned} \frac{x_2^2}{x_2^1} &= p^{*1}, \\ p^{*1}x_2^1 + x_2^2 &= p^{*1} + 2. \end{aligned}$$

Solving for the demand functions, we have

$$\begin{aligned} x_2^1 &= \frac{p^{*1} + 2}{2p^{*1}}, \\ x_2^2 &= \frac{p^{*1} + 2}{2}. \end{aligned}$$

Consumer 3's demand is characterized by

$$\begin{aligned} \frac{x_3^2}{Ax_3^1} &= p^{*1}, \\ p^{*1}x_3^1 + x_3^2 &= p^{*1} + 1. \end{aligned}$$

Solving for the demand functions, we have

$$\begin{aligned} x_3^1 &= \frac{p^{*1} + 1}{(A+1)p^{*1}}, \\ x_3^2 &= \frac{A(p^{*1} + 1)}{(A+1)}. \end{aligned}$$

We can solve for p^{*1} by using the market clearing condition for commodity 2:

$$\frac{2p^{*1} + 1}{2} + \frac{p^{*1} + 2}{2} + \frac{A(p^{*1} + 1)}{(A+1)} = 4.$$

Solving, we have

$$\frac{3A + 5}{5A + 3}.$$

(c) Consumer 2 is a net purchaser of commodity 2 if we have

$$\frac{p^{*1} + 2}{2} > 2.$$

This can be written equivalently as

$$\frac{3A + 5}{5A + 3} = p^{*1} > 2,$$

or

$$3A + 5 > 10A + 6,$$

or

$$7A < -1.$$

Obviously, this is inconsistent with A being positive, so there are no positive values of A for which consumer 2 is a net purchaser of good 2.