

Department of Economics  
The Ohio State University  
Final Exam—Econ 805

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**Directions:** *Answer all questions, and show all work. Answer parts I and II in separate booklets.*

**Part I:** (33 points, questions 1 and 2)

**1. (18 points)**

Consider a duopoly industry. The market inverse demand is given by  $P(q) = 1 - q$ , where  $q = q_1 + q_2$  is total industry output and where  $q_1$  and  $q_2$  are the outputs of firm 1 and firm 2, respectively. Both firms have constant marginal cost functions denoted by  $c_1$  and  $c_2$ . It is common knowledge that  $c_1 = 0$ . Ex ante, firm 2's costs are random, where  $c_2 = 0$  with probability  $\frac{1}{2}$ , and  $c_2 = 1$  with probability  $\frac{1}{2}$ . (Firm 2 observes its own costs, while firm 1 only knows the ex ante distribution.)

(a) Solve for the Cournot-Nash equilibrium of the simultaneous move game. Namely, what are  $q_1$ ,  $q_2(0)$ , and  $q_2(1)$ ? What is the expected price? What are the expected profits of the two firms?

(b) Assume now that firm 1 is the *Stackelberg Leader* (thus, moves first) and firm 2 is the *Follower* (moves second). As in part (a), solve for firms' quantities, profits, and the expected market price.

(c) How would you change your answer in (a) if  $c_2 = 0$  with probability  $\frac{1}{2}$ , and  $c_2 = 0.6$  with probability  $\frac{1}{2}$ ? [The higher cost is 0.6 rather than 1.]

**2. (15 points)**

Consider a duopoly industry with a differentiated (but close substitutes) products,  $X$  and  $Y$ . The demand for  $X$  and  $Y$  are given by

$$\begin{aligned} X &= 1 - P_X + \frac{P_Y}{2}, \text{ and} \\ Y &= 1 - P_Y + \frac{P_X}{2}. \end{aligned}$$

Each firm has a constant marginal cost,  $c \in [0, 1]$ . The two firms are engaged in Bertrand price competition.

(a) Set up the maximization problem of each firm.

(b) Solve for the Bertrand-Nash equilibrium, in terms of  $c$ . What are the prices and quantities of  $X$  and  $Y$ ?

(c) What are the firms' profits? If your answer is that profits are zero, explain why. If your answer is that profits are positive, explain how this is possible, given the fact that the two firms have the same constant marginal cost.

**Part II:** (67 points, questions 3 and 4)

**3. (33 points)**

The following economy has  $n$  consumers and  $k$  commodities. Each consumer's utility function satisfies strict monotonicity, strict quasi-concavity, and continuity. Suppose that  $(p^*, x^*)$  is a competitive equilibrium, and that  $x^{**}$  is a feasible allocation that is **not** Pareto optimal.

For each of the following statements, either prove the statement or provide a counterexample. Carefully explain. [You can use the theorems proven in class without proving them here.]

(a) For **all**  $i$ , we have

$$u_i(x_i^*) \geq u_i(x_i^{**}).$$

(b) At least one consumer prefers her competitive equilibrium bundle to any other consumer's bundle. That is, for **some**  $i$ , we have

$$u_i(x_i^*) \geq u_i(x_h^*) \text{ for all } h.$$

(c)

$$p^* \cdot \sum_{i=1}^n x_i^* \geq p^* \cdot \sum_{i=1}^n x_i^{**}.$$

**4. (34 points)**

Consider the following economy, with two consumers and two commodities. Consumer 1 has the endowment vector,  $\omega_1 = (1, 1)$ , and the utility function,

$$u_1(x_1) = \log(x_1^1) + A \log(x_1^2),$$

where  $A$  is a positive constant. Consumer 2 has the endowment vector,  $\omega_2 = (1, 1)$ , and the utility function,

$$u_2(x_2) = \log(x_2^1) + \log(x_2^2).$$

(a) Define a competitive equilibrium for this economy.

(b) Calculate the competitive equilibrium price vector, as a function of the parameter,  $A$ .

(c) For what value of  $A$  is  $(x_1^1, x_1^2) = (\frac{1}{2}, \frac{3}{2})$  on the contract curve?