Department of Economics
The Ohio State University
Final Exam Answers–Part II–Econ 805

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3. (33 points)
The following economy has n consumers and k commodities. Each consumer's utility function satisfies strict monotonicity, strict quasi-concavity, and continuity. Suppose that (p^*, x^*) is a competitive equilibrium, and that x^{**} is a feasible allocation that is **not** Pareto optimal.

For each of the following statements, either prove the statement or provide a counterexample. Carefully explain. [You can use the theorems proven in class without proving them here.]

(a) For **all** i, we have

$$u_i(x_i^*) \geq u_i(x_i^{**}).$$

(b) At least one consumer prefers her competitive equilibrium bundle to any other consumer's bundle. That is, for **some** i, we have

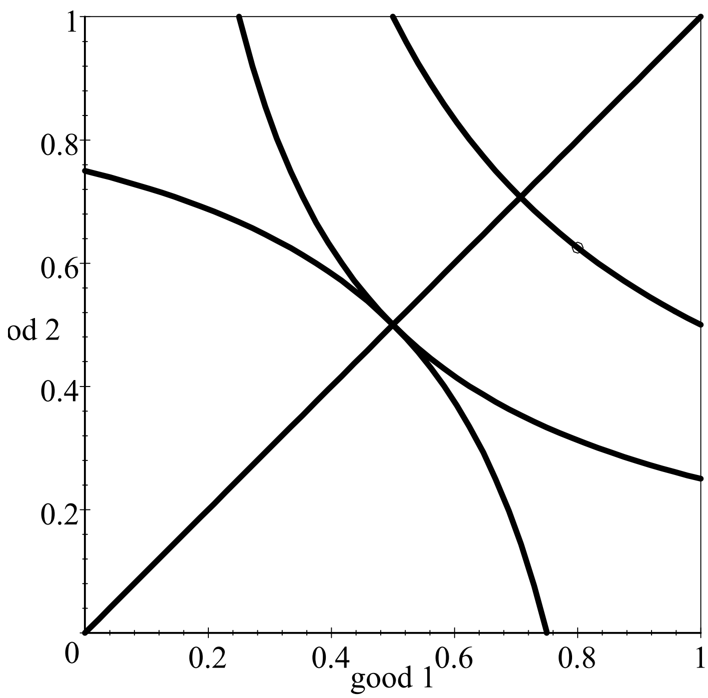
$$u_i(x_i^*) \geq u_i(x_h^*) \text{ for all h.}$$

(c)

$$p^* \cdot \sum_{i=1}^{n} x_i^* \geq p^* \cdot \sum_{i=1}^{n} x_i^{**}.$$

ANSWER:

(a) This statement is false. There must be some allocation that Pareto dominates x^{**}, but it does not have to be x^*. For example, in the following diagram, x^* is the allocation determined by the point (.5, .5), and x^{**} is the allocation determined by the point (.8, .625). Since x^* is on the contract curve, it is a competitive equilibrium allocation, and since x^{**} is not on the contract curve, it is not Pareto optimal. However, consumer 1 prefers x^{**} to x^* .



(b) This statement is true. To see that there is some consumer i such that we have

$$u_i(x_i^*) \geq u_i(x_h^*) \text{ for all h,}$$

we know that each x_i^* solves

$$\begin{aligned} & \max u_i(x_i) \\ \text{s.t. } p^* \cdot x_i &\leq p^* \cdot \omega_i \\ x_i &\geq 0. \end{aligned} \tag{1}$$

Let i be Bill Gates, the consumer with the highest income, evaluated at the equilibrium price vector (the right side of (1)). This consumer can afford any bundle that any other consumer chooses. Since x_i^* solves his utility maximization problem, then it must yield at least as much utility as anyone else's bundle.

(c) This statement is true. Because x^* is a competitive equilibrium allocation, and utility satisfies strict monotonicity, we have

$$p^* \cdot \sum_{i=1}^{n} x_i^* = p^* \cdot \sum_{i=1}^{n} \omega_i. \tag{2}$$

Because x^{**} is a feasible allocation, it follows that, for each commodity j, we have

$$\sum_{i=1}^{n} \omega_i^j \geq \sum_{i=1}^{n} x_i^{j**} \tag{3}$$

From (3), we have

$$p^* \cdot \sum_{i=1}^{n} \omega_i \geq p^* \cdot \sum_{i=1}^{n} x_i^{**}. \tag{4}$$

Combining (2) and (4), the desired inequality follows.

4. (34 points)

Consider the following economy, with two consumers and two commodities. Consumer 1 has the endowment vector, $\omega_1 = (1,1)$, and the utility function,

$$u_1(x_1) = \log(x_1^1) + A\log(x_1^2),$$

where A is a positive constant. Consumer 2 has the endowment vector, $\omega_2 = (1,1)$, and the utility function,

$$u_2(x_2) = \log(x_2^1) + \log(x_2^2).$$

(a) Define a competitive equilibrium for this economy.

(b) Calculate the competitive equilibrium price vector, as a function of the parameter, A.

(c) For what value of A is $(x_1^1, x_1^2) = (\frac{1}{2}, \frac{3}{2})$ on the contract curve?

ANSWER:

(a) A competitive equilibrium is a (normalized) price vector, (p,1), and an allocation, $(x_1^{1*}, x_1^{2*}, x_2^{1*}, x_2^{2*})$, such that

(i) (x_1^{1*}, x_1^{2*}) solves (equalities because of monotonicity)

$$\begin{aligned} & \max \log(x_1^1) + A\log(x_1^2) \\ & \text{subject to} \\ px_1^1 + x_1^2 &= p + 1 \end{aligned}$$

(ii) (x_2^{1*}, x_2^{2*}) solves (equalities because of monotonicity)

$$\begin{aligned} & \max \log(x_2^1) + \log(x_2^2) \\ & \text{subject to} \\ px_2^1 + x_2^2 &= p + 1 \end{aligned}$$

(iii) markets clear

$$\begin{aligned} x_1^{1*} + x_2^{1*} &= 2 \\ x_1^{2*} + x_2^{2*} &= 2. \end{aligned}$$

(b) First, we solve for consumer 1's demand function. Simultaneously solve the MRS condition and the budget equation,

$$\begin{aligned} \frac{x_1^2}{Ax_1^1} &= p \\ px_1^1 + x_1^2 &= p+1 \end{aligned}$$

to derive the demand functions

$$x_1^1 = \frac{p+1}{(1+A)p} \quad \text{and} \quad x_1^2 = \frac{A(p+1)}{(1+A)}.$$

Next we solve for consumer 2's demand function. Simultaneously solve the MRS condition and the budget equation,

$$\begin{aligned} \frac{x_2^2}{x_2^1} &= p \\ px_2^1 + x_2^2 &= p+1 \end{aligned}$$

to derive the demand functions

$$x_2^1 = \frac{p+1}{2p} \quad \text{and} \quad x_2^2 = \frac{(p+1)}{2}.$$

To solve for the equilibrium price, use market clearing for good 2:

$$\frac{A(p+1)}{(1+A)} + \frac{(p+1)}{2} = 2,$$

which yields

$$p = \frac{3+A}{1+3A}.$$

(c) To evaluate whether $(\frac{1}{2}, \frac{3}{2})$ is on the contract curve, it is necessary and sufficient to see whether the marginal rates of substitution for the two consumers are equal (and the allocation must be nonwasteful). Therefore, we have

$$\frac{x_1^2}{Ax_1^1} = \frac{x_2^2}{x_2^1}. \tag{5}$$

Plug consumer 1's bundle, $(\frac{1}{2}, \frac{3}{2})$, into (5). Consumer 2's bundle must be whatever resources are left, $(\frac{3}{2}, \frac{1}{2})$. Thus, equation (5) becomes

$$\frac{3}{A} = \frac{1}{3},$$

which implies $A = 9$.