The Ohio State University
Department of Economics

Econ. 805 Prof. James Peck
Winter 2000

Midterm Examination

Directions: Answer all questions, show all work, and label all diagrams.

(1) **25 points**

A monopolist has many potential customers, represented by the interval, [0,1]. For customer $x \in [0,1]$, consuming one unit of the product provides a benefit of v, but the customer must pay a transportation cost of tx to purchase the product, where t is a positive parameter. The monopolist has a constant marginal cost, c, and no fixed costs.

Assume that customers are uniformly distributed across the interval, [0,1], according to the density function, $f(x) = 1$. In other words, if everyone in the interval [0,x] purchases, the monopolist sells x units of the product. Also assume the following:

assumption 1: $(v-c)/2 \leq t \leq (v-c)$.

(A) Suppose that the monopolist must charge a constant price, p, but that it can also offer to pay a "transportation subsidy" to its customers, as a function of their location, denoted by s(x). That is, a customer at "location" x that pays the price, p, and incurs the transportation cost, tx, also receives a payment from the monopolist, s(x). Therefore, her utility would be $(v - p - tx + s(x))$ if she purchases the product, and zero if she does not purchase. What are the profit maximizing values of p, s(x), and monopoly profits? (Hint: think of price discrimination)

(B) Suppose that the monopolist must charge a constant price, p, with no transportation subsidy. Therefore, a customer at location x would receive utility of $(v - p - tx)$ if she purchases the product, and zero if she does not purchase. Calculate the profit maximizing value of p, the total quantity sold, and monopoly profits.

(2) **25 points**

The following two firms are engaged in quantity competition. For i = 1,2, firm i's total cost of producing y_i units of output is $c_i y_i$. (Thus, the firms may have different marginal costs.) Letting Y denote total output of the two firms, $Y = y_1 + y_2$, the market (inverse) demand function is given by: $p(Y) = 1 - Y$.

Calculate the Cournot-Nash equilibrium quantities produced by the two firms, and the equilibrium price. (Assume that c_1 and c_2 are such that both firms produce a positive quantity.)

(3) **25 points**

This question concerns a pure exchange economy with K commodities and n consumers, where all utility functions are strictly monotonic, strictly quasi-concave, and continuous.

For each of the following statements, if the statement is true, then prove it. If the statement is false, then provide a counterexample. A carefully drawn, labeled, and explained Edgeworth Box diagram is enough for a counterexample.

(A) If x and x** are strongly Pareto optimal, then* $u_i(x_i^*) \geq u_i(x_i^{**})$ ***for all i.***

(B) If x and x** are strongly Pareto optimal, then* $u_i(x_i^*) \geq u_i(x_i^{**})$ ***for some i.***

(C) If $u_i(x_i^*) = u_i(x_i^{**})$ ***for all i****, and if x** $\neq$ *x**, then x** ***cannot*** *be strongly Pareto optimal.*

(4) **25 points**

Consider the following pure exchange economy with 3 consumers and two commodities. Consumer 1 has the endowment vector (1,1) and the utility function

$$u_1(x_1^1, x_1^2) = \log(x_1^1) + \log(x_1^2) .$$

Consumer 2 has the endowment vector (1,0) and the utility function

$$u_2(x_2^1, x_2^2) = 2\log(x_2^1) + \log(x_2^2) .$$

Consumer 3 has the endowment vector (0,1) and the utility function

$$u_3(x_3^1, x_3^2) = \log(x_3^1) + 2\log(x_3^2) .$$

(A) (10 points) Define a competitive equilibrium for this economy.

(B) (15 points) Calculate the competitive equilibrium price and allocation.