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Department of Economics

Econ. 805 Prof. James Peck
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Midterm Questions and Answers

(1) **25 points**

A monopolist has many potential customers, represented by the interval, [0,1]. For customer $x \in [0,1]$, consuming one unit of the product provides a benefit of v, but the customer must pay a transportation cost of tx to purchase the product, where t is a positive parameter. The monopolist has a constant marginal cost, c, and no fixed costs.

Assume that customers are uniformly distributed across the interval, [0,1], according to the density function, $f(x) = 1$. In other words, if everyone in the interval [0,x] purchases, the monopolist sells x units of the product. Also assume the following:

assumption 1: $(v-c)/2 \leq t \leq (v-c)$.

(A) Suppose that the monopolist must charge a constant price, p, but that it can also offer to pay a "transportation subsidy" to its customers, as a function of their location, denoted by s(x). That is, a customer at "location" x that pays the price, p, and incurs the transportation cost, tx, also receives a payment from the monopolist, s(x). Therefore, her utility would be (v - p - tx + s(x)) if she purchases the product, and zero if she does not purchase. What are the profit maximizing values of p, s(x), and monopoly profits? (Hint: think of price discrimination)

(B) Suppose that the monopolist must charge a constant price, p, with no transportation subsidy. Therefore, a customer at location x would receive utility of (v - p - tx) if she purchases the product, and zero if she does not purchase. Calculate the profit maximizing value of p, the total quantity sold, and monopoly profits.

ANSWER: (A) The monopolist can **perfectly** price discriminate, charging everyone their full willingness to pay. The highest net payment, $p - s(x)$, that consumer x is willing to make is $v - tx$. The monopolist can extract this payment by setting $p - s(x) = v - tx$. The simplest way is to set $p=v$ and $s(x) = tx$. The additional profits received from consumer x is: $v - tx - c$. Since $v - c > t$ (assumption 1), it follows that the monopolist wants to sell to everyone, including the consumer located at $x = 1$. In other words, charge people their valuation, v, but pay their transportation costs. The key is noticing that the ability to tailor the transportation subsidy to the individual allows perfect price discrimination.

(B) We will set up an expression for the monopolist's profit, as a function of the price, p. For an interior solution, an interval of consumers [0,x] will purchase, where x solves: $v - p - tx = 0$, so we have $x(p) = (v - p)/t$. Thus, profits are: $\pi(p) = (p - c)(v - p)/t$. Setting the derivative with respect to p equal to zero and solving for p, we have:

$(p - c)(-1/t) + (v-p)/t = 0$, which implies $p = (v + c)/2$. Therefore, the quantity sold is $x(p) = (v - p)/t = (v - (v+c)/2)/t = (v - c)/(2t)$. Notice that assumption 1 guarantees that $x(p) < 1$, so we were correct to look for an interior solution. Profits are: $(p-c)x = [(v+c)/2 - c](v - c)/2t = (v - c)^2 / 4t$.

(2) **25 points**

The following two firms are engaged in quantity competition. For i = 1,2, firm i's total cost of producing y_i units of output is $c_i y_i$. (Thus, the firms may have different marginal costs.) Letting Y denote total output of the two firms, $Y = y_1 + y_2$, the market (inverse) demand function is given by: $p(Y) = 1 - Y$.

Calculate the Cournot-Nash equilibrium quantities produced by the two firms, and the equilibrium price. (Assume that c_1 and c_2 are such that both firms produce a positive quantity.)

ANSWER: Firm 1's profit function is: $\pi_1 = (1 - y_1 - y_2)y_1 - c_1 y_1$. Differentiating with respect to y_1 and setting it equal to zero, we have firm 1's reaction function, $1 - y_1 - y_2 - c_1 + y_1 (-1) = 0$, which can be simplified to:

$$y_1 = (1 - y_2 - c_1)/2 .$$

Performing the same operations for firm 2 (or by noticing the symmetry of the problem and reversing subscripts 1 and 2), we have firm 2's reaction function:

$$y_2 = (1 - y_1 - c_2)/2 .$$

Substituting the right side of firm 1's reaction function for y_1 in firm 2's reaction function, we have

$$y_2 = (1 - [(1 - y_2 - c_1)/2] - c_2)/2 .$$

Solving for y_2, we have: $y_2 = (1 + c_1 - 2 c_2)/3$. Plugging this expression into firm 1's reaction function, we have: $y_1 = (1 + c_2 - 2 c_1)/3$. The equilibrium price is: $p = 1 - y_1 - y_2 = 1 - (1 + c_2 - 2 c_1)/3 - (1 + c_1 - 2 c_2)/3$. This can be simplified to: $(1 + c_1 + c_2)/3$.

(3) **25 points**

This question concerns a pure exchange economy with K commodities and n consumers, where all utility functions are strictly monotonic, strictly quasi-concave, and continuous.

For each of the following statements, if the statement is true, then prove it. If the statement is false, then provide a counterexample. A carefully drawn, labeled, and explained Edgeworth Box diagram is enough for a counterexample.

(A) If x and x** are strongly Pareto optimal, then* $u_i(x_i^*) \geq u_i(x_i^{**})$ ***for all i.***

(B) If x and x** are strongly Pareto optimal, then* $u_i(x_i^*) \geq u_i(x_i^{**})$ ***for some i.***

(C) If $u_i(x_i^*) = u_i(x_i^{**})$ ***for all i****, and if x* ≠ x**, then x** ***cannot*** *be strongly Pareto optimal.*

ANSWER: (A) This statement is **false**. Starting with just about any two Pareto optimal points will be a counterexample. For example, two consumers, two goods, Cobb-Douglas utility functions, and an aggregate endowment of 1 unit of each good. The contract curve is the diagonal of the Edgeworth box, so let x* be given by $x_1^* = (1/4, 1/4)$ and $x_2^* = (3/4, 3/4)$, and let x** be given by $x_1^{**} = (3/4, 3/4)$ and $x_2^{**} = (1/4, 1/4)$. The statement is false, since consumer 1 strictly prefers x_1^{**} to x_1^*.

(B) This statement is **true**. If not, then **every** consumer strictly prefers x**, but then x** would Pareto dominate x*, contradicting the Pareto optimality of x*.

(C) This statement is **true**. If $u_i(x_i^*) = u_i(x_i^{**})$, then the two bundles are on the same indifference curve. Consider instead the allocation, x′, in which everyone receives the midpoint of the line segment connecting x* and x**, x′ = (x* + x**)/2. Since x* and x** are feasible, then x′ is feasible. If $x_i^* = x_i^{**}$, then obviously this consumer receives the same utility under x_i^* and x_i', since it is the same bundle. For all consumers i such that $x_i^* \neq x_i^{**}$, and we know there is at least one such consumer, then consumer i strictly prefers x_i', because the line segment cuts through the upper contour set, by strict quasi-concavity. Therefore, x′ Pareto dominates x*, so x* cannot be Pareto optimal.

(4) **25 points**

Consider the following pure exchange economy with 3 consumers and two commodities. Consumer 1 has the endowment vector (1,1) and the utility function

$$u_1(x_1^1, x_1^2) = \log(x_1^1) + \log(x_1^2).$$

Consumer 2 has the endowment vector (1,0) and the utility function

$$u_2(x_2^1, x_2^2) = 2\log(x_2^1) + \log(x_2^2).$$

Consumer 3 has the endowment vector (0,1) and the utility function

$$u_3(x_3^1, x_3^2) = \log(x_3^1) + 2\log(x_3^2).$$

(A) (10 points) Define a competitive equilibrium for this economy.

(B) (15 points) Calculate the competitive equilibrium price and allocation.

ANSWER: (A) A competitive equilibrium is a price vector, (p_1 , p_2), and an allocation, $(x*_1^1, x*_1^2, x*_2^1, x*_2^2, x*_3^1, x*_3^2)$, such that:

1) x_1* solves $\max \log(x_1^1) + \log(x_1^2)$

Subject to $p_1 x_1^1 + p_2 x_1^2 = p_1 + p_2$ (Equality due to monotonicity),

$x_1 \geq 0$,

2) x_2* solves $\max 2\log(x_2^1) + \log(x_2^2)$

Subject to $p_1 x_2^1 + p_2 x_2^2 = p_1$ (Equality due to monotonicity),

$x_2 \geq 0$,

3) x_3* solves $\max \log(x_3^1) + 2\log(x_3^2)$

Subject to $p_1 x_3^1 + p_2 x_3^2 = p_2$ (Equality due to monotonicity),

$x_3 \geq 0$,

4) market clearing:

$$x_1^1 + x_2^1 + x_3^1 = 2,$$
$$x_1^2 + x_2^2 + x_3^2 = 2.$$

(B) To make life easier, we will normalize the price of good 2 to be 1, so the price vector is (p,1). We first derive the demand functions for the three consumers. The two relevant equations are the budget equation and the marginal rate of substitution equation (MRS = p). For consumer 1, we have:

$$\frac{x_1^2}{x_1^1} = p \quad \text{and} \quad px_1^1 + x_1^2 = p + 1.$$

Solving for the demands, we have: $x_1^1 = (p+1)/2p$ and $x_1^2 = (p+1)/2.$

For consumer 2, we have: $\dfrac{2x_2^2}{x_2^1} = p$ and $px_2^1 + x_2^2 = p$

Solving for the demands, we have: $x_2^1 = 2/3$ and $x_2^2 = p/3.$

For consumer 3, we have: $\dfrac{x_3^2}{2x_3^1} = p$ and $px_3^1 + x_3^2 = 1$

Solving for the demands, we have: $x_3^1 = 1/3p$ and $x_3^2 = 2/3.$

Now we pick one of the market clearing equations to solve for p. Good 2 is easier, so we have: (p+1)/2 + p/3 + 2/3 = 2. Solving for p, we get p = 1. (We could have guessed that, because of the symmetry between goods 1 and 2, but it is better to solve it.)

Now we plug p = 1 to get the final allocation:
$(x*^1_1, x*^2_1, x*^1_2, x*^2_2, x*^1_3, x*^2_3)$ = (1 , 1 , 2/3 , 1/3 , 1/3 , 2/3).