

Department of Economics
The Ohio State University
Econ 805–Double Marginalization Problem

Prof James Peck

A monopolist manufacturer produces a product at constant marginal cost, c , and chooses a wholesale price (a price at which retailers can order the product from the manufacturer), p^w . Given the wholesale price, p^w , each of n retailers can order as much of the product they want, where the quantity ordered by retailer i is given by q_i and the total quantity is given by Q . All output ordered by retailers is sold to consumers, with the retail price, p^r , determined by the supply, Q , and the demand curve, $D(p^r) = a - bp^r$. In other words, the manufacturer selects the wholesale price and the retail market is determined by quantity competition. Retailers have no costs, except for the wholesale price paid to the manufacturer for each unit.

(a) Solve for the equilibrium wholesale price, retail price, quantity ordered by each firm, and profits for each retailer and the manufacturer.

(b) How does the equilibrium retail price vary with n , and is the price greater than or less than the price a vertically integrated monopolist would charge? (A vertically integrated firm is a firm that acts as both producer and retailer.) Notice that n basically measures the degree of competition in the retail market. Provide some intuition for this result.

(c) Suppose that there is only one retailer, $n = 1$, and suppose that the manufacturer can offer the retailer a general nonlinear schedule specifying the payment made to the manufacturer as a function of the quantity ordered (as a take-it-or-leave-it offer). What profit would the manufacturer receive from its optimal contract?

Answer: (a) The optimal wholesale price depends on the solution to the retailer subgame, so we will solve that first, for a fixed wholesale price, and then go back to the monopolist's problem. Inverting the demand equation, we have

$$p^r = \frac{a - Q}{b},$$

from which we derive the profit function for retailer j ,

$$\pi_j = \frac{a - Q}{b} q_j - p^w q_j \tag{1}$$

Differentiating (1) with respect to q_j , we have the first order condition

$$\frac{a - Q}{b} + q_j \left(-\frac{1}{b}\right) - p^w = 0.$$

Now we can impose symmetry to determine the supply of each retailer, q .

$$\frac{a - nq - q}{b} = p^w,$$

or

$$q = \frac{a - bp^w}{n + 1}. \quad (2)$$

To finish the problem, we must determine the equilibrium wholesale price. The monopolist's profits, taking into account how retail quantities depend on the wholesale price, are given by

$$\pi^m = (p^w - c)Q = \frac{n(p^w - c)(a - bp^w)}{n + 1}. \quad (3)$$

Differentiating (3) with respect to p^w and simplifying, we have the first order condition,

$$(p^w - c)(-b) + (a - bp^w) = 0,$$

from which we derive

$$p^w = \frac{a + bc}{2b}. \quad (4)$$

Substituting (4) into (2), we find the quantities,

$$q = \frac{a - b\frac{a+bc}{2b}}{n + 1} = \frac{a - bc}{2(n + 1)} \quad (5)$$

$$Q = \frac{n(a - bc)}{2(n + 1)} \quad (6)$$

The retail price is

$$p^r = \frac{a - \frac{n(a-bc)}{2(n+1)}}{b} = \frac{a}{b} - \frac{n(a - bc)}{2b(n + 1)}. \quad (7)$$

From (1), we compute the profits of retailer j as

$$\pi_j = \frac{(a - bc)^2}{4b(n + 1)^2}. \quad (8)$$

From (3), we compute the monopolist's profits as

$$\pi^m = \frac{n(a - bc)^2}{4b(n + 1)}. \quad (9)$$

(b) First, notice that for demand to be positive when the retail price equals the marginal production cost, we have $a - bc > 0$. This condition must hold in order for the market to be viable. From (7), it follows that

$$\frac{\partial p^r}{\partial n} = -\frac{(a - bc)}{2b(n + 1)^2} < 0.$$

In other words, more retailer competition leads to a lower retail price. This makes sense when you realize that the wholesale price does not depend on n .

In the limit, as n approaches infinity, the retail price equals the wholesale price, and retailers receive zero profits. A vertically integrated monopolist would solve

$$\max_Q \left(\frac{a-Q}{b} \right) Q - cQ, \quad (10)$$

yielding the solution,

$$Q = \frac{(a-bc)}{2}, p^r = \frac{a+bc}{2b}. \quad (11)$$

Thus, in the limit, as n approaches infinity, the retail price, total quantity, and monopolist's profits are identical to the vertically integrated case. With a small number of retailers, however, the retail price is greater than the price a vertically integrated monopolist would charge. This phenomenon is called "double marginalization." The monopolist's wholesale price reflects a monopoly markup over marginal cost, and imperfect retail competition creates another markup of the retail price over the wholesale price.

(c) With $n = 1$, and when the manufacturer's revenues must be $p^w Q$, the monopolistic manufacturer and monopolistic retailer do not cancel each other's power, but instead create a situation that is worse than dealing with just one monopolist. However, the double marginalization problem can be avoided if the manufacturer extracts a lump sum "franchise fee," and then offers the product at a marginal wholesale price equal to c . Let the payment from the retailer to the manufacturer be $F + cQ$. Then the retailer's problem is

$$\max_Q \left(\frac{a-Q}{b} \right) Q - F - cQ. \quad (12)$$

Since the solution to (12) is the same as the solution to (10), the retailer chooses the vertically integrated monopoly quantity and retail price, given by (11). The highest value of F that the retailer will accept will yield the retailer zero profits:

$$\begin{aligned} F &= \left(\frac{a-Q}{b} \right) Q - cQ = \left(\frac{a - \frac{(a-bc)}{2}}{b} \right) \frac{(a-bc)}{2} - c \frac{(a-bc)}{2} \\ &= \frac{(a-bc)^2}{4b}. \end{aligned} \quad (13)$$

Thus, the monopolist receives the same profits that the vertically integrated monopolist would receive. Since it is impossible to imagine a higher profit, this contract must be optimal. Intuitively, the manufacturer sells the monopoly rights to the retailer and allows the retailer to order output at cost.