Department of Economics
The Ohio State University
Final Exam Answers–Econ 805

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1. (30 points)

A pedestrian slips and injures himself on a sidewalk. There are n people nearby who observe the accident. The pedestrian needs immediate medical attention, but he will be fine if at least one of the n bystanders dials 911 on her cell phone. Simultaneously and independently, each of the n bystanders must decide whether or not to call 911. If player i calls for help, she receives a payoff of $v - c$, where v is the benefit of knowing that the pedestrian will be OK and c is the personal cost of making the call. If player i does not call for help, her payoff is v if at least one of the other bystanders calls for help, and her payoff is 0 if no one calls.

(a) (15 points) Find the symmetric mixed-strategy Nash equilibrium of this game.

(b) (15 points) Derive an expression for the probability that the pedestrian receives medical attention, as a function of n. Is he better off with a large or a small crowd of witnesses?

Answer:

(a) In a symmetric MSNE, each bystander will call 911 with the same probability, which I will denote by $1 - p$, so the probability that a bystander does not call is given by p. In order to be willing to mix between either action, a bystander must be indifferent between the two choices. That is, her expected payoff from calling, given by

$$v - c,$$

must equal her expected payoff from not calling, given by

$$\begin{aligned} & pr(\text{at least one of } n-1 \text{ others calls}) \cdot v \\ = & [1 - pr(\text{none of } n-1 \text{ others calls})] \cdot v \\ = & v - vp^{n-1}. \end{aligned}$$

Equating these two payoffs and solving for p, we have

$$p = \left(\frac{c}{v}\right)^{1/(n-1)}.$$

Bystanders call with probability $1 - \left(\frac{c}{v}\right)^{1/(n-1)}$ and do not call with probability $\left(\frac{c}{v}\right)^{1/(n-1)}$.

(b) The probability that the pedestrian receives help is $[1 - pr(\text{none of the } n \text{ bystanders calls})]$, which is given by

$$\begin{aligned} & 1 - p^n \\ = \;& 1 - \left(\frac{c}{v}\right)^{n/(n-1)}. \end{aligned}$$

The exponent is decreasing in n, and because $c < v$, it follows that $\left(\frac{c}{v}\right)^{n/(n-1)}$ is increasing in n. Therefore, the entire expression is decreasing in n. The pedestrian is better off with a smaller crowd.

2. (30 points)

Consider the following game, based on a sequential-move, first-price auction with two bidders. First, nature draws player 1's valuation, v_1, from a uniform distribution over the unit interval $[0, 1]$. Player 1 observes v_1, then makes a bid, b_1. Next, nature draws player 2's valuation, v_2, from a uniform distribution over the unit interval $[0, 1]$. Assume that v_1 and v_2 are drawn independently. Player 2 observes her valuation **and player 1's bid**, but not player 1's valuation, after which she makes a bid, b_2. Whichever player i makes the higher bid wins the auction, receiving a payoff of $v_i - b_i$, and the other player receives a payoff of zero. In case of a tie, assume that player 2 wins the auction.

(a) (15 points) Find a pure-strategy weak perfect Bayesian equilibrium (WPBE) for this game. Remember to fully specify the strategies and player 2's beliefs. Explain why the conditions for a WPBE are satisfied.

(b) (15 points) Find a (Bayesian) Nash equilibrium in which player 2 receives a higher ex ante expected payoff than she receives in the WPBE you found in part (a).

Answer:

(a) In a WPBE, player 2's bid must be sequentially rational, given her valuation, her observation of player 1's bid, and her beliefs about player 1's valuation. (Player 1's valuation determines the node in player 2's information set.) What makes this problem easy is the fact that player 2's payoff depends on player 1's bid but does not depend on player 1's valuation, so beliefs do not affect player 2's sequentially rational bid. Player 2 will match player 1's bid when $v_2 \geq b_1$ and she will bid less than player 1's bid otherwise (in this case, the exact bid is irrelevant as long as it is below player 1's bid). Here is player 2's strategy:

$$\begin{aligned} b_2(b_1, v_2) &= b_1 \quad \text{if} \quad v_2 \geq b_1 \\ b_2(b_1, v_2) &= v_2 \quad \text{if} \quad v_2 < b_1. \end{aligned}$$

Now that we have determined player 2's strategy, we see that player 1 will win the auction whenever his bid is greater than v_2. We can express player 1's payoff as a function of his bid:

$$\begin{aligned}\pi_1(b_1) &= pr(v_2 < b_1)(v_1 - b_1) \\ &= (b_1)(v_1 - b_1).\end{aligned}$$

Differentiating with respect to b_1, setting the expression equal to zero, and solving for b_1, we have player 1's strategy:

$$b_1(v_1) = \frac{v_1}{2}.$$

Now for player 2's beliefs. Based on player 1's strategy, we can use Bayes' rule to conclude that player 1's valuation is always exactly twice his bid, as long as his bid is not more than one-half. That is, we have for $b_1 \leq \frac{1}{2}$,

$$\begin{aligned}\mu(v_1|b_1, v_2) &= 1 \quad \text{for} \quad v_1 = 2b_1 \\ \mu(v_1|b_1, v_2) &= 0 \quad \text{for} \quad v_1 \neq 2b_1.\end{aligned}$$

Bids greater than one-half are off the equilibrium path, so any beliefs about player 1's valuation would be consistent.

(b) Here is a BNE in which player 2's strategy is a best response to player 1's strategy, but in which player 2's strategy is not sequentially rational:

$$\begin{aligned}b_1(v_1) &= 0 \quad \text{for all } v_1, \\ b_2(b_1, v_2) &= b_1 \text{ for all } b_1, v_2.\end{aligned}$$

In words, player 1 always bids zero and player 2 always matches player 1's bid. Given player 2's strategy, player 1 never wins the auction, so bidding zero is a best response. Given player 1's strategy, player 2 always wins the auction at a price of zero, so her strategy of matching player 1's bid is a best response. (It is not sequentially rational since it would not be optimal to match a bid greater than v_2.) Clearly this BNE yields player 2 a higher payoff than the answer in part (a).

3. (40 points)

For the following Cournot duopoly, each of the two firms has zero cost (type L) with probability $\frac{2}{3}$, and constant marginal cost of c (type H) with probability $\frac{1}{3}$. Assume that before choosing its output, each firm observes its own cost type but not the other firm's cost type, and assume that cost realizations are independent across firms. The market price is given by the inverse demand function,

$$p = 1 - q_1 - q_2.$$

(a) (25 points) Assuming that c is small enough so that type H firms produce positive output in equilibrium, find the Bayesian Nash Equilibrium of this game.

(b) (15 points) For what values of c will type H firms produce zero output in equilibrium? Find the BNE that results when c is high enough so that type H firms produce zero output.

Answer:

(a) A BNE is a quantity for each firm for each cost type: $((q_1^L, q_1^H), (q_2^L, q_2^H))$. When firm 1 is type L, its payoff function is

$$\pi_1^L = \frac{2}{3}(1 - q_1^L - q_2^L)q_1^L + \frac{1}{3}(1 - q_1^L - q_2^H)q_1^L.$$

The expectation is over the unobserved cost of firm 2. Differentiating with respect to q_1^L, equating the expression to zero, and solving, we have firm 1,L's best response function:

$$q_1^L = \frac{1}{2} - \frac{q_2^L}{3} - \frac{q_2^H}{6}. \tag{1}$$

When firm 1 is type H, its payoff function is

$$\pi_1^H = \frac{2}{3}(1 - q_1^H - q_2^L)q_1^H + \frac{1}{3}(1 - q_1^H - q_2^H)q_1^H - cq_1^H.$$

The expectation is over the unobserved cost of firm 2. Differentiating with respect to q_1^H, equating the expression to zero, and solving, we have firm 1,H's best response function:

$$q_1^H = \frac{1-c}{2} - \frac{q_2^L}{3} - \frac{q_2^H}{6}. \tag{2}$$

Note: this computation assumes that we have an interior solution with $q_1^H > 0$.

Because of the symmetry of the problem, we can impose the conditions

$$q_1^L = q_2^L = q^L \quad \text{and} \quad q_1^H = q_2^H = q^H.$$

From (1), we have

$$\begin{aligned} \frac{4q^L}{3} &= \frac{1}{2} - \frac{q^H}{6} \\ q^L &= \frac{3}{8} - \frac{q^H}{8}. \end{aligned} \tag{3}$$

Substituting (3) into (2), we have

$$\begin{aligned} q^H &= \frac{1-c}{2} - \frac{1}{3}\left(\frac{3}{8} - \frac{q^H}{8}\right) - \frac{q_2^H}{6} \\ q^H\left(1 - \frac{1}{24} + \frac{1}{6}\right) &= \frac{1}{2} - \frac{c}{2} - \frac{1}{8} \\ q^H\left(\frac{9}{8}\right) &= \frac{3}{8} - \frac{c}{2} \\ q^H &= \frac{1}{3} - \frac{4}{9}c. \end{aligned} \tag{4}$$

Substituting (4) into (3) and simplifying yields

$$q^L = \frac{1}{3} + \frac{1}{18}c.$$

(b) From (4), we see that a type H firm supplies a positive quantity whenever $c < \frac{3}{4}$. When $c = \frac{3}{4}$, a type H firm supplies zero output, and a type L firm supplies

$$q^L = \frac{1}{3} + \frac{1}{18}c = \frac{1}{3} + \frac{1}{18} \cdot \frac{3}{4} = \frac{3}{8}.$$

(You could also see this by substituting $q^H = 0$ into (3)).

When $c > \frac{3}{4}$, a type H firm is at a corner solution and strictly prefers to supply zero output, so the BNE is the same as it is for the case $c = \frac{3}{4}$. We have $q^L = \frac{3}{8}$ and $q^H = 0$.