

Equilibrium in Dynamic Games

In extensive form games, a strategy is a specification of what action a player chooses at each information set he/she controls. A Nash equilibrium of an extensive form game is a profile of strategies such that no player can receive a higher expected payoff with any other strategy, just as for normal form games.

The set of N.E. of an extensive form game is always the same as for its corresponding normal form game.

However, the explicit timing of play raises a new issue: when is a strategy credible?

Consider the game Γ_1 , which has the following normal form representation:

		player 2	
		left	right
player 1	top	2, 1	0, 0
	bottom	1, 2	1, 2

Check that (top, left) and (bottom, right) are both Nash equilibria for both the matrix and the game tree.

For the matrix game, it is true that player 2 is playing a weakly dominated strategy in (bottom, right), but it is hard to argue that player 2's strategy is not rational.

For the game tree, we can see that, whenever player 2 is called upon to move, a rational player would choose left. Player 1 should anticipate this, and choose top.

The (bottom, right) Nash equilibrium involves a threat by player 2 to choose right, but that threat is not credible.

Backwards induction for games of perfect information: Starting with each node just before the terminal nodes, assign the node the payoff vector that the player controlling that node would choose. This eliminates that player's choice and converts the node into the terminal node of a reduced game. By applying this procedure to the new game, we eventually assign a payoff to the initial node. This procedure constructs a Nash equilibrium in which all moves are credible.

For example, see game Γ_3 below.

Zermelo's theorem: Every finite game of perfect information has a pure strategy Nash equilibrium that can be derived through backwards induction. If no player has the same payoffs at any two terminal nodes, only one equilibrium can be derived in this manner.

For more general games than games of perfect information, sequential rationality is imposed by the concept of *subgame perfection*.

Def: A subgame of an extensive form game is a subset of the game tree having the following properties:

1. It begins with an information set containing a single node, contains all successor nodes (direct and indirect), and only contains these nodes.
2. If a node is in the subgame, then all other nodes belonging to the same information set is in the subgame. (You can't cut through an information set.)

Note: A subgame of an extensive form game itself satisfies the definition of a game. A strategy for the original game *induces* a strategy for the subgame.

(draw some pictures of subgames and non-subgames)

Def: A profile of strategies in an extensive form game is a *subgame perfect Nash equilibrium* if it induces a Nash equilibrium in every subgame (including the original game).

A related issue is “rules vs. discretion.” It could be beneficial to commit to do something that you will not want to do, should the situation arise.

If the technology to make commitments exists, then that changes the game and could make a threat credible.

For example, in game Γ_1 , (2,1) is the unique SPNE payoff. But if player 2 first commits not to choose left, then that arc is removed from the tree before player 1 has a chance to move. Player 1 now chooses “b” and the payoffs are (1,2).