

The game Γ_2 has two WPBE outcomes.

Equilibrium 1:

player 1 plays R (if nature moves left),

player 1 plays r (if nature moves right).

player 2 plays left.

Beliefs at the only multiple node information set are determined by Bayes' rule: $\mu(x_1) = 1$, and $\mu(x_2) = 0$.

It is easy to see that player 1's strategy is sequentially rational, and that player 2's strategy is sequentially rational, given the beliefs.

Equilibrium 2:

player 1 plays L (if nature moves left),

player 1 plays r (if nature moves right).

player 2 plays right.

Beliefs at the only multiple node information set are not determined by Bayes' rule, because the probability of reaching the information set is zero, given the strategies. Some beliefs supporting the equilibrium are: $\mu(x_1) = 0$, and $\mu(x_2) = 1$.

To see that strategies are sequentially rational, given the beliefs, notice that player 2 is rationally choosing right given her beliefs, since a payoff of 0 is better than a payoff of -1 . Player 1 rationally chooses L, because 2 is better than -1 , and choosing r strictly dominates ℓ .

Notes:

1. Many beliefs are consistent and support the outcome of equilibrium 2.
2. It is not sensible to think that player 1 will choose a dominated strategy when nature moves right, and indeed, he does not. However, off the equilibrium path it is consistent with the definition of WPBE for player 2 to believe that player 1 is choosing a dominated strategy.

Further refinements:

Attempts to strengthen WPBE have been made, such as PBE and sequential equilibrium. However, it is often hard to check whether we have a (PBE or) sequential equilibrium, and these refinements do not always rule out the “bad” equilibria.

The problem is that beliefs about information sets that are “off the equilibrium path” cannot be pinned down by Bayes’ rule, but the beliefs are crucial in determining what is on and off the equilibrium path.

Forward Induction

Suppose that before playing the following version of the battle of the sexes, player 1 has an opportunity to publicly *burn a dollar*.

		player 2	
		left	right
player 1	top	3, 1	0, 0
	bottom	0, 0	1, 3

Will player 1 burn the dollar if he does not intend to play top?

Since the answer is no, player 1 has the opportunity to signal this intention, effectively ruling out equilibria where he plays bottom.

Given that player 1 can achieve a payoff of 2 by burning a dollar, what intention is signalled by not burning a dollar?