

Department of Economics  
The Ohio State University  
Econ 805–Homework #1  
due Thursday, January 22

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1. A consumer's *offer curve* is the locus of tangencies between the budget lines and indifference curves as relative prices change. (See Mas-Colell, pp. 517-518.)

(i) For a two-person, Edgeworth Box economy, explain why any intersection of the two offer curves is a competitive equilibrium.

(ii) What is the equation for the following consumer's offer curve?

$$u(x^1, x^2) = \log(x^1) + \log(x^2),$$

$$(\omega^1, \omega^2) = (1, 1).$$

2. Using an Edgeworth box diagram, provide a counterexample to the second welfare theorem when utility functions are continuous and strictly monotonic, but not necessarily quasi-concave. That is, find an example of a Pareto optimal allocation that cannot be supported as a competitive equilibrium allocation (for any redistribution of initial endowments).

3. Suppose there is no market for some commodity (say commodity  $K$ ). Since consumers cannot trade commodity  $K$ , then everyone consumes his/her endowment of that good. A competitive equilibrium is a price vector for the  $K-1$  tradable goods,  $p = (p^1, \dots, p^{K-1})$ , and an allocation for all  $K$  commodities, satisfying market clearing and the following consumer optimization problem:

$\max u_i(x_i)$   
subject to

$$\sum_{j=1}^{K-1} p^j x_i^j \leq \sum_{j=1}^{K-1} p^j \omega_i^j$$

$$x_i^K = \omega_i^K$$

$$x_i \geq 0.$$

Assume that all utility functions are continuous and strictly monotonic.

a) Will the competitive equilibrium be fully Pareto optimal, where dominating allocations can involve reallocations of commodity  $K$  as well as the other commodities?

b) Will the C.E. be constrained Pareto optimal, where only the first  $K-1$  commodities can be reallocated?

4. Consider a pure exchange economy with two consumers,  $i = 1, 2$ , and two goods. The consumers' utility functions are given by

$$u_i(x_i^1, x_i^2) = (x_i^1)(x_i^2)$$

and endowments are given by

$$\omega_i = (a_i, b_i) \text{ for } i = 1, 2.$$

Let  $p$  denote the relative price of good 1 in terms of good 2.

- (i) Compute the excess demand function for this economy.
- (ii) Define a competitive equilibrium for this economy.
- (iii) Find the equilibrium price ratio.
- (iv) Determine an equation for the contract curve (the set of Pareto optimal points in the Edgeworth box, expressed as consumer 1's consumption of good 2 as a function of his consumption of good 1).