Department of Economics
The Ohio State University
Econ 805–Homework #2
due Thursday, February 5

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1. Consider the following economy with two consumers, two equally likely states of nature, and one good per state. For $i = 1, 2$, consumer i has the utility function

$$V_i(x_i^1, x_i^2) = \frac{1}{2}\log(x_i^1) + \frac{1}{2}\log(x_i^2).$$

The endowment vectors are $\omega_1 = (2, 0)$ and $\omega_2 = (0, 1)$. Suppose that before the state of nature is revealed, the consumers trade state-1 contingent consumption for state-2 contingent consumption.

(a) Define a competitive equilibrium.

(b) Calculate the competitive equilibrium price vector and allocation.

2. For the economy of problem 1, suppose that before the state of nature is revealed, the consumers trade state-1 and state-2 Arrow securities. Then the state is revealed, securities are redeemed, and we have a spot market.

(a) Define a competitive equilibrium.

(b) Calculate the competitive equilibrium price vector and allocation.

3. Now modify the previous economy to include a good that must be consumed before the state of nature is revealed. For $i = 1, 2$, consumer i has the utility function over consumption at date 1, consumption at date 2 in state 1, and consumption at date 2 in state 2 given by

$$V_i(x_i^1, x_i^{2,1}, x_i^{2,2}) = \log(x_i^1) + \frac{1}{2}\log(x_i^{2,1}) + \frac{1}{2}\log(x_i^{2,2}).$$

The endowment vectors are $\omega_1 = (1, 2, 0)$ and $\omega_2 = (1, 0, 1)$. Suppose that before the state of nature is revealed, the consumers trade date-1 consumption, state-1 Arrow securities, and state-2 Arrow securities. Consumer i's income on this date-1 market results from selling her endowment ω_i^1, and the consumption she purchases, x_i^1, is consumed immediately. Then the state is revealed, securities are redeemed, and there is a spot market on which date-2 consumption is traded.

(a) Define a competitive equilibrium.

(b) Calculate the competitive equilibrium price vector and allocation.

4. In the following Rothschild-Stiglitz model, all consumers have initial wealth of 10 and a potential accident with damages of 5, so the state-contingent initial endowment is (10,5). All consumers have von Neumann-Morgenstern utility functions with a "Bernoulli" utility of certain consumption given by

$u(W) = \log(W)$. For high risk consumers, the probability of an accident (state 2) is given by $\pi^H = \frac{1}{2}$, and for low risk consumers, the probability of an accident (state 2) is given by $\pi^L = \frac{1}{4}$. The population fraction of high risk consumers is given by λ.

(a) What is the unique "candidate" equilibrium contract offered to the high risk consumers?

(b) What is the unique "candidate" equilibrium contract offered to the low risk consumers?