Department of Economics
The Ohio State University
Econ 805–Homework #3 Answers

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1. Mas-Colell, exercise 7.D.1.

Answer: A strategy must specify what actions are taken at each of a player's information sets. Thus, the number of strategies is the product,

$$\prod_{n=1}^{N} M_n.$$

2. Consider the following game. The players are three voters, who must vote either YES or NO (no abstentions). The referendum will succeed if at least two of the voters vote YES, and it will fail otherwise. Suppose that player 1 receives a payoff of 2 if the referendum succeeds, and a payoff of -2 if it fails. Players 2 and 3 receive a payoff of -1 if the referendum succeeds, and a payoff of 1 if it fails. Assume that we have a private ballot, where players cannot see how the other players vote until the ballots are counted.

(a) What is the normal form of this game? That is, for $i = 1, 2, 3$,write out the strategy sets, S_i, and the payoff functions, π_i.

(b) Express the normal form game using matrices. (For a three player game, you can write it as a collection of matrices, one matrix for each strategy of player 3. Player 3 chooses the matrix, player 1 chooses the row, and player 2 chooses the column. Each box of each matrix has three numbers, corresponding to the payoffs of the three players.)

(c) Represent this game in extensive form.

Answer: (a) For $i = 1, 2, 3$, we have $S_i = \{yes, no\}$. The payoff functions are given by

$$\begin{aligned}
\pi_1(yes, yes, yes) &= 2 \\
\pi_1(yes, yes, no) &= 2 \\
\pi_1(yes, no, yes) &= 2 \\
\pi_1(yes, no, no) &= -2 \\
\pi_1(no, yes, yes) &= 2 \\
\pi_1(no, yes, no) &= -2 \\
\pi_1(no, no, yes) &= -2 \\
\pi_1(no, no, no) &= -2
\end{aligned}$$

and

$$
\begin{aligned}
\pi_2(yes, yes, yes) &= -1 \\
\pi_2(yes, yes, no) &= -1 \\
\pi_2(yes, no, yes) &= -1 \\
\pi_2(yes, no, no) &= 1 \\
\pi_2(no, yes, yes) &= -1 \\
\pi_2(no, yes, no) &= 1 \\
\pi_2(no, no, yes) &= 1 \\
\pi_2(no, no, no) &= 1
\end{aligned}
$$

The payoff function for player 3, π_3, is identical to π_2.

(b)

		player 3: yes player 2 yes	 no	player 3: no player 2 yes	 no
player 1	yes	$2, -1, -1$	$2, -1, -1$	$2, -1, -1$	$-2, 1, 1$
	no	$2, -1, -1$	$-2, 1, 1$	$-2, 1, 1$	$-2, 1, 1$

(c) You should draw and label the game tree, but due to my computer limitations, I will just describe it here. At the initial node, player 1 has an arc going down to the left, which we label "yes" and an arc going down to the right, labeled "no." Draw a circle around the two successor nodes, and label the information set "player 2" to indicate that it is player 2's move. Each node has a left arc labeled "yes" and a right arc labeled "no." Draw a single circle around all of these four successor nodes, and label the information set "player 3" to indicate that it is player 3's move. Each node has a left arc labeled "yes" and a right arc labeled "no." The payoffs at the eight terminal nodes, from left to right, are: $(2, -1, -1), (2, -1, -1), (2, -1, -1), (-2, 1, 1), (2, -1, -1), (-2, 1, 1), (-2, 1, 1), (-2, 1, 1)$.

3. Mas-Colell, exercise 8.D.4.

Answer: (a) Lets interpret the description as saying that any nonnegative demand is allowed, and denote the demands by the two players as d_1 and d_2. There are no strictly dominated strategies. Going back to the definition, in order for d_1 to be dominated by some d_1', it must be the case that $\pi_1(d_1, d_2) < \pi_1(d_1', d_2)$ for all d_2. However, player 1 receives a payoff of zero no matter what he does when $d_2 = 100$, so the required inequality does not hold. The same argument applies to player 2.

(b) The weakly dominated strategies for player 1 are demands of zero or greater than 100. To show that this satisfies the definition, we show that there exists a d_1' such that we have $\pi_1(d_1, d_2) \leq \pi_1(d_1', d_2)$ for all d_2, with strict inequality for some d_2. Let $d_1' = 50$. Since the left side of the inequality is zero for demands of zero or over 100, the inequality is always satisfied. The inequality is strict for $d_2 = 50$, so these strategies are weakly dominated.

To see that strategies satisfying $0 < d_1 \leq 100$ are not dominated, notice that when player 2 happens to choose a demand so that the two demands add up to exactly 100, player 1's payoff is given by $\pi_1(d_1, 1 - d_1) = d_1$. Any other strategy by player 1 yields a strictly lower payoff, so the inequality for weak domination is not satisfied.

(c) Any (d_1, d_2) where we have $d_1 \geq 0, d_2 \geq 0$, and $d_1 + d_2 = 100$ is a Nash equilibrium. Raising the demand lowers your payoff to zero, and lowering your demand lowers your payoff, so there is no beneficial deviation. We also have (less interesting) Nash equilibria for any (d_1, d_2) where we have $d_1 \geq 100$ and $d_2 \geq 100$, since both players receive a payoff of zero whatever they do. For any other combination of strategies, at least one of the players is choosing a demand strictly less than 100, and the other player can receive a higher payoff by making the two demands add up to exactly 100.

4. Mas-Colell, exercise 8.D.5.

Answer: This is a classic example of a model of spacial competition, which has been used to model choice of product characteristics in oligopoly and choice of platform in politics.

(a) First we show that both vendors locating at the midpoint forms a N.E. As it stands, each vendor sells to half the market. If a vendor moves away from the midpoint, say to a position $x < \frac{1}{2}$, then the customer who is indifferent between the two vendors is located at position midway between x and $\frac{1}{2}$, or $\frac{x}{2} + \frac{1}{4}$. Thus, the vendor sells to everyone located at a position less than $\frac{x}{2} + \frac{1}{4}$, but this is less than $\frac{1}{2}$. The same argument applies to a deviation to a position $x > \frac{1}{2}$. [Notice how the problem changes if the price is not regulated. Now the closer the vendors are located, the more fiercely they must compete on price.]

Now we can show that both firms locating at the midpoint is the only N.E. in pure strategies. We will suppose that there can be a different N.E., and contradict the supposition by showing that one of the firms could profitably deviate. If they are both locating at the same position, $x < \frac{1}{2}$, then a firm choosing location $x+\varepsilon$ would win more than half the customers for small enough ε. If they are both locating at the same position, $x > \frac{1}{2}$, then a firm choosing location $x-\varepsilon$ would win more than half the customers. Now suppose that there is a N.E. with the two firms at different locations. One firm could increase its payoff by moving closer to its rival (but not quite going all the way to the rival's location).

(b) We will find a contradiction to every possible configuration of locations. If all three firms are at the same location, receiving one third of the market, one firm could move slightly to the left or right, and receive at least half of the market. If two firms are at the same location, and the third firm is at a different location, then the third firm could move closer to the other two, but not quite all the way, and increase its payoff. If all three firms are at different locations,

$x_1 < x_2 < x_3$, then firm 1 could choose location $x_2 - \varepsilon$, which increases its payoff for small enough ε.