

Department of Economics
The Ohio State University
Econ 805–Homework #4 Answers

Prof James Peck
Winter 2009

1. Mas-Colell, exercise 8.E.2.

Hint: Guess that the symmetric equilibrium is linear in a bidder's valuation type, of the form $b_i = Av_i$. Then find the value of A by computing bidder j 's best response.

Answer: If there are I bidders (two bidders is just a special case) using the bidding strategy $b_j = Av_j$ for all $j \neq i$, then the payoff for bidder i for bid b is given by

$$\begin{aligned}\pi_i(b) &= pr(Av_j < b \text{ for } j \neq i)[v_i - b] \\ &= \left[\frac{b}{a\bar{v}} \right]^{I-1} [v_i - b].\end{aligned}$$

The best response of bidder i is found by differentiating with respect to b , setting the expression equal to zero, and solving for b .

$$\begin{aligned}0 &= (I-1)b^{I-2}[v_i - b] + b^{I-1}(-1) \\ b &= \left(\frac{I-1}{I} \right) v_i.\end{aligned}$$

This verifies that the best response is linear, so $A = (I-1)/I$. For $I = 2$, the equilibrium strategy is to bid half your value. As I increases, bids become closer and closer to the bidder's value.

2. Mas-Colell, exercise 9.B.9

Just find the “best” subgame perfect Nash equilibrium, in the sense of maximizing the sum of the payoffs of the two players over the two rounds.

Answer: Notice that (a_2, b_2) and (a_3, b_3) are the only pure strategy N.E. for the one-shot game. Therefore, in any S.P.N.E. of the twice-repeated game, one of these two action combinations must be played at the second round. The best outcome possible (in terms of the sum of payoffs) for the first round occurs when the players choose (a_1, b_1) . If we can construct a S.P.N.E. in which the payoffs are $(10, 10)$ in round 1 and $(5, 5)$ in round 2, we therefore cannot possibly do better.

Construct the equilibrium as follows. Player 1 plays a_1 in round 1. In round 2, player 1 plays a_2 after the “history” (a_1, b_1) , and plays a_3 after all other histories. Player 2 plays b_1 in round 1. In round 2, player 2 plays b_2 after the “history” (a_1, b_1) , and plays b_3 after all other histories.

To see that the strategies defined above constitute a S.P.N.E., first notice that no matter what happens in round 1, the play in round 2 is either (a_2, b_2) or (a_3, b_3) , so it is a N.E. for the subgame corresponding to round 2. Next, notice that the full game is in equilibrium as well. For example, if player 1 were to change his move in round 1, his payoff would increase by 2 or 3 in round 1, but the deviation would cause player 2 to play b_3 in round 2, causing player 1's payoff to decrease by at least 4 in round 2.

3. Mas-Colell, exercise 9.C.7.

Answer: (a) After B, player 2 must play D to be consistent with subgame perfection, and after T, player 2 must play U. Given that, player 1 must play B. The unique S.P.N.E. is given by $\{(B), (D \text{ at the left node}, U \text{ at the right node})\}$. However, there is one more N.E. in pure strategies which is not a S.P.N.E. : Player 1 plays T and player 2 plays U at both nodes.

(b) Now suppose that player 2 cannot observe player 1's move, so draw an information set around the two nodes she could face. Now there is a unique N.E. in pure strategies. Player 1 plays T and player 2 plays U.

(c) See the posted game tree for the extensive form of this game. Basically, nature gives a signal to player 2, which you can think of as a less than perfectly accurate estimate of whether player 1 chose B or T. Surprisingly, no matter how accurate the signal is, the answer from part (a) is completely overturned. The unique W.P.B.E. corresponds to the non-credible N.E. from part (a). Player 1 chooses T, and player 2 chooses U at both of her information sets. The beliefs are determined by Bayes' rule, given by

$$\begin{aligned}\mu(x_1) &= 0, \\ \mu(x_2) &= 1, \\ \mu(x_3) &= 0, \\ \mu(x_4) &= 1.\end{aligned}$$

Given the strategies, beliefs are consistent. Player 1 always plays T, so player 2 believes that player 1 played T no matter what signal she observes. Even when player 2 observes the signal "B," she attributes this to player 1 choosing T and nature garbling the information, since player 1 never chooses B. Given these beliefs, player 2 is being sequentially rational by playing U. Since player 2 always plays U, player 1 is being sequentially rational by playing T.

Now lets see that the above W.P.B.E. is the only one. If player 1 plays T, then player 2's beliefs and actions are uniquely determined as described above. If player 1 plays B as part of a W.P.B.E., then player 2 will assign probability 1 to being in the left node of her information set (by Bayes' rule), and will therefore choose D. However, player 1's choice of B is no longer sequentially rational, because switching to T guarantees him a payoff of 5.

